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Engineering Science Series

A TREATISE ON HYDRAULICS

ENGINEERING SCIENCE SERIES

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A TREATISE ON HYDRAULICS
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A TREATISE

ON

HYDRAULICS

BY

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PREFACE TO THE REVISED EDITION

In this revision, the attempt has been made to cut down the size of the volume and yet retain most of the material, contributing additional data to bring the book as nearly as possible up to date.

This work is intended primarily as a textbook of practical hydraulics; but sufficient experimental data and hydraulic tables have been added to make it useful to the practicing engineer in solving his everyday hydraulic problems.

Hydraulics is a science based almost entirely upon experimental data. No two problems are ever quite the same and each new problem must be considered by itself in the light of previous experiments. That textbook after all is most useful which includes a large amount of experimental data. The previous edition of this book appealed to its readers because of the great amount of experimental data. It has been attempted to include in this volume practically all of the reliable experimental work which has been carried out in the more simple phases of hydraulics. No attempt has been made to cover the more complicated problems met with, but generally they can be resolved into components, which can be solved by the various methods given in this book.

The author gratefully acknowledges his obligation to Edward P. Hamilton, Assistant Engineer in the office of the Proprietors of the Locks and Canals for most of the burden and responsibility of this revision; to the Proprietors of the Locks and Canals on Merrimack River for the inspiration of its former engineers, James B. Francis and Hiram F. Mills, its hydraulic library and the time and opportunity for this revision; to the many authors, designers and others for the use of their material; to Howard M. Turner, Consulting Engineer, 12 Pearl Street, Boston, Mass., for his contribution on the theory of water wheels; to S. Stanley Kent, Assistant Engineer of the Proprietors of the Locks and

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ARTHUR T. SAFFORD

LOWELL, MASS.
July, 1926.

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"La théorie des fluides, au contraire, doit necessairement avoir pour base l'experience —"

—D'Alembert, 1744.

The theory of fluids, on the other hand, must necessarily be based upon experiment.

HYDRAULICS

CHAPTER I

THE HISTORY OF HYDRAULICS

The modern science of hydraulics is based almost wholly upon experiment. Many of the fundamental formulas are theoretical in form, but are modified by empirical coefficients. The science has developed in proportion to the amount of attention devoted to experimental knowledge. It was not until about 1800 that this had become sufficient to allow quantitative results to be foretold, and then only for the more simple conditions of flow. Most of the early students of hydraulics approached it from a mathematical point of view, or simply reasoned out what they thought should be the behavior of water under certain conditions, and formed their theorems accordingly.

Rome had an extensive and excellent system of water works, comprising underground conduits, open aqueducts, and pipes, through which water was brought to the city from the surrounding districts. *Frontinus*, who was, in his earlier years, a Roman governor of Britain, and who, in the later years of his life, was appointed superintendent of the water works of Rome, left us a description of the system in the year 97 A.D. He lists all the various springs and aqueducts, and describes the operation of the system. From this book we can obtain a good idea of the knowledge of hydraulics in Roman times. It was not realized then that discharge was determined both by cross section and by velocity; cross section alone was thought to be the determining factor. A gaging of the flow in an aqueduct consisted simply of measuring the width and the depth of the water. While it was realized that the velocity varied, its import was not understood, nor was there any known means of determining it. Water was doled out to consumers through orifices of size and shape determined by law. It was realized that two orifices, otherwise the same, must be set at the same depth below the surface in order to discharge the same amount.

After the decline and fall of Rome, all scientific knowledge was lost for many centuries. The first serious consideration given to hydraulics came from *Leonardo da Vinci* (1452–1519), who, while he is known to history largely as a painter, was an ardent scientist and one of the first engineers. He was a firm believer in study by experiment, and he made many experiments in various branches of science. One of his manuscripts that has survived shows that he had a very clear knowledge of the various conditions governing flow through an orifice. In his later years, he was engineer for numerous river and irrigation projects in Italy.

The Benedictine *Castelli* (1577–1644), a pupil of Galileo, probably was the first clearly to understand that discharge depended upon both velocity and cross section. He also realized that flow through an orifice varied with the head. His pupil *Torricelli* (1608–1647), the inventor of the barometer, discovered that the flow varied with the square root of the head, rather than with the first power. *Huygens* (1629–1695) has been credited with first establishing the value of the gravity constant, g. After this discovery, all the necessary knowledge existed to complete the basic formula, $V = \sqrt{2gh}$, and by 1732 we find that this law was understood and applied.

Pascal (1623–1662) came to the conclusion in about 1650 that pressure was transmitted equally in all directions through a fluid at rest.

Guglielmini (1655–1710), professor at the University of Bologna, and later at Padua, published a book on measuring water in 1690. While it showed no new knowledge of the subject, it gave in one volume a description of all the methods of measuring water known at that time, and it may well be considered the first real work on hydraulics. Guglielmini followed this work with another on river flow (1697), the first book on that subject, and one long famous. It covers the flow of water in rivers, river control and regulation, erosion of channels, navigation canals, and the drainage of lowlands. Among its illustrations is one of a hydraulic jump. One of Guglielmini's theories shows the errors into which abstract reasoning in hydraulics may lead. Since the velocity through an orifice increased with the square root of the head, he reasoned that this must also hold good in rivers, and that the velocity would be zero at the surface and at a maximum at the bottom. Observation showed him that the

velocity was not zero at the surface, but he overcame this objection to his theory by stating that the velocity in the lower levels induced motion at the surface. Guglielmini had no means of observing the velocity below the surface, so that his error has something to excuse it. *Pitot* (1695–1771), through the invention in 1730 of the tube bearing his name, was able to measure velocities below the surface, and hence to show the fallacy of Guglielmini's theory.

Daniel Bernoulli (1700–1782), a member of a famous family of scientists, made the next great step in hydraulics, that of applying the law of the conservation of energy to the flow of water. This may first have been suggested by his father, *John Bernoulli*, but Daniel is usually given credit for it. It was nearly a century, however, before this principle was put to practical use.

Hitherto little had been known of the relation between the flow of a river and the slope of its water surface. In 1750 the Berlin Academy offered a prize for a method of determining this relation, but met with no response. Seven years later a German, *Brahms*, published a book in which the first understanding of this relation is shown. In 1775 *Antoine de Chezy* (1718–1798) made a report on a proposed canal, in which the formula known by his name, $V = C\sqrt{RS}$, appeared. This formula is still the basis used for determining flow in pipes and in open channels.

The abbé *Bossut* (1730–1814) was the first to undertake hydraulic experiments in a systematic manner. He made a large number of experiments on flowing water in pipes and in open channels, on water wheels, on the impact of jets, and on the resistance of water to moving bodies. What is perhaps of equal importance, he published the original data of these experiments in his work on hydraulics. Many of the experiments were on a small scale, but they were of great value, particularly in showing the way whereby further knowledge of hydraulics must be gained.

Colonel Du Buat (1732–1787), of the French Engineer Corps, made a number of experiments during the years 1780–83 by order of the French Government. His experiments covered pipes, a small open channel, and a small weir. For this latter he gave the discharge formula $Q = 3.46\, LH^{3/2}$, and he also gave another expression correcting for the velocity of approach. Du Buat gave a complicated formula for the flow through pipes and through open channels, but here he made a serious error, saying that the

resistance to flow was independent of the roughness of the channel, since particles of water filled the interstices and the only friction was that of water on water. Save for this error, his understanding of hydraulics was very complete; modern hydraulics may be said to date from the publication of his treatise in 1786. We find his book referred to in this country as the standard work on hydraulics as late as 1849.

Venturi (1746–1822) was professor at Modena and later at Pavia. He made many experiments on orifices and short tubes of various shapes, including the flaring tube which is the basis of the Venturi meter, invented by Clemens Herschel. He also put the hydraulic jump to practical use in draining lowlands.

In the year 1700 the general behavior of water was fairly well understood, and the basic relation between head and velocity was known, but the knowledge of the quantitative flow of water was very limited, in fact, almost lacking. The progress during the next century was very great, and by 1800 we find that hydraulics had become an exact science, still needing much refining, to be sure, but in its essentials correct. Bossut had given the coefficients to use for orifices and short tubes, de Chezy the formula for flow in open channels, and Du Buat those for flow in pipes and over weirs. Bernoulli's theorem was known, although it was not as yet applied, and even the hydraulic jump had been put to practical use. A man who was fully abreast with the science of hydraulics as it existed in 1800 had at his disposal practically all of the working tools necessary to the hydraulics of the present day. Many of them were crude, and a few faulty, but on the whole, with the addition of practical experience, they were sufficient to solve almost any hydraulic problem.

Eytelwein (1764–1849) translated Du Buat's works into German with certain additions of his own. He gave de Chezy's formula for flow in open channels. This formula became popular in Germany and elsewhere, and was known as Eytelwein's formula, but in France it gave way to a more complicated one deduced by *Girard* in 1803, $RS = aV + bV^2$. It was later realized that the term "aV" in this last formula had little effect when the velocity became moderately high, and this was often dropped, giving a return to the simpler Chezy formula.

Poncelet (1788–1868) and *Lesbros* (1788–1867), officers in the French Corps of Engineers, made a series of experiments on the

discharge of orifices and weirs in 1827–1828. These experiments were continued by Lesbros from 1829 to 1834. These were the first really scientific hydraulic experiments. Hitherto all experiments had been on what was really a laboratory scale. Poncelet and Lesbros' work, however, was on a large scale and was carried out very completely and exactly, so much so that their results, within their limits, have never been improved upon.

Darcy (1803–1858), a French engineer, was one of the first to devote careful study to the flow in pipes under actual working conditions. In the course of his work he studied the velocity distribution in pipes with a movable Pitot tube, worked through stuffing boxes. He came to the conclusion, in about 1850, that Du Buat's theory was wrong, and that the roughness of the pipe walls had a very decided effect upon the flow.

Weisbach (1806–1871) was the author of a very complete manual of engineering, first published in 1850, which enjoyed great popularity in its day. While he made a number of hydraulic experiments on a small scale, Weisbach largely compiled the experiments of others in his hydraulic section. Some of his experiments are still quoted in hydraulic textbooks. They were made on too small a scale, however, to be of any great value.

Uriah Boyden (1804–1879) was one of the earliest American hydraulic engineers. He had considerable to do with the early development of hydraulic turbines, and was the inventor of the hook gage. He put Venturi's flaring tube to practical use in his "diffuser," the forerunner of the modern draft tube.

James B. Francis (1815–1892) was the most famous of the early American hydraulic engineers. In his position as engineer of the first large water-power system, he realized the necessity of additional research, particularly under the conditions usually obtaining in practice. Among his achievements were the first scientific water-wheel tests made in this country, the development of the inward-flow turbine, and the determination of the Francis weir formula. While the results obtained by this weir formula were only some four per cent different from those obtained by Du Buat's weir formula of 1786, the formula is based upon a much more exact knowledge of the behavior of the water and a precise measurement of the quantities dealt with, quantities which were much greater than any that had hitherto been used.

Henri Bazin (1829–1917) was one of the greatest hydraulic engineers. As a young man he worked under Darcy, and, after the latter's death, he continued the experiments alone. These experiments were on the flow in open channels. Bazin applied Darcy's theory of the effect of the roughness of the wall to open channels and finally derived a formula which corrected for this effect, by the use of a coefficient which varied with the material forming the walls. Ganguillet and Kutter based their formula for flow in open channels to a large extent upon Bazin's experiments.

Bazin's chief fame, however, is based upon his experiments with weirs. The work of Poncelet and Lesbros had been followed by that of *Castel* at Toulouse in 1835–36, and *Boileau* at Metz in 1845–52. Francis, meanwhile, was experimenting in this country, and his work was continued by *Fteley* and *Stearns* on the Sudbury aqueduct of the Boston Water Works in 1877–1879. Bazin's exhaustive studies commenced in 1886 and lasted for a number of years. His experiments, due to their variety and precision, are perhaps the most useful which have yet been made, and formulas based upon them are adapted to a greater variation of conditions than are those of Francis and Fteley and Stearns.

The development of hydraulics until about 1700 was very slight, and consisted solely in a gradual comprehension of a few of the basic laws. The next century saw a very great advance, so that by 1800 the art had become an exact science. During the last century there were no great discoveries save the realization of the effect of the roughness of the channel upon the flow, but there was a steady development, and a very great addition to the experimental knowledge.

Modern hydraulics began in Italy and continued there for some two centuries. It then shifted to France, where it continued to develop until the modern science was evolved. The great majority of those who have contributed to the advancement of the art have been Frenchmen, and the French government has always been generous in furnishing funds and facilities for conducting hydraulic experiments. Experimental hydraulics originated in France, and until about the middle of the last century but little worthwhile work was done outside of that country.

The science of hydraulics requires a number of special instruments with which various observations are made. The two com-

mon methods of measuring the height of a water surface, the vertical staff gage, or its equivalent, a rule measuring down to the water surface, and the glass gage-tube are obvious and must have existed from early times. Bernoulli's *Hydrodynamica* (1738) shows a cherub measuring the height of a water column in a glass tube with a pair of dividers.

The Romans had instruments with which they were able to run lines of levels. Guglielmini's *De Fluminum Natura* (1697) shows a levelman sighting through an instrument at a rod held by an assistant. The combination of a telescope with a cross-hair and a level-vial may have come into use about this time. It is said to have been invented by Huygens, but it was not made practical until de Chezy discovered a method of grinding the vial to a true arc. By 1737 the level existed in practically its modern form, with tripod and a four-screw levelling head. A crude form of transit with both vertical and horizontal arcs was also in use at this time.

The most important hydraulic instruments are those which measure the velocity of flow. The earliest and most obvious is the surface float. Subsurface floats and rod floats soon followed, and it is not known when they first came into use. The first scientific work with rod floats was done by Francis in 1856.

The hydraulic pendulum, said to have been invented by Castelli, was one of the early instruments used. It consisted of a ball, slightly heavier than water, on the end of a string. The angle of the string from the perpendicular when the ball was immersed in flowing water, was read on an arc and was taken as a measure of the velocity. Pitot's tube has already been mentioned. At first it was largely a scientific toy and was not much used until Darcy turned his attention to it in about 1850.

A small, freely revolving wheel with flat blades around its rim was used to measure the surface velocity of a stream, but it was not of much practical value, since surface velocity could usually be measured by floats quite as easily.

Bruning's tachometer (1789) consisted of a flat plate opposed to the current. A scale arm and weight served to record the amount of the pressure against the plate and furnished a measure of the velocity. *Woltmann*, an engineer of Hamburg, invented his "moulinet" in 1790. This consisted of a small propeller wheel on a horizontal shaft, mounted on bearings and connected with a

mechanism which recorded its revolutions. This was the forerunner of the modern current meter, which has since been made in many forms, all of which, however, are the same in principle.

We have seen that the knowledge of hydraulics gradually increased in the course of centuries until it was approaching an exact science by 1800. The various special instruments had paralleled this development, so that by this same time all our modern instruments were in existence in a more or less crude form. One thing was yet needed to further the progress of the art, and that was a field of application. Before the industrial revolution the field of application of hydraulics was comparatively small. Rivers were regulated to some slight extent, largely in Italy, and navigation canals were built, particularly in France, A few of the large cities had crude water-supply systems of a sort, but sewerage systems were unknown. Water power was developed only in small mills, and the quantity of flow was usually more than ample to serve the mill, so that there was no particular need for a means of determining the quantity of flow.

In the nineteenth century, standards of living advanced tremendously. All cities and many smaller towns built water-supply works and sewerage systems. Water-power developments increased rapidly, and to such an extent that on many rivers the question of the division of water became of considerable importance, necessitating exact measurements of large quantities of water flowing in canals or in races. The utilization of all the available fall on rivers brought up the question of the backwater effect caused by dams, *e.g.* how high a dam could be raised without damaging the next mill upstream or without flooding valuable lands. In the earlier days most hydraulic works had been on a comparatively small scale, and cut and try methods usually sufficed, but larger and more expensive works required a careful examination of the various factors entering into the case in order to determine the feasibility of the project. The need for an exact hydraulic science had risen, and the last century saw a tremendous growth in the art, particularly in the direction of refinement and amplification of existing knowledge, rather than in the discovery of any revolutionary principles.

Out of all the work and thought devoted to hydraulics during the last two thousand years, there have gradually been evolved the following basic theorems and formulas, which, with slight

modifications based upon later experiments, are still the laws which govern the flow of water.

The fundamental law of flow (Castelli, 1628):

$$Q = AV.$$

Pascal's principle (1650):

Pressure is transmitted equally in all directions through a mass of fluid at rest.

The velocity of falling water (Torricelli — Huygens — Bernoulli, 1732):

$$V = \sqrt{2\,gh}.$$

Bernoulli's theorem (1738):

The law of the conservation of energy applies to the flow of water. Pressure can freely be converted into velocity and velocity into pressure.

The flow of water in pipes and in open channels (de Chezy, 1775):

$$V = C\sqrt{RS}.$$

The Francis weir formula (1852):

$$Q = 3.33\,LH^{3/2}.$$

These and other principles of less fundamental importance are discussed in detail in later chapters.

CHAPTER II

INTRODUCTION

1. Definition. Hydraulics is the science of fluids in motion. Hydrostatics is the science of fluids at rest. The term hydraulics, as used by engineers, includes the principles of hydrostatics; and, in effect, it means the practical mechanics of water.

The science of hydraulics comprises: pressures due to water; the stability of structures subjected to water pressures; the flow of water; the measurement of the flow of water; the proportioning of channels to convey water; the conversion of energy contained in water (by virtue of its velocity, elevation, or pressure) into useful work; the raising of water by pumps; the transmission of energy by water; the storage of water; the purification of water for domestic and commercial uses; the design of vessels to move in water; and various special applications which have been developed by experiment and practice.

This book will be confined chiefly to questions of water pressure, the stability of simple structures subjected to water pressure, the flow of water, the measurement of the flow of water, and the fundamental principles of water motors.

2. Limited Application of Theoretical Formulas. Theoretical hydromechanics, which treats of fluids without viscosity, hence without friction, while leading to definite and reliable results when dealing with fluids at rest, may yield results regarding fluids in motion that may be not only useless to the engineer, but even positively misleading. The formulas expressing the laws of fluids in motion, derived from such purely theoretical considerations, have become in many instances merely a framework on which are hung the results of experiment. While the formulas are rational in form and serve to define the limits of application, on account of the wide range of numerical coefficients, they have become in fact strictly empirical. In measuring the flow of water, the engineer performs experiments to find the value of modifying elements for the problem under consideration.

3. Primary Units. In American and English hydraulic practice, the customary primary units are the foot, pound, pound per

square foot, second, cubic foot, and cubic foot per second. In Continental practice, or where the metric system is used, the corresponding units are the meter, kilogram, kilogram per square meter, second, cubic meter, and cubic meter per second.

TABLE I

CONVERSION TABLES FOR THE PRINCIPAL UNITS

One foot	= 0.3048 meters	One cubic foot	= 7.481 U. S. gallons
One meter	= 39.37 inches	One horsepower	= 0.746 kilowatts
One inch	= 2.540 centimeters	One acre-foot	= one foot depth over one acre = 43,560 cubic feet
One pound (Av.)	= 0.4536 kilograms		

Miners' inch = a measure of discharge used in western American hydraulic practice, originally the discharge through an orifice one inch square, the head and other important conditions varying in different localities. Each state, by law, now defines the miners' inch in equivalent cubic feet per second. It varies from about 0.02 to 0.028 cubic feet per second.

Rainfall, evaporation, and runoff are frequently stated in terms of inches in depth on the drainage area.

One inch of mercury = 13.6 inches of water
= 0.491 pounds per square inch

4. The Weight of Water. *Variation with Temperature.* The weight of a cubic foot of water for temperatures used in hydraulics may usually be taken as 62.4 pounds (for rough calculations 62.5); and of sea water, 64.0 pounds. The weight of a cubic meter of water is commonly taken as 1000 kilograms at 4° C.

Above 39.3° F., the temperature of maximum density, water decreases in density as the temperature rises. Below 39.3° F., the density decreases as the temperature falls to freezing.

The specific gravity of distilled water at 39.3° F. being 1.000, the specific gravity of sea water is about 1.025.

5. Increase of Weight of Water due to Impurities. Water entirely free from foreign substances is rarely found in nature. Unless water contains considerable air, its specific gravity is always higher than that of distilled water of the same temperature. When dealing with fresh water, except in the most refined calculations, and at unusual temperatures, variation both in temperature and in specific gravity may usually be disregarded. Sewage may weigh as much as $\frac{1}{2}$ per cent and in extreme cases 1 per cent more than distilled water. Such variations are commonly neglected, chiefly because the degree of precision attainable in hydraulic engineering does not warrant their consideration.

6. The Pressure of the Atmosphere. Atmospheric pressure, which is due to the weight of the air, varies with meteorological conditions and with differences in altitude. For hydraulic problems, it generally is sufficiently accurate to use the mean value of atmospheric pressure at sea level, or 14.7 pounds per square inch (p_a), which is commonly designated as one *atmosphere.*

Vacuum is usually stated in head *below* atmospheric pressure in inches of mercury. It is the difference between atmospheric pressure and absolute intensity of pressure in a closed vessel.

7. The Value of *g*. In American and English practice, velocity is usually expressed in feet per second, and acceleration in feet per second *per second;* in the metric system, in meters per second, and in meters per second *per second.* The acceleration g due to the force of gravity is a minimum at sea level at the equator; it increases with an increase of latitude, and decreases with a rise in altitude. The value of g used in this book will be 32.16 ft./sec^2 ,which is practically the value for latitude 42°. It varies from 32.25 at the poles to 32.09 at the equator.

8. Precision in Computation. In the appendix is a four-place table of logarithms. The precision of the data upon which a problem is based should indicate the required nicety in calculation. In general, logarithms should be used to one more place than the number of significant figures desired in the result. Answers should be computed only to the number of significant figures justified by the original data. Work should invariably be checked, not only to eliminate numerical mistakes, *but also to make certain above all things that the answer is reasonable.*

PROBLEMS

1. Convert a discharge (volume of flow) of 15 cubic meters per second into its equivalent in cubic feet per second.

2. Convert a discharge of 14 cubic feet per second into its equivalent (*a*) in U. S. gallons per minute; (*b*) in gallons per day of 24 hours.

3. Convert intensity of pressure of 5.5 atmospheres into its equivalent: (*a*) in pounds per square inch; (*b*) in pounds per square foot; (*c*) in head in feet of fresh water; (*d*) in head in feet of sea water; (*e*) in inches of mercury; (*f*) in meters of fresh water.

4. A tank 16 feet internal diameter and 40 feet high is filled with water at 40° F. (*a*) How many U. S. gallons will it contain? (*b*) How many pounds? (*c*) How many liters? (*d*) If the temperature were 190° F., how many pounds?

5. How many pounds of water at ordinary temperature will be contained in a 48-inch (internal diameter) pipe 1 mile long?

CHAPTER III

FLUID PRESSURE

9. Properties of *Fluids.* *Fluids* are substances whose parts oppose little or no resistance to distortion of form. They include gases, liquids, and vapors.

A *perfect fluid*, that is, a frictionless fluid, cannot exert resistance to shearing or tensional stresses, and takes the exact form of a vessel in which it may be contained. Actual fluids do exert more or less resistance to change of form; the measure of this opposition is termed the *viscosity*. The lower the viscosity, the more nearly the actual fluid approaches the perfect fluid. In fluids considered in engineering, the viscosity is slight. With fluids in motion, the resistance to motion increases with an increase of viscosity. In engineering, water at rest may be treated, with negligible error, as a perfect fluid.

10. Gases. A gas is a compressible fluid; practically, it is a fluid in which slight changes of pressure produce very sensible changes in volume. The law of gaseous pressures may be stated briefly: *if the temperature is constant, the pressure multiplied by the volume is a constant.*

11. Liquids. A liquid is a fluid which changes volume very slightly even with considerable variation in pressure, and when the pressure is entirely removed does not sensibly dilate; but, like all elastic substances, it is actually somewhat compressible. The liquid chiefly considered in hydraulics is water.

12. Vapors. A liquid may be changed to a gas by the application of heat or by decreasing the pressure sufficiently. A gas may be changed to liquid form by the abstraction of heat or by the application of pressure. A gas on the point of becoming a liquid, or a liquid on the point of becoming a gas, is called vapor. Rain clouds are composed of water vapor.

13. Compressibility of Water. Water at its maximum density has a coefficient of compressibility of about 0.00005 [1] for each atmosphere of added pressure, and a modulus of elasticity E in

[1] *Smithsonian Physical Tables*, p. 83.

compression of about 296,000. The coefficient of compressibility decreases and the modulus increases with a rise in temperature; at 212° F. they are about 0.00004 and 360,000, respectively. Thus, a cubic foot of water weighs 62.4 pounds under one atmosphere; but a cubic foot of water under a pressure of 11 atmospheres weighs 62.4 $(1 + 0.00005 \times 10)$ or 62.43 pounds, a very slight increase. This correction for compression, like that for temperature, implies a higher degree of precision than is ordinarily required in hydraulic problems. Water is, therefore, considered as incompressible in engineering.

14. The Free Surface of a Liquid. That surface of a body of liquid at rest which is in contact with a gas or a vapor is the free surface. It is, therefore, a surface of equal pressure normal to the line of action of the force of gravity at any point on the surface of the earth. For ordinary engineering work it may be considered horizontal.

15. Pascal's Law. The basic theorem of the laws of fluid pressure is PASCAL'S LAW. ***In a fluid, the pressure at a given point is normal to the surface on which it acts, and of equal intensity in all directions.***

16. Intensity of Pressure. The *intensity of pressure*, p, is the pressure per unit of area between contiguous fluid surfaces, or between a fluid and the face of a solid, and is equivalent to the weight of a column of the fluid supported by that unit area. The intensity at any point is therefore measured by the vertical distance from the point to the free surface of the fluid, called *the head*, h, and *the weight*, w, of a cubic unit of the fluid. Intensity of pressure is independent of the shape, area, or direction of the surface presented to the liquid. If all factors are given in corresponding units of the same system, intensity of pressure may be expressed by the following fundamental formula:

$$p = hw. \tag{1}$$

17. Conversion Formula for Intensity of Pressure. The terms commonly used to express intensities of pressure are in several different units; such, for example, as feet or inches of water or of mercury, or of oil of various specific gravities, in pounds per square inch or square foot, kilograms per square meter or square centimeter. The following formula may be used in converting

expressions for intensities of pressure into units of the same system or other systems:

(2) $$p = hw;$$

h = the head corresponding to an intensity of pressure p; h being the height of a column of any given liquid, and measured in any linear unit.

p = the intensity of pressure that will support a column of the given liquid of height h; p being stated in any unit of weight and any unit of area.

w = the weight of a prism of the given liquid in the same units of weight as p, having for its height one unit of length in which h is measured, and for the cross-sectional area, the unit area in which p is stated.

18. Intensity of Water Pressure. The weight of a cubic foot of water may be taken as 62.4 pounds (see § 4). Therefore the relation between intensity of pressure p, and head h in feet of water is, by (2),

(3) p (in pounds per square foot) $= h\,62.4$, and $h = \dfrac{p}{62.4}$;

p (in pounds per square inch) $= h\,\dfrac{62.4}{144} = 0.4333\,h$;

and

(4) $$h = \frac{p}{0.4333} = 2.308\,p.$$

19. The Term *Head* in Hydraulics. The word *head* has various special meanings in hydraulics, but their distinctions have become dulled by a necessarily varied usage. In any particular case, the intended meaning must be clearly specified. The following special meanings may be distinguished:

The velocity head of water moving with a given velocity is the equivalent height through which a body must fall to acquire the same velocity.

The pressure head is the difference in intensity of pressure between two points in a body of water; or merely the intensity of pressure at any point.

The head due to elevation is the difference in elevation between any point in a body of water and any other point or reference plane either within or without the body.

The head due to atmospheric pressure. This usually is not effective since it also in most cases is present as a back pressure opposing the pressure or motion, but its possible effect must be remembered.

Lost head in flowing water represents the energy irrecoverably converted into heat in overcoming resistances.

Heads or pressures are measured from some reference point usually fixed by the conditions of a problem. Intensities of pressure, unless otherwise stated, are measured from atmospheric pressure as zero.

20. Differences in Intensity of Pressure. Effective Pressure. In computing the effective intensity of pressure at a given point in a fluid, the modifying effect of the action of external forces, such as atmospheric pressure, compressed air, rarefied air, weights, or mechanical force, must be thoroughly considered. Obviously, there are numerous instances in which the effect of atmospheric pressure may be disregarded without error because it acts equally on opposite sides. Yet there are many cases in which the fluid pressure on one side of a surface is greatly intensified by the production of a vacuum on the other which brings the atmospheric pressure into action. Effective pressure is determined, therefore, by differences in absolute pressure and elevation.

EXAMPLE. A vessel, shown in Fig. 1, is filled with water to a point 100 feet above the axis of the cylinder. It is open only at D to the atmosphere.

At any point on the line AA' the *absolute* intensity of pressure on the inside, p, in pounds per square inch, is

$$p = 100 \times 0.433 + p_a = 43.3 + 14.7 = 58.0$$

This pressure acts with equal intensity in all directions.

At A, the *effective* intensity of pressure on the shell, that is, the difference between the inside or outside pressures is

$$100 \times 0.433 + p_a - p_a,$$

or simply 43.3 pounds per square inch.

At B the absolute intensity of pressure on the inside is

$$96 \times 0.433 + 14.7 = 56.3 \quad \text{pounds per square inch.}$$

The intensity of pressure in the small pipe at B is equal to that in the large cylinder at the same point.

21. Total Normal Pressure. Intensity of pressure should be considered a means of determining a force rather than as a force itself. ***Intensity of pressure multiplied by the area of a surface,*** however, ***is the total pressure on a given surface,*** which is a force. To define a force its *magnitude, direction,* and *point of application* must be known.

The total normal fluid pressure on an immersed surface or solid is the summation of all the forces due to fluid pressure on

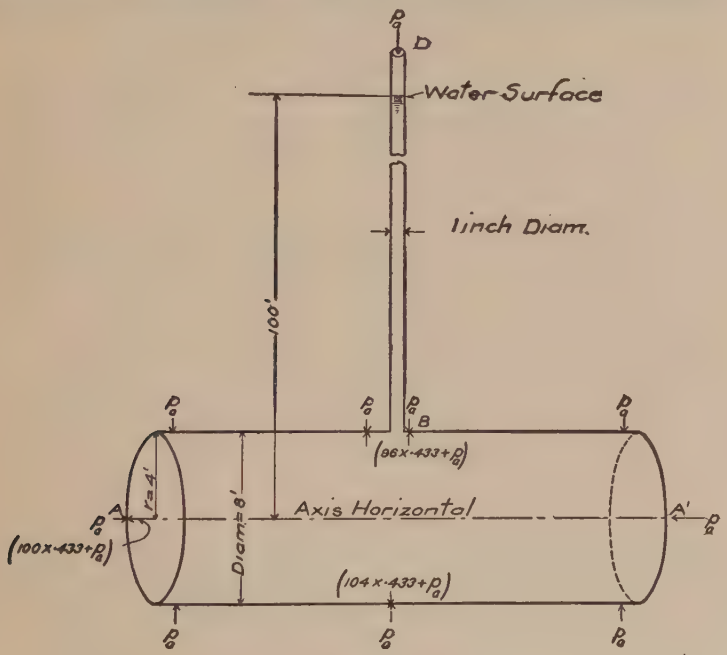

FIG. 1

its faces. On irregular surfaces, the total normal pressure *per se* has no significance, because it cannot be defined. In an investigation of the effects of fluid pressures on irregular surfaces or solids, they should be divided into suitably small areas that present as nearly as may be regular plane surfaces. The magnitude, direction, and point of application of the total normal pressure or force on each area should be computed and considered a single force. The problem then consists in summing up these individual forces, to determine the total resultant pressure in any required direction.

22. Uniform and Non-Uniform Pressures. It is desirable to consider (1) surfaces subjected to pressures of uniform intensity, and (2) surfaces subjected to pressures of varying intensity. When all points of a surface immersed in a fluid are equally distant from the free surface, the intensity of pressure is uniform over the whole surface.

23. Pressures Assumed as Uniform. In engineering practice, many cases occur in which, without impairing necessary precision, the intensity of pressure may be assumed to be uniform. The following groups include the important cases:

(1) When the surface is in contact with a gas or a vapor; for example, in a compressed air reservoir.

(2) When the surface under liquid pressure is very small, *i.e.*, an elementary area.

(3) When all parts of the surface are under such high intensities of liquid pressure that the assumption of a uniform pressure will not introduce error in excess of the limits fixed by the other factors of a problem.

This group includes water pipes under pressure, certain forms of steam boilers, gates and valves, and many receptacles for liquids. The uniform intensity which is usually assumed is the effective pressure either on the lowest point of the area, or on its center of gravity.

24. An Immersed Plane Surface Parallel to the Free Surface of the Water. Consider an irregular plane surface $BCDE$, area A, lying in a horizontal plane h feet below the free surface of the water. See Fig. 2. Let $a_1, a_2, \ldots a_n$, be elementary areas. The intensity of pressure at any point, by equation (1), is given by the relation $p = hw$.

The magnitude of P. The normal pressure on any element (§ 21) is hwa_n. The total normal pressure P is the sum of the normal pressures on all elementary areas. Since

$$\lim \Sigma(a_1 + a_2 + \cdots + a_n) = \text{area } A,$$

we may write

$$P = \lim \Sigma(a_1 + a_2 + \cdots + a_n)p = p \int\int dx\, dy = pA = hwA.$$

EXAMPLE. If $h = 20$ feet and $A = 10$ square feet,

$$P = 20 \times 62.4 \times 10 = 12{,}480 \quad \text{pounds.}$$

The direction of P. Since the pressure on every elementary area must be normal to the surface, it is vertical and downward. Hence P, which is the resultant of this system of parallel forces, must have the same direction.

The point of application of P. Since the intensity of pressure is uniform over the area A, and since P is the resultant of a system of parallel forces normal to A, the point of application of P, called the center of pressure must be the center of gravity of the area A.

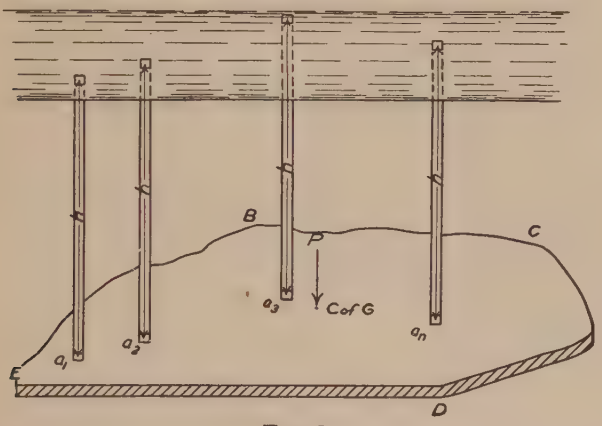

FIG. 2

25. Pressure on an Irregular Plane Surface. Consider an irregular plane surface subjected on one side to a uniform pressure and inclined at an angle θ with the horizontal. The total normal pressure is $P = pA$; it can be resolved into a horizontal component, $pA \sin\theta$, and a vertical component, $pA \cos\theta$. The projection of the area A on the vertical plane is $A \sin\theta$, and on the horizontal plane is $A \cos\theta$. Instead of determining the total normal pressure P and then resolving it to find a desired component, it is evident that we can determine this component directly by multiplying p by the projection of the area A onto a plane normal to this desired component. See Fig. 3.

RULE. ***The total normal pressure on an immersed plane surface subjected to pressure of uniform intensity equals the intensity of pressure multiplied by the area. The total component in any direction equals the intensity of pressure multiplied by the area of the projection of the surface on a plane perpendicular to the given direction.***

26. Curved Plate under Uniform Pressure. A plate, curved in one plane only, is subjected to a uniform pressure on one side. It is desired to find the pressure against it in the direction XY

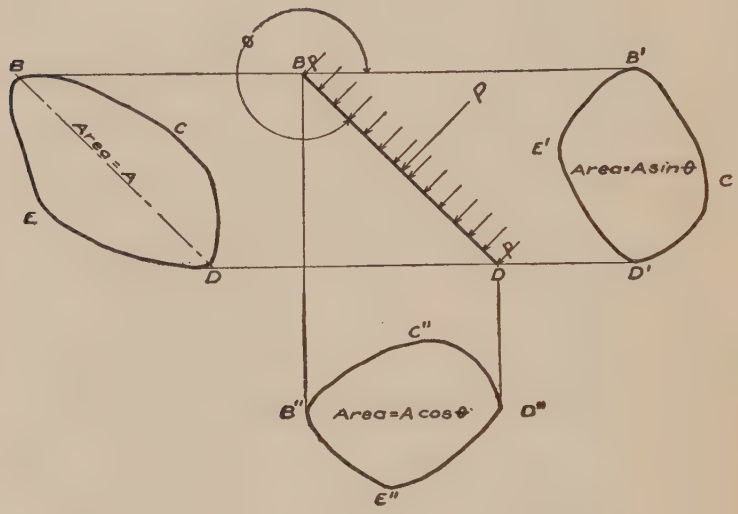

FIG. 3

(Fig. 4). The plate, $ABCD$, can be divided up into an indefinite number of elemental areas, a_1, a_2, ..., a_n, inclined to the direction XY at angles θ_1, θ_2, ..., θ_n, and so small that they can be treated as if they were plane. The normal pressures against these elemental areas are pa_1, pa_2, ..., pa_n, respectively. By § 25, we have seen that the pressure against one of these areas in the direction XY is the pressure times the area times the sine of the angle at which it is inclined to the direction XY. The total pressure against $ABCD$ in the direction XY, then, is

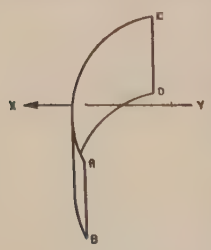

FIG. 4

$$\lim \Sigma(pa_1 \sin \theta_1 + pa_2 \sin \theta_2 + \cdots + pa_n \sin \theta_n),$$

or

$$p \lim \Sigma(a_1 \sin \theta_1 + a_2 \sin \theta_2 + \cdots + a_n \sin \theta_n).$$

But this last expression is nothing but p times the projection of the area $ABCD$ onto a plane normal to the direction XY.

For purposes of simplification *ABCD* has been considered as being curved in one plane only, but it is evident that no matter how warped and irregular it is, the preceding result still holds.

RULE. *On any curved surface subjected to pressure of uniform intensity, the total component pressure in any direction equals the intensity of pressure multiplied by the area of the projection of the surface on a plane perpendicular to the given direction.* Its point of application is on a line perpendicular to the projection and through the center of gravity of that projection.

27. Internal Pressures of Uniform Intensity in Thin Pipes and other Cylindrical Vessels. Hoop Tension. Thickness of Shell. Consider a pipe or cylindrical vessel (Fig. 5), subjected to an internal water pressure of uniform intensity p.

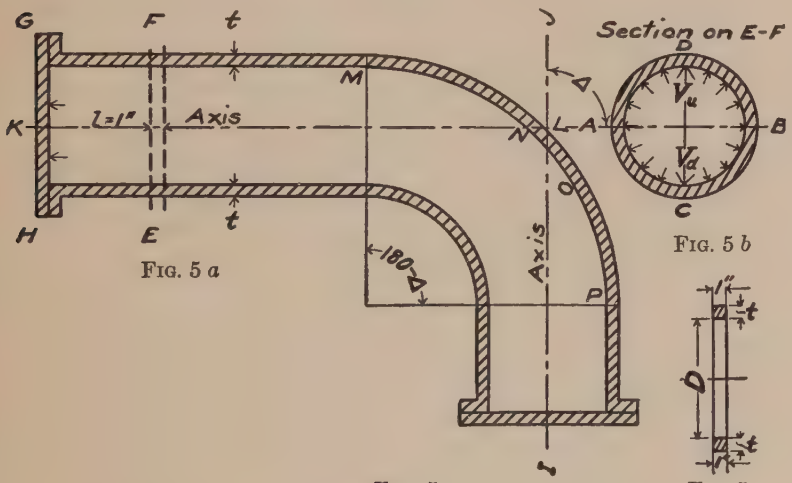

FIG. 5 *a*

FIG. 5 *b*

FIG. 5

FIG. 5 *c*

Let us suppose that the shell is thin and of homogeneous material, and let us use the following notation: D = the internal diameter in inches, r = the radius in inches, t = the uniform thickness of the shell in inches, p = the effective intensity of pressure, that is, the difference between the absolute interior and exterior intensities of pressure, in pounds per square inch.

Since the shell is homogeneous and symmetric, and since the pressure is uniform, the forces tending to rupture the material in a plane parallel to the axis of the pipe are symmetric about any

diameter. Consider a strip one inch long; and let AB be any diameter. The total upward pressure V_u perpendicular to AB (see Fig. 5 *b*), due to the normal pressures on the area $ADB \times 1$ is $p \times D \times 1$, since $D \times 1$ is the horizontal projection of one-half the circumference ADB. There is also an equal perpendicular force V_d opposite in direction, due to the normal pressure on $ACB \times 1$. The tendency of these two forces to pull the pipe apart at A and B is resisted by two strips of material having a cross-sectional area of $2t \times 1$ (see Fig. 5 *c*). Since V_u is uniformly distributed on the projection of ABD, the stresses at A and B are equal.

If S denotes the tensile stress in pounds per square inch produced in the material at A and B, the total resistance of the material, that is, hoop tension at A and B, is $2 \times S \times t \times 1$, which, if the pipe does not rupture, must equal the force tending to pull the pipe apart at A and B. Therefore

$$p\,2\,r = 2\,St; \qquad \text{or} \qquad pr = St.$$

Hence the tensile stress S is pr/t, in pounds per square inch.

The requisite thickness t, in inches, is pr/S_a, where S_a denotes the allowable tensile stress in pounds per square inch.

The bursting intensity of pressure p_b is $S_u t/r$, where S_u denotes ultimate tensile strength in pounds per square inch.

28. Formula in Practical Use for the Thickness of Cast-Iron Pipes. Brackett's formula, used in determining the thickness of shell in the specifications of the New England Water Works Association, is

$$t = \frac{(p + p')r}{3300} + 0.25 \qquad \text{inches,}$$

where

p = intensity of internal pressure in pounds per square inch,

p' = additional pressure in pounds per square inch allowed for water hammer,

3300 = allowable unit tensile stress for cast iron, being $\frac{1}{5}$ of 16,500, the ultimate tensile strength of cast iron in pounds per square inch,

0.25 inch = extra allowance for deterioration in use, or eccentricity of casting.

The following table gives the values of p' used in allowing for water hammer.[1]

Nominal Diameter of Pipe in Inches	p' in Pounds per Square Inch
4, 6, 8, and 10	120
12 and 14	110
16 and 18	100
20	90
24	85
30	80
36	75
42 to 60	70

Note. The allowance for water hammer is in most cases greater than the actual fluid pressure. In Chapter XVII will be found two formulas for computing water hammer.

In actual practice, however, rather than determining the requisite thickness by a formula, one of the eight classes specified by the American Waterworks Association is used. These classes vary in thickness: Class A, the lowest, is used up to 100 feet head, and class H, up to 800 feet.

29. Thickness of Riveted Steel Pipes. A formula used for determining the thickness for riveted steel pipes is

$$t = \frac{pr}{S_a},$$

where S_a denotes the allowable unit tensile stress, usually 16,000 pounds per square inch, which is one-fourth of the ultimate strength, on the net cross-sectional area, that is, on the area after deducting the rivet holes; or 16,000 multiplied by the efficiency of riveted joints, which varies from 0.50 to 1.00.

Minimum thickness. When t as computed by this formula gives a value less than $\frac{3}{16}$ inch to $\frac{1}{4}$ inch, a minimum t, determined by practice, must be used instead of the calculated t, since ordinary wear would destroy a thinner pipe in a few years.

Note. A value of p must be used sufficiently high to include the water hammer. This value will usually be less than the value used for cast iron, but may be greater.

[1] From *Standard Specifications of the New England Water Works Association*, adopted Sept. 10, 1902. Journal N. E. W. W. Association, vol. 16 (1902), pp. 89, 343.

30. Cylindrical Vessels. The formula for pipe-thickness holds for any cylindrical vessel or such parts thereof as may be, or may be assumed to be, under uniform intensity of pressure. Allowance for water hammer need not be made when provision is made to preclude its occurrence.

31. The Total Pressure in a Cylindrical Shell on an End Closed by a Plate, or on the Surface of a Bend. The total pressure parallel to the axis on the flat plate GH in Fig. 5 a, is $p\pi r^2$.

The projection of the curved surface MNO (see Fig. 5 a) on a plane perpendicular to the axis is πr^2; hence the total pressure parallel to the axis on this curved surface is $p\pi r^2$.

There is therefore a thrust on the end, and on the bend, of $p\pi r^2$ pounds, which, if not suitably resisted, will set up a longitudinal tensile stress in the shell, pull apart the joints, or cause the pipe to creep. In some cases special external devices are required to prevent this.

32. The Longitudinal Tensile Stress in a Cylindrical Shell. If t is the thickness, the cross-sectional area of the shell is

$$\pi(r + t)^2 - \pi r^2 = 2\,\pi rt + \pi t^2 = 2\,\pi rt\left(1 + \frac{t}{2\,r}\right).$$

See Fig. 6. If S denotes actual unit tensile stress, and S_a denotes allowable unit tensile stress in the material of the shell, the resistance of the material is

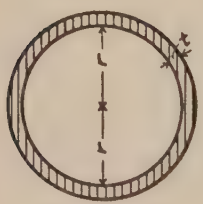

Fig. 6

$$S\,2\,\pi rt\left(1 + \frac{t}{2\,r}\right) = \text{the thrust } p\pi r^2.$$

Hence

$$S = \frac{p\pi r^2}{2\,\pi rt\left(1 + \frac{t}{2\,r}\right)};$$

but $1 + t/(2\,r)$ will never be much over one, and therefore may be neglected. Hence

$$S = \frac{\pi r^2 p}{2\,\pi rt} = \frac{pr}{2\,t}, \quad \text{and} \quad t = \frac{pr}{2S_a}.$$

The intensity of bursting pressure is

$$p_b = \frac{2\,S_u t}{r},$$

if S_u denotes the ultimate unit strength of the material.

33. A Hollow Sphere Subjected to Internal Pressure. The formulas for longitudinal stress in a cylindrical shell apply to a sphere; that is,

$$P = p\pi r^2 = S_a 2\,\pi r t; \quad S = \frac{pr}{2t}; \quad t = \frac{pr}{2\,S_a}.$$

34. Resultant of the Axial Thrusts on a Bend. The total axial pressure, parallel to KL on the curved surface MNO (see Fig. 5 a),

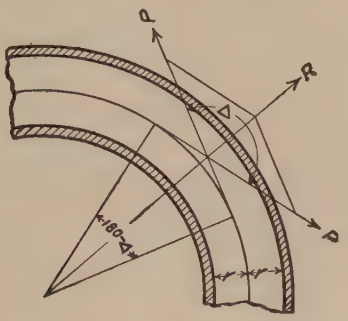

Fig. 7

is $p\pi r^2$. At right angles along the axis IJ there is an equal force, $p\pi r^2$.

Note. Figure 5 a is a special case of a right-angle bend; for a general case, see Fig. 7.

The single force along the bisector of Δ which may replace these two equal forces P, *i.e.*, their resultant, is

$$R = P\cos\frac{\Delta}{2} + P\cos\frac{\Delta}{2} = 2\,P\cos\frac{\Delta}{2}.$$

35. Collapsing Pressures on Cylindrical Shells. When the external exceeds the internal intensity of pressure, the two opposite forces normal to any diameter tend to crush the shell, or cause it to collapse.

The formula for a true cylindrical shell, if t is given, derived by reasoning similar to that used in § 27, is

$$S_c = \frac{pr_0}{t},$$

where S_c denotes the actual compressive unit stress in the material in pounds per square inch, and when t and r_0 are given in inches.

If S_{ca} denotes the allowable compressive stress, t the necessary thickness, and r_0 the outside radius of the cylinder, we may write

$$t = \frac{pr_0}{S_{ca}}.$$

The formula differs only from the hoop-tension formula (§ 27) in that the stress is compressive instead of tensile. It would indicate that S_c is independent of the length, but this conclusion, however, is not confirmed by experience. The formula, moreover, is applicable only so long as the body maintains its true cylindrical form. If the body is inaccurately constructed, or is locally deformed, the stresses set up not only become complex but also intensify with increase in deformation, as is shown by numerous cases of the collapse of cylindrical vessels, even though the stresses due to the maximum possible intensity of pressure as computed by rational formulas have indicated an ample factor of safety.

36. Experiments on Collapsing Pressure. An expression may be deduced for the unit compressive stresses in a cylindrical shell subjected to an external pressure tending to collapse it. This is applicable, however, only to truly cylindrical shells; and, since such a condition is seldom met with in practice, the expression is of little value.

Sir William Fairburn, in 1858, published the results of some experiments on the collapse of thin cylinders. He deduced a formula for collapsing pressure, but experience has shown it to be unsatisfactory.

Professor R. T. Stewart in 1906 published [1] the results of a number of experiments on commercial lap-welded tubes. These tests were numerous and trustworthy.

The collapse of pipe due to an external force is seldom met with in hydraulics, except in the case of large, plate-steel penstocks, where a sudden removal of the water has subjected them to an atmospheric pressure. Such a failure must be prevented, not by designing for this additional stress, but by eliminating the possibility of it by the use of air inlets and air-relief valves.

[1] Transactions A. S. M. E., vol. 27 (1906).

37. The Hydraulic Press. The hydraulic press, first put into use by Bramah (1795), will serve to illustrate further the principles of fluid pressures. See Fig. 8.

Let the plunger area a be 1 square inch, and let the ram area A be 1000 square inches. Suppose that the force F on plunger at C in addition to the force due to atmospheric pressure is 100

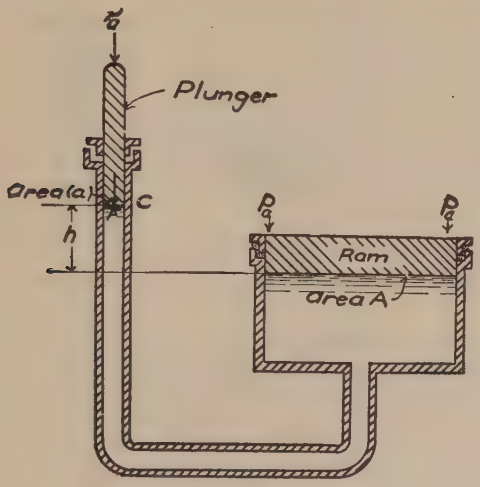

FIG. 8

pounds. The intensity of pressure due to F at C is $100/a$, or 100 pounds per square inch. Let h be one foot.

The intensity of pressure at C due to F and the atmosphere is $100 + p_a$, where p_a denotes the atmospheric pressure per square inch. Since this is transmitted undiminished to the ram, the intensity of pressure on the bottom of the ram corrected for difference in elevation h is $100 + p_a + 1 \times 0.433$. Since there is atmospheric pressure on top of the ram, the effective intensity of pressure on the bottom of the ram is $100 + 0.433 = 100.433$ pounds per square inch. Since the area $A = 1000$ square inches, the total vertical upward pressure on the ram $= 100.433 \times 1000 = 100{,}433$ pounds.

38. The Water Piezometer. If a tube, open at the top, is inserted in a body of still water, the height of the column of water inside the tube will correspond to the intensity of pressure

at the point where the tube is inserted. Such a tube used for measuring pressure is usually called a piezometer. See Fig. 9 *a*.

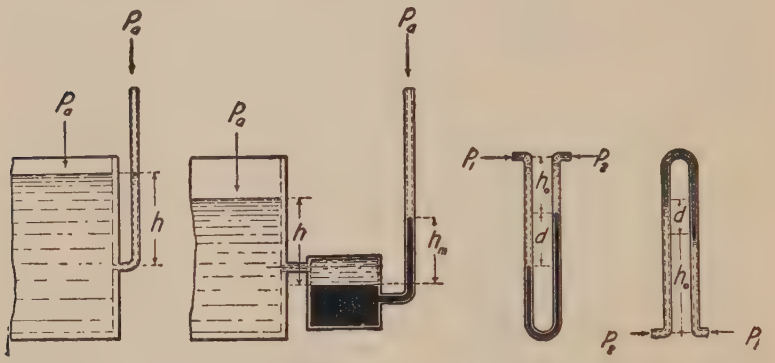

Fig. 9 *a* Water Piezometer — Fig. 9 *b* Mercury Gage — Fig. 9 *c* Fig. 9 *d* Differential Gages

39. The Mercury Piezometer, or Mercury Pressure Gage. If a tube inserted in a body of still water is led to a closed reservoir containing mercury, into which a second tube is inserted, the mercury will stand in the second tube at a height h_m above the point where the first tube is inserted in the water, equal to h divided by the specific gravity of the mercury. See Fig. 9 *b* for illustration not to scale. The height of mercury column will be

$$h_m = \frac{h}{13.5956} = 0.07355\, h,$$

where h_m denotes the height of the mercury column in feet, and h denotes the height of the equivalent water column in feet. One inch in height of mercury is equivalent to $1/(12 \times 0.07355)$, or 1.133 feet of water. For the details of a mercury gage, see Fig. 50.

40. The Differential Gage with a Heavy Liquid. If a U-tube (see Fig. 9 *c*), partly filled with a liquid heavier than water, has also water or other lighter liquid under different intensities of pressure in each leg, on top of the heavier liquid, the tops of the two columns of heavier liquid will have a difference in elevation, or a *deflection*, d, which depends upon the difference between the pressures p_1 and p_2, and the relative densities of the two liquids.

If the liquids are quiet, the equation of equilibrium, using heads to represent pressures, may be written in the form

$$s\left(\frac{p_1}{w} + h_0 + d\right) = s\left(\frac{p_2}{w} + h_0\right) + s'd,$$

where s denotes the specific gravity of the lighter liquid, s' denotes the specific gravity of the heavier liquid, and w denotes the weight of a unit column of the lighter liquid. Hence we have

$$\frac{p_1}{w} - \frac{p_2}{w} = d\left(\frac{s' - s}{s}\right);$$

but p_1/w and p_2/w are the elevations at which the lighter liquid would stand in a simple piezometer and can be expressed as h_1 and h_2. Then we may write

$$h_1 - h_2 = d\left(\frac{s' - s}{s}\right).$$

Suppose that the heavy liquid in the tube is carbon tetrachloride, having a specific gravity of 1.25, so that

$$\frac{s' - s}{s} = \frac{1.25 - 1}{1} = 0.25;$$

then we have

$$h_1 - h_2 = 0.25\, d.$$

In this case, the difference in gage height recorded by the differential gage is four times that which actually exists. It will be seen that the more nearly the specific gravity of the heavier liquid approaches that of the lighter, the greater will be the reduction of the observed difference in gage height from that actually existing.

41. The Differential Oil Gage. If a U-tube, partly filled with oil and inverted, has water or other liquid heavier than the oil under difference of pressure sustaining the oil in each leg (see Fig. 9 *d*), the difference d in the heights of the two water columns will be an exaggeration of the difference in the pressure heads due to p_1 and p_2.

The relation between the difference in height of the two columns and the difference between the two pressure heads may be

determined as in § 40. The specific gravity of the lighter liquid must, however, be determined very exactly in order to secure accurate results, and the problem is further complicated by the tendency of oil to adhere to glass. It is necessary, then, to calibrate a differential oil gage by direct comparison with a water column.

Williams, Hubbell, and Fenkell, who first put the differential oil gage to practical use, found that kerosene was a very satisfactory oil to use.[1]

PROBLEMS

1. A vessel similar to the one shown in Fig. 1 has the vertical tube filled with water to a height of 408 feet above the bottom of the cylinder. (*a*) Compute the absolute intensity of pressure in pounds per square inch on the inside along the bottom line. (*b*) Compute the total axial component pressure tending to blow off the ends of the horizontal cylinder.

2. A rectangular solid 10 feet by 10 feet by 10 feet is submerged in water. Its top surface is horizontal, and is 30 feet below the water surface. (*a*) What is the total normal pressure on the top? (*b*) on the bottom?

3. A steel cylindrical tank with its axis vertical is 30 feet high and 10 feet in diameter, inside dimensions. It contains water 12 feet deep, the remaining space being filled with compressed air at 100 pounds per square inch as indicated by an ordinary steam gage placed at the top.

(*a*) Compute the absolute intensity of pressure on the top and bottom of the inside of the tank.

(*b*) What are the total effective vertical component pressures on the top and bottom?

4. Compute the thickness of a cast-iron pipe 30 inches internal diameter under a head of 180 feet of water, using the N. E. W. W. Association formula and making the customary allowance for water hammer.

5. For the same head and water hammer allowance as in problem 4, compute the necessary thickness of a 30-inch riveted steel pipe, assuming that the allowable stress is 15,000 pounds per square inch, and that the joint efficiency is 70 per cent. FACTOR SAFETY

6. Given a 48-inch pipe with a quarter bend, and with an internal intensity of pressure of 100 pounds per square inch. What is the resultant of the axial thrusts on the bend?

7. Water under a pressure of 20 pounds per square inch stands in the upper part of one leg of a differential gage (as shown in Fig. 9 *c*), and at 10 pounds in the other leg. What will be the deflection of the heavier liquid, (*a*) if its specific gravity is 13.6? (*b*) if its specific gravity is 1.40? (*c*) if its specific gravity is 1.95? What will happen if a liquid lighter than water instead of heavier is placed in this kind of a tube?

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[1] Williams, Hubbell, and Fenkell, *Experiments on the Flow of Water in Pipes*, Transactions A. S. C. E., 1902.

8. A liquid having a specific gravity of 0.95 is placed in the top of a differential oil gage (Fig. 9 *d*); the oil is supported on water having a specific gravity of 1.005. The deflection is 0.4 foot. What is the difference in intensity of pressure in the water in the two legs in pounds per square inch?

9. A hydraulic jack with a 2-inch ram and a $\frac{3}{4}$-inch plunger is to lift a weight of 1.5 tons. The leverage of the handle is 12 to 1. What force must be applied to the handle?

10. The plunger of a hydraulic press is 1 inch in diameter. The diameter of the ram is 7 inches. The leverage is 6 to 1. What weight could be lifted on the ram if the force acting at the end of the handle is 90 pounds?

CHAPTER IV

FLUID PRESSURES OF VARYING INTENSITY

42. A Plane Surface Subjected to Pressures of Varying Intensity. If an immersed plane surface is not parallel to the free surface of the liquid, the intensity of pressure, since it is propor-

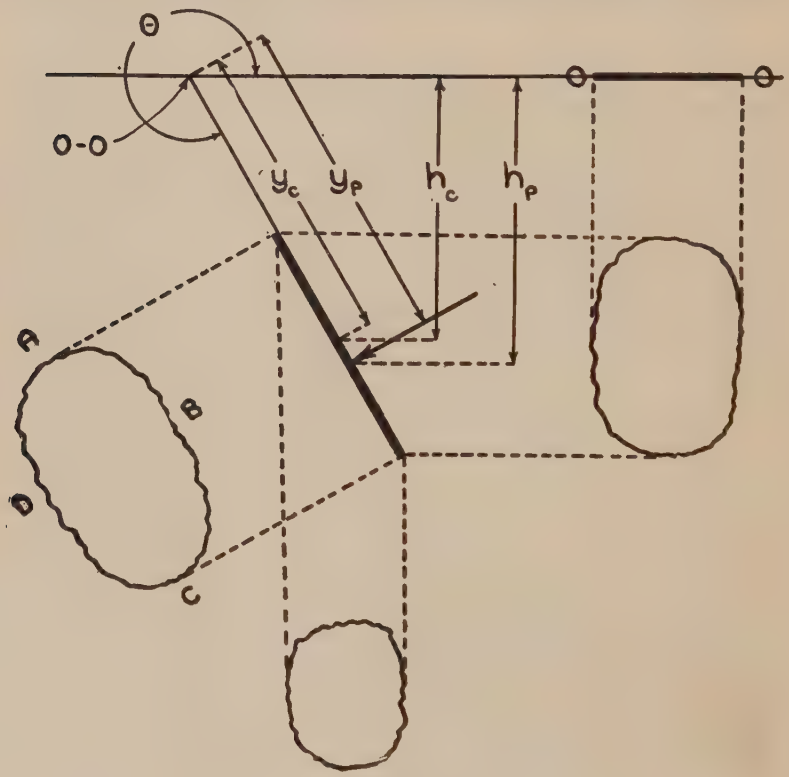

FIG. 10

tional to the head at any point, will have a different value for every part of the surface.

The irregular plane surface *ABCD*, area *A*, immersed in a liquid and inclined at any angle θ with the free surface, will represent a general case from which may be deduced the magnitude of the total normal pressure on any plane surface, and of its

components, its direction, and its point of application. See Fig. 10.

Let $a_1, a_2, \cdots, a_n$ be elements of area of $ABCD$; and $h_1, h_2, \cdots, h_n$ the heads upon these areas. The intensity of pressure on a_1 is $p_1 = h_1 w$; on a_2 is $p_2 = h_2 w$; on a_n is $p_n = h_n w$. The total normal pressure on a_1 is $p_1 a_1 = h_1 w a_1$; on a_2 is $p_2 a_2 = h_2 w a_2$; on a_n is $p_n a_n = h_n w a_n$.

The total normal pressure P on the area $ABCD$ is the sum of such pressures; in the limit, it is given by the integral formula

$$P = w \int\int dh\, da = w h_c \int da = w h_c \int\int dx\, dy.$$

Let the intersection of the plane $ABCD$ with the free surface, which is the line $O-O$, be taken as a center of moments. Then y_c is the shortest distance from $O-O$ to the center of gravity of $ABCD$. We have

$$h_c = y_c \sin\theta,$$

and hence

$$\begin{aligned} P &= w y_c \sin\theta \int\int dx\, dy \\ &= w y_c \sin\theta\, A \\ &= w h_c A. \end{aligned}$$

In other words, the total pressure on the area is equal to the area multiplied by the unit pressure at its center of gravity. The direction of this pressure is, of course, normal to the plane $ABCD$.

The point of application of P. The distance from any center of moments to the point of application of a system of parallel forces equals the sum of the moments of all the forces divided by the sum of all the forces. Using the same center of moments $O-O$, we may write

$$y_p = \frac{\int\int y^2\, dx\, dy}{\int\int y\, dx\, dy} = \frac{I'}{M} = \frac{I + A y_c^2}{A y_c}$$

$$= y_c + \frac{I}{A y_c}$$

$$= y_c + \frac{A k_0^2}{A y_c}$$

$$y_p = y_c + \frac{k_0^2}{y_c},$$

where I = moment of inertia of an area about an axis parallel to $O - O$ through its center of gravity $= A k_0^2$;

I' = moment of inertia of area about any other parallel axis;

$I + Ay^2$ = general equation for finding I' about an axis distant y from the given axis of gravity;

k_0 = radius of gyration about a parallel axis through the center of gravity. For a rectangle this is $a^2/12$, where a is the side normal to $O - O$.

NOTE. These distances must be measured along the same plane, either that of the surface itself, or the surface may be projected onto a vertical plane.

SUMMARY. The magnitude of *the total normal pressure on any immersed plane surface equals the intensity of pressure at its center of gravity multiplied by the area* of the surface.

Direction of the total normal pressure. Since the direction is fixed by the position of the surface, P is always perpendicular to the surface and toward it.

The point of application of the total normal pressure, or of its components, is through a point on the immersed surface at a distance measured parallel to the surface and distant k_0^2/y_c below the center of gravity. When the pressure is of uniform intensity, $y_p = y_c$.

43. Formulas for Regular Areas. For regular areas, the precise values of P and y_p are most easily found by integrating between their proper limits the expressions:

$$P = y_c wA \sin\theta = w \sin\theta \int\int y \, dx \, dy; \quad \text{and} \quad y_p = \frac{\int\int y^2 \, dx \, dy}{\int\int y \, dx \, dy}.$$

44. Composite Areas. If an area is composed of one or more regular areas, the precise value of P and y_p for each area may be found as above, and the total normal pressure and its point of application computed by the ordinary methods of composition of forces.

45. Center of Moments. A very convenient center of moments is the line of intersection of the immersed surface, or the surface produced, with the free surface of the liquid, which will be taken in the following examples, and designated $O-O$.

46. A Vertical Wall having its Upper Edge at or above the Water Surface. A wall with vertical faces, as shown in Fig. 11, forms the partition between two compartments of a reservoir.

One is empty, the other has water 20 feet deep. Compute the total normal pressure on the face in contact with the water.

In computations for retaining walls, dams, and similar structures, *it is customary to consider a section* 1 *foot long.* If the structure is homogeneous and symmetric, the computations for

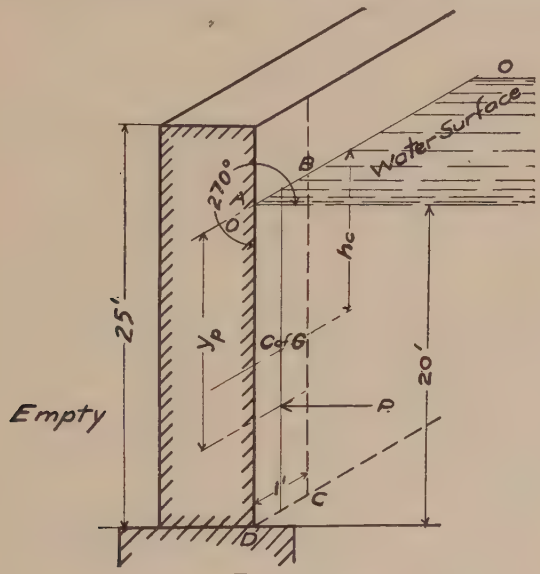

FIG. 11

a one-foot strip may be applied to the entire structure by multiplication. If atmospheric pressure p_a exists on both faces, its influence is negligible.

The magnitude of P. The immersed surface $ABCD$ has an area 20 feet $\times$ 1 foot = 20 square feet. Then, neglecting algebraic signs, we find

$$h_c = y_c \sin 270° = 10 \quad \text{feet.}$$

Therefore

$$P = h_c w A = 10 \times 62.4 \times 1 \times 20 = 12{,}480 \quad \text{pounds.}$$

The direction of P is 180°.

The horizontal and vertical components of P.

$$H = P \cos 180° = 12{,}480 \quad \text{pounds.}$$
$$V = P \sin 180° = 0.$$

The point of application of P (y_p). Since the figure is symmetrical, the point of application, or center of pressure, is on a vertical

line bisecting the surface $ABCD$, and distant from $O-O$ by the amount

$$y_p = y_c + \frac{k_0^2}{y_c} = \tfrac{2}{3}\, a,$$

since $k_0^2 = a^2/12$, and $y_c = a/2$.

Therefore y_p, measured in feet below $O-O$, is

$$y_p = 10 + \frac{20}{6} = \frac{2}{3} \times 20 = 13\tfrac{1}{3} \qquad \text{feet.}$$

47. A Vertical Wall having its Upper Edge below the Water Surface. A masonry dam, as shown in Fig. 12, has a vertical upstream face. The water is 30 feet deep on the upstream face, and there is no water pressure on the downstream face. Compute the total normal pressure on the upstream face. Consider a section 1 foot long. Since atmospheric pressure p_a acts on both faces, its influence is negligible.

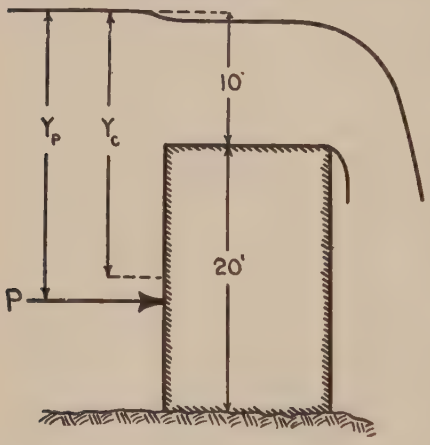

FIG. 12

The magnitude of P. The immersed surface has an area 20 feet $\times$ 1 foot = 20 square feet.

$$h_c = y_c \sin 270° = 20 \qquad \text{feet.}$$

Therefore

$$P = h_c w A = 20 \times 62.4 \times 20 = 24{,}960 \qquad \text{pounds.}$$

The direction of P is 0°.

The horizontal and vertical components of P.

$$H = P \cos 0° = 24{,}960 \qquad \text{pounds.}$$
$$V = P \sin 0° = 0.$$

The point of application of P (y_p). The point of application is on a vertical line bisecting the surface distant from the water surface by the amount

$$y_p = y_c + \frac{k_0^2}{y_c}.$$

But $k_0^2 = a^2/12$. Therefore

$$y_p = 20 + \frac{20 \times 20}{12 \times 20} = 20 + 1\tfrac{2}{3} = 21.67 \qquad \text{feet.}$$

48. An Inclined Gate or Valve Covering a Circular Opening. Compute the total normal pressure on an inclined gate covering

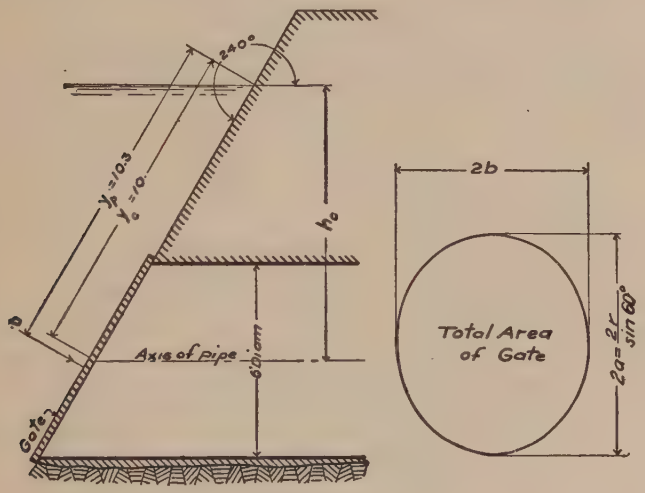

Fig. 13

a 6-foot circular pipe, as shown in Fig. 13. Given $y_c = 10$ feet. Neglect p_a.

The magnitude of P. The area of the opening A is an ellipse in which $a = r/\sin 60°$, and $b = r$. Hence

$$A = \frac{\pi r^2}{\sin 60°} = \pi r^2 \times 1.155 = 32.65 \qquad \text{square feet.}$$

$$h_c = y_c \sin 240° = 10 \times 0.866 = 8.66 \qquad \text{feet.}$$

Therefore

$$P = h_c w A = 10 \sin 60° \times 62.4 \times 32.65 = 17{,}640 \qquad \text{pounds.}$$

The direction of P is 330°.

Horizontal and vertical components of P.

$$H = 17{,}640 \cos 330° = 17{,}640 \times 0.866 = 15{,}270 \qquad \text{pounds.}$$

$$V = 17{,}640 \sin 330° = 17{,}640 \times 0.5 = 8820 \qquad \text{pounds.}$$

The point of application of P. The point of application of P is on a vertical through the center of gravity of the opening, and below it a distance k_0^2/y_c. But $k_0^2 = a^2/4$. Therefore

$$y_p = 10 + \frac{(3 \times 1.155)^2}{4 \times 10} = 10 + \frac{12}{40} = 10.30 \qquad \text{feet.}$$

If in computing y_p, uniform pressure were assumed, y_p would be 10 instead of 10.3, an appreciable error, but only 3 per cent.

49. A Solid with Water on Two Opposite Parallel Surfaces. The rectangular solid, as shown in Fig. 14, has two vertical faces, each 33 feet high and 8 feet wide; the water stands 30 feet deep

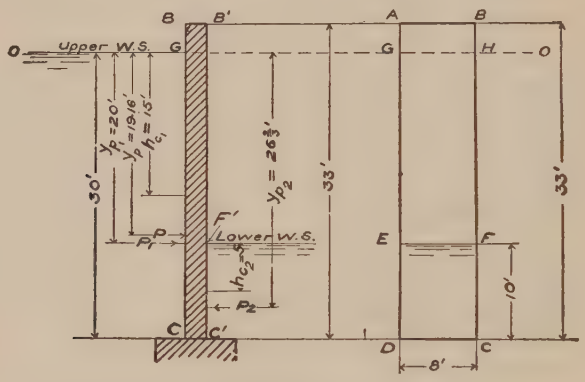

FIG. 14

on one side and 10 feet deep on the other. Compute the total normal pressures on the opposite faces, and the resultant pressure on the wall.

Let A_1 be the immersed surface $CDGH$; and let P_1 be the total normal pressure on A_1. Let A_2 be the immersed surface $C'D'E'F'$; and let P_2 be the total normal pressure on A_2.

The total normal pressure on A_1 (P_1).

$$P_1 = h_{c_1} w A_1 = 15 \times 62.4 \times 30 \times 8 = 224{,}640 \qquad \text{pounds.}$$

The direction of P_1 is 0°.

The point of application of P_1.

$$y_{p_1} = \tfrac{2}{3} \times 30 = 20 \qquad \text{feet below water surface, } O - O.$$

The total normal pressure on A_2(P_2).

$$P_2 = h_{c_2} w A_2 = 5 \times 62.4 \times 10 \times 8 = 24{,}960 \qquad \text{pounds.}$$

The direction of P_2 is 180°.

The point of application of P_2.

$y_{P_2} = \frac{2}{3} \times 10 = 6\frac{2}{3}$ feet below the lower water surface;

or 20 feet + $6\frac{2}{3}$ feet below the upper water surface, *i.e.*, 26.67 feet below $O-O$.

The resultant pressure (P).

Since P_1 and P_2 are parallel opposite forces,

$$P = P_1 - P_2 = 224{,}640 - 24{,}960 = 199{,}680 \quad \text{pounds.}$$

The direction of P is horizontal, and since P_1 is greater, the direction is 0°.

The point of application of P. Taking moments about $O-O$, we have

$$y_p = \frac{224640 \times 20 - 24960 \times 26.67}{224640 - 24960} = 19.16^{1} \quad \text{feet below } O-O.$$

NOTE. In computing water pressures against dams, walls, and similar structures, the difference between the highest possible elevation to which the water can ever rise, and the lowest point of the dam, however deep in the ground, should in general be used. Moreover, the beginner should note that the difference between P_1 and P_2 is not identical with the pressure on $EFGH$. The total pressure on $CDEF = 25 \times 62.4 \times 10 \times 8 = 124{,}800$ pounds; while the pressure on $C'D'E'F' = 5 \times 62.4 \times 10 \times 8 = 24{,}960$ pounds, — one-fifth as much; and they do not offset each other.

50. A Concrete Overfall Dam. The dam shown in section in Fig. 15 is of solid concrete, which is assumed to weigh 140 pounds per cubic foot. Consider a section one foot long. Although there may be water on the downstream side, its effect is usually neglected.

The stability of the dam is to be determined, under the action of the following forces:

The weight W of one foot of dam.

The total water pressure P against the Plane AD.

The upward pressure P_2 against the base of the dam AB caused by the percolation of water under it.

The resistance of the foundation.

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[1] Since the constant 62.4 pounds for the weight of a cubic foot of water is exact only at one certain temperature (*i.e.* approximately 4° C.), and the measured distances are subject to instrumental inaccuracies in actual cases, the fourth significant figure in this number cannot be relied on.

The question of upward pressure against the base of a dam is difficult to determine exactly. Every endeavor should be made to eliminate the percolation of water, but the fact that it may nevertheless occur must be taken into account. The failure of

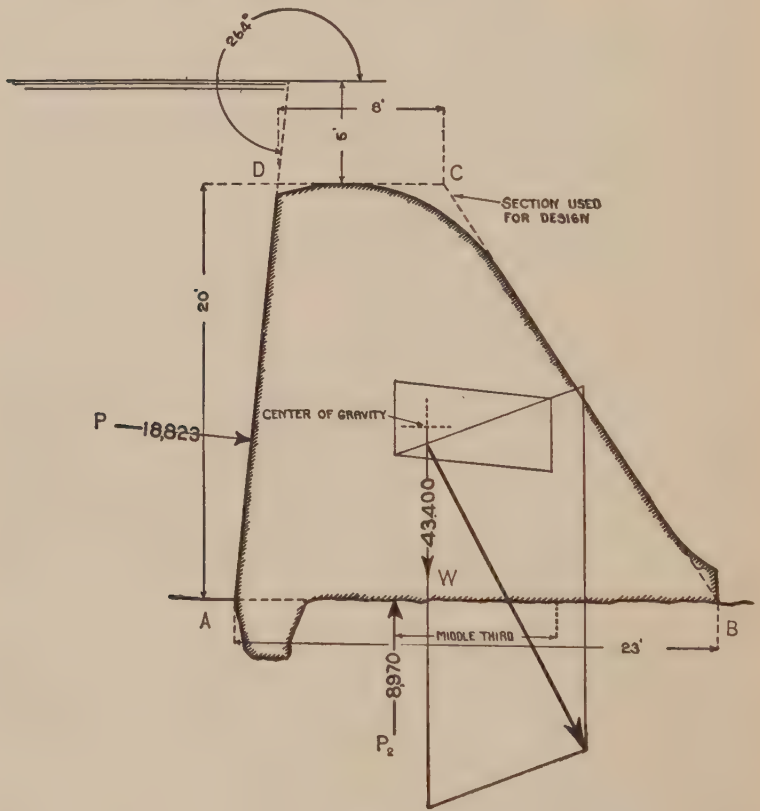

FIG. 15

many masonry dams is directly traceable to neglect of this upward pressure. Various assumptions are made as to how much this pressure will be. These will vary with conditions of the foundation and the cut-off wall, but a good general assumption in the case of a dam founded upon solid ledge is that the pressure under the upstream end of the dam is equal to half the actual head at this point, and decreases to zero under the downstream end.

51. Conditions of Stability of a Masonry Dam. To insure stability and permanency, a dam must be built with suitable provision against the following dangers:

I. Sliding bodily as a whole on its foundation, or in part along any plane in the structure.

II. Excessive stresses in the material in the structure or excessive pressure on its foundations.

III. The percolation of water through, under, or around the dam.

I. *Sliding.* The force tending to move the dam bodily downstream along the plane AB is

$$P \cos 6° + P_2 \cos 270° + W \cos 270°.$$

Obviously W always acts vertically and P_2 also, so long as the line AB is horizontal, which usually holds true, at least for purposes of design. The actual force, then, resolves itself to $P \cos 6°$.

$$P = 20 \times \sec 6° \left[\frac{20}{2} + 5\right] 62.4 = 18{,}823 \quad \text{pounds}$$

$$P \cos 6° = 18{,}710 \quad \text{pounds}$$

The force resisting this sliding is the frictional resistance between the dam and its foundation. The weight of the dam is

$$\left[8 \times 20 + \frac{15 \times 20}{2}\right] 140 = 43{,}400 \quad \text{pounds.}$$

There is, however, the upward pressure P_2 on the base of the dam which tends to counteract this force; it is

$$P_2 = 23 \times \frac{\frac{25}{2} + 0}{2} \times 62.4 = 8{,}970 \quad \text{pounds.}$$

The effective value of W is

$$43{,}400 - 8{,}970 = 34{,}430 \quad \text{pounds.}$$

The coefficient of friction to be used varies with conditions, but for masonry on ledge it is usually taken as 0.6; hence the frictional resistance is

$$34{,}430 \times 0.6 = 20{,}658 \quad \text{pounds,}$$

or 20,000 pounds, to an approximation within the limits of error. This frictional resistance is greater than $P \cos 6°$, and the dam is stable against sliding.

II. *Stresses in the material or in the foundation.* Masonry, except reinforced concrete, should never be designed to withstand tensile stress. This applies also, of course, to the joint between the base of the dam and the foundation.

If the resultant of all the forces acting upon the dam falls within the middle third of the base of the dam there will be pressure on the foundation throughout the length of the base, and no tension will exist. If the resultant cuts the base at the downstream end of the middle third, the pressure at the upstream end of the base will be zero; if it cuts the base beyond this point, tension will exist at the upstream end. It is obvious that internal tensile stresses within the dam itself must not be allowed, and that any horizontal plane in the dam section may be considered as the base and the position of the resultant of the forces acting on that portion of the dam above this plane may be determined.

The resultant of all the forces acting upon a dam must cut the base of the dam within the area of the middle third.

The position of the resultant force can perhaps be most easily determined by graphic construction and composition of the forces acting (Fig. 15), but it can also be determined by figuring the moments of all the forces about the downstream end of the dam.

The unit pressure on the base must not exceed at any point the allowable compressive stress. The pressure will be greatest at the toe of the dam. It can be computed by the expression [1]

$$P_f = \frac{R \sin \alpha}{L}\left(1 \pm \frac{6\,e}{L}\right),$$

the intensity of pressure in pounds per square foot at either end of the base.

Here e is the distance from the gravity axis of the base to the point where the resultant cuts the base. If e is measured down-

[1] This formula is taken over by similarity from the formula used to determine the extreme fiber stresses in a strut eccentrically loaded, viz.

$$S = \frac{R \sin \alpha}{a} \pm \frac{Mc}{I}$$

where

a = area of section; here, $a = L \times 1$
c = any distance from neutral axis; here, $c = L/2$
I = moment of inertia of a; here, $I = L^3/12$
M = moment of external forces; here, $M = Re \sin \alpha$
S = intensity of stress at any point distant c from the neutral axis, and is tension whenever $(R/a) \sin \alpha$ is less than Mc/I.

stream from the gravity axis, the maximum P_f is at the downstream toe; if measured upstream, the maximum is at the upper toe.

α = the angle of inclination of the resultant.

III. *Percolating water.* Water percolating under or around a dam may cause its failure. If the sides and foundation of the dam are earth, percolating water may wash them out, and undermine the dam.

Some percolation is almost certain to exist, but it should be reduced to an absolute minimum. Watertight cut-off walls of masonry or sheet piling should be used. In extreme cases, drains under the dam will carry off the leakage and remove the pressure. The allowance of half the pressure at the upstream end and zero at the downstream end of the dam, used in the example, is for an ideal condition with a tight cut-off wall. For foundations of seamy rock the allowance must be higher. Whatever be the foundation conditions, upward pressure must not be neglected, but its possible presence in greater or less degree must be taken into account in the design.

The question of the design of dams is not included in the scope of a textbook such as this, but it is attempted in this problem to show the general factors entering into such design. The question of the factor of safety against sliding to be used varies with conditions, but it is perhaps preferable to include it in the determination of the maximum head over the crest of the dam.

The capacity of the spillway not infrequently determines the stability of the dam. It must be sufficient to pass the maximum possible flood without raising the water high enough to endanger the stability of the dam. The two main steps in the design of a dam, then, are as follows:

I. Determination of the maximum possible flood.

II. Design of a dam section for the depth of water over the crest necessary to pass this flood such that it will be stable against sliding, and that the resultant of all the forces acting on the dam will cut the base of the dam within the middle third.

52. A Framed Dam. In contrast is shown, in Fig. 16, a dam which depends very little upon its weight for stability. Such dams, which are now built in wood, steel, and reinforced concrete, must be designed as framed structures. See Fig. 16.

There are many types of dams in addition to the two examples shown. Masonry reservoir dams often have a separate spillway so that the dam proper is designed as a non-overflow section.

Arched masonry dams depend partly upon gravity for their stability, but their strength is largely due to the fact that, presenting an arched face against the pressure, they transmit this pressure to their abutments in the form of a thrust. Arch dams require sites with high rock walls on either side.

Multiple arch dams are essentially the same as frame dams, except that the deck between the piers, instead of being a flat

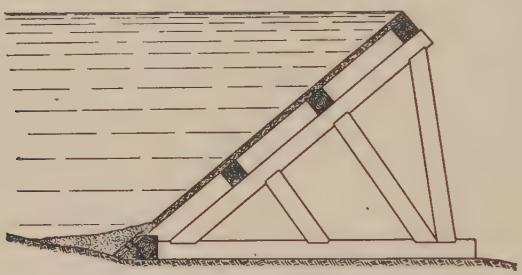

Fig. 16. — Timber Frame Dam

concrete slab requiring reinforcing steel, is arched and in compression only. This type of dam has the disadvantage, however, that if one section should fail, the rest are apt to topple over like a row of dominoes, due to the unbalanced end thrust. Earth fill dams, when well built, are perhaps the most permanent form of dam. If, however, an earth fill dam is overtopped by water, it is certain to fail.

53. Curved Surfaces Subjected to Fluid Pressures of Varying Intensity. On a curved surface as on a plane the intensity of pressure varies as the distance from the free surface of the fluid. Inasmuch as the normal forces on the different elements of area of the curved surface do not form a system of parallel forces, no single force can be said to be the total resultant force. However, the total horizontal or vertical fluid pressure, or the total fluid pressure in any direction, may be determined.

If the highest degree of precision is required, and the curved surface is regular, the magnitude, direction, and point of application of the components may be found by the integration of the

proper differential equations. If the surface is not regular, and if the variation in intensities of pressure cannot be neglected, the problem is usually solved by approximate methods. Two cases will illustrate approximate methods as applied to curved surfaces.

54. A Deep Vertical Cylindrical Tank. In a deep tank filled with water, there is a considerable variation in the intensity of fluid pressure. In determining the thickness of the shell, its surface may be divided for computation into sections, of which the dimensions will be more or less fixed by commercial considerations. The pressure may be assumed constant on each of these sections, and the necessary thickness of material t may be computed for successive sections by the formula for hoop tension.

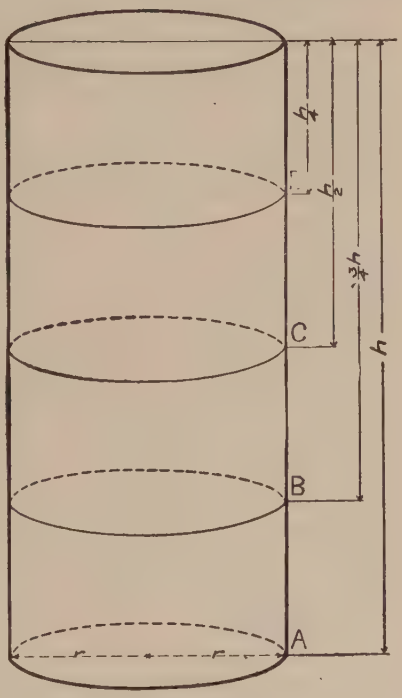

FIG. 17

Let Fig. 17 represent a deep cylindrical tank. Use the intensity of pressure at the point A for the cylindrical surface between A and B, and the pressure at the points, B, C, and D for each successive ring.

If the cylindrical tank of radius r (inches), shown in Fig. 17, is made of steel or of any other homogeneous metal, and is filled with water, the necessary thickness is

$$t \text{ at } A = \frac{pr}{S_a} = \frac{0.433 \times h \times r}{S_a},$$

which is the required thickness of the circular plate of width AB;

$$t \text{ at } B = \frac{pr}{S_a} = \frac{0.433 \times 3\,h \times r}{4\,S_a},$$

which is the required thickness of the circular plate of width BC.

In like manner, the thickness of each successive set of plates may be computed.

55. A Wooden-Stave Tank. If the tank is of wooden staves hooped with metal, the requisite net cross section a of the material necessary to hold together the area defined by A and B may be determined as follows. The force tending to rupture the cylinder AB (using the intensity of pressure at A) equals $0.433\,h \times 2\,r \times AB$. The resistance of the material in the hoops equals $2 \times a \times S$, where a denotes the net cross section of material in the hoops in the section AB; and S denotes the actual unit tensile stress in the material.

Since the resistance of the material of the hoops must be equal to the force tending to rupture the hoops, we have

$$0.433 \times h \times 2\,r \times AB = 2 \times a \times S.$$

Therefore, if a is given, we may write

$$S = \frac{0.433 \times h \times r \times AB}{a} = \text{unit stress in tension in the hoops.}$$

If S_a, the allowable tensile stress, is given, the required cross-sectional area a in the material in the hoops for section AB, is given in square inches by the equation

$$a = \frac{0.433 \times h \times AB \times r}{S_a} = \frac{p \times AB \times r}{S_a}.$$

This is merely another application of the computation of hoop tension (§ 27).

PROBLEMS

1. On one side of a sluice gate the water stands 8 feet above the bottom; on the other side, 4 feet; the gate is 6 feet wide. What is the total pressure against the grooves in which the gates slide? Where is the center of this pressure?

2. A flap valve, opening inwardly, automatically controls the water-supply in a small boiler. When the surface of the water is 3 feet above the center of the valve, and the steam gage reads 20 pounds, at what height must the water stand in pipes outside, in order to cause flow?

3. A cast-iron sluice gate is placed at the bottom of a dam; its dimensions are 4 inches thick, by $1\frac{1}{2}$ feet wide, by 6 feet high; the water is 50 feet deep. If the coefficient of friction of gate on its runners is 0.23, if the gate is exposed to air on the downstream side, what force will be required just to lift it? Assume the iron to weigh 480 pounds per cubic foot.

4. Compute the horizontal and vertical components of the normal pressure on an immersed rectangular area 6 feet by 8 feet, inclined 30° to the vertical, one of the longer edges being in the water surface.

5. Find the magnitude and point of application of the normal pressure on an immersed rectangle with sides 4 feet by 6 feet, with shorter side in the water surface.

6. The partition wall between the compartments of a settling basin is 12 feet high. When on one side the water is within 1 foot of the top and the other side is empty, if masonry weighs 160 pounds per cubic foot, and 60 per cent of its total weight represents its ability to resist sliding, how thick must it be? Consider a strip 1 foot wide.

7. A small masonry dam 20 feet high, 4 feet thick at top, 10 feet thick at bottom, is symmetrical in cross section. The masonry weighs 140 pounds per cubic foot. When filled to the top, what will be the resultant pressure on base, and how far from a perpendicular line through the center of gravity of the section will the resultant cut the base? Consider a strip one foot wide. Is this section stable?

8. A circular plate 4 feet in diameter is immersed so that the head on its center is 40 feet, its plane making an angle of 35° with the vertical. Compute the horizontal and vertical pressures on one side of it.

9. The upstream face of a dam has a slope of 60° with the horizontal. A penstock, 6 feet in diameter, covered by a gate, is so set in the dam that the center of the pipe is 20 feet below the surface of the water, measured along the face of the dam. Find the total normal pressure on the gate and its point of application.

10. A masonry dam is 5 feet thick at the top, 40 feet thick at the bottom, and 40 feet high; upstream face vertical. Assume that the water stands 6 feet higher than the top, and that there is no water on the downstream face. Take the weight of masonry as 160 pounds per cubic foot and consider the pressure of water on the top of the dam. Determine (*a*) factor of safety against sliding if the coefficient of friction is 0.6; (*b*) distance from upstream toe to point where resultant cuts the base; (*c*) pressure in pounds per square foot at upstream and downstream toes.

11. A standpipe is 20 feet in internal diameter and 30 feet high; it is to be of riveted steel plates and to be entirely filled with water. The horizontal joints are to be 7.5 feet apart vertically; and all joints are to have an efficiency of 75 per cent. The allowable unit stress for steel in tension may be taken as 15,000 pounds per square inch. Compute the necessary thickness of each row of plates.

12. If the standpipe in problem 11 were to be made of wooden staves 2 inches thick, compute the required number of hoops made of $\frac{1}{2}$-inch diameter steel rods, and determine the spacing of the hoops.

CHAPTER V

THE EQUILIBRIUM OF FLOATING SOLIDS

56. Equilibrium of a Floating Solid. To secure complete equilibrium of a totally immersed or floating solid, *the resultant horizontal and the resultant vertical components of all forces acting on it must be zero; and it must have no tendency to rotate about any axis.*

Every face of a solid totally immersed in water, and every face below the water line of a floating solid, is subjected to water pressure.

If the surface of the water in which the body is immersed is exposed to the atmosphere or compressed gases, or is under pressure due to mechanical forces, the intensity of pressure at any point on the solid is the sum of the intensity of pressure due to the head of water, and that due to forces acting on the water surface.

Whenever the intensity of pressure due to any force is transmitted with unchanged intensity to every face of the solid, the neglect of such a pressure will cause no error. In the following computations concerning the stability of bodies either immersed or floating in still water, the fluid pressures due to water alone will be considered, the atmospheric pressure being neglected.

57. Horizontal Pressures. It has been shown that the pressure against an irregular surface is equivalent to the pressure against the projection of the surface upon a plane normal to the desired force. Immerse an irregular solid in a fluid. Project one face of this solid onto a vertical plane. Project the diametrically opposite face upon another plane parallel with the first. The two projected planes will be identical, and, since their centers of gravity are the same distance below the water surface, the pressures against these planes will be the same. The two opposing forces acting on the two faces of the solid are equal, so there is no movement. The above will be found to hold no matter where the two opposing vertical projections are taken.

In an immersed solid there is no tendency to translation in a horizontal direction.

58. Vertical Pressures. Consider the same solid cut into elementary vertical prisms, shown in Figs. 18 *a* and *b*.

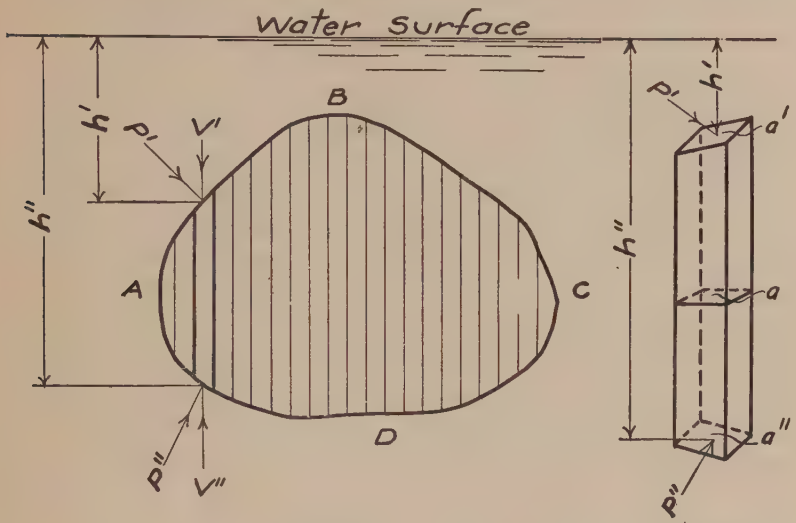

FIG. 18 *a*

FIG. 18 *b*

If a' and a'' denote the elementary areas forming the ends of the prism, their common horizontal projection is a, the horizontal cross-sectional area of the prism.

Let V' and V'' denote the vertical components of P', and P'', respectively. Then

$$V' = h'wa, \qquad V'' = h''wa,$$

and the resultant vertical fluid pressure on the prism is

$$V'' - V' = (h'' - h')wa.$$

But $(h'' - h')a$ is the volume of the prism, and also the volume of water displaced by it, and $(h'' - h')wa$ is the weight of water displaced by the prism.

The magnitude of the resultant vertical pressure; buoyancy. Since the sum of the volumes of all the prisms equals the volume of the solid, the algebraic sum of all the vertical components, which is the resultant vertical fluid pressure, *equals the volume of the solid*, or, if it is floating, that part which is immersed, *multiplied by the weight of a cubic unit of water, or the weight of an equal volume of water*. Its effect is a tendency to lift the solid. The

resultant vertical pressure $V'' - V'$ is therefore usually called the ***buoyancy,*** and will be designated by B.

Direction of the resultant vertical pressure. Since the upward vertical fluid pressure is always greater than the downward vertical fluid pressure, the direction of B is vertically upward.

59. Center of Buoyancy. The center of gravity of the displaced water is called the center of buoyancy, and designated by b. If the body is wholly immersed and homogeneous, the center of gravity of the solid and the center of buoyancy are identical. If it is hollow or of varying density, or not wholly immersed, the center of gravity of the solid and the center of buoyancy may not coincide.

60. Loss of Weight. The weight W of the body is a downward vertical force; and the buoyancy is an upward vertical force. W and B form, therefore, a set of parallel, opposite forces.

If B is less than W, a totally immersed body will be in equilibrium only when an additional vertical upward force equal to the difference between the weight of the body and the buoyancy is applied to it; that is, the body is supported by some external force. If the body is weighed while suspended in water, its apparent weight will be W minus B. Since W is its weight in air, its loss of weight is B.

If B is greater than W, an upward vertical force, B minus W, will be exerted upon an immersed body; and to keep it immersed a vertical downward force equal in magnitude to B minus W must be applied to maintain equilibrium.

If unrestrained, this vertical upward pressure, B minus W, will cause the body to rise until it floats in equilibrium; which occurs when the weight of the displaced water, no longer of equal volume with the solid, equals W. In this position, B equals W; and the apparent weight of the solid is zero. Its loss of weight is B.

The loss of weight of an immersed or floating body is merely the buoyant effect of the water, and equals the weight of the displaced water.

61. Depth of Flotation. Draught. The depth of flotation, or the ***draught,*** of a floating body may be determined by computing: first, a volume of displaced water equal in weight to the body, including all loads, and the vertical resultant of all forces acting upon it; and then the depth of immersion, measured from the water surface to the lowest point of the solid, necessary to pro-

duce the required volume of displacement. The depth of flotation is constant only for a few shapes; usually, it varies as the body is tilted.

62. Stability. The stability of an immersed or floating body against overturning may be determined, as in any problem in statics, by taking moments of all the forces acting upon the body, using any convenient axis of reference. For bodies of irregular shape the necessary computations, especially those required to determine the normal water pressures or their components, may be somewhat intricate. This work may be simplified in many instances by an investigation of the relative position of the center of gravity (or more precisely the point of application of all vertical forces) and the position of the center of buoyancy.

If the algebraic sum of the horizontal and vertical components of all forces equals zero, and if the center of gravity and center of buoyancy are in the same vertical line, the body will be in complete equilibrium.

63. Buoyancy of a Floating Body. The buoyancy B of a floating body is equal in magnitude and opposite in direction to the sum of all the vertical downward forces, including the weight of the body and all loads. This sum for convenience will be called the weight W.

The center of buoyancy will be denoted by b. The center of gravity or the point of application of all vertical forces will be denoted by G.

64. The Axis of Flotation. If a floating body is in complete equilibrium, a vertical through G and b (both computed for the position in which it is then floating) is called the ***axis of flotation.*** If the axis of flotation divides the body into symmetrical parts, the body will float upright.

The centers of gravity and of buoyancy, when the body floats upright, are the original centers of gravity and of buoyancy, designated by G_0 and b_0.

65. The Angle of Heel. Change of Trim. The angular displacement in any given direction of the axis of symmetry measured from the vertical is called the ***angle of heel,*** θ. The length of the water line multiplied by the sine of the angle of heel is called the ***change of trim.***

66. The Righting Couple. The Upsetting Couple. If a floating body is tilted, so that G_0 and b_0 are not in the same vertical line, W and B, which are equal in magnitude, parallel, but opposite in direction, will form a couple whose magnitude is equal to Bc, or to Wc, where c is the perpendicular distance between the lines of action of B and W. The effect of this couple will be either to restore the body to an upright position or to cause further tilting.

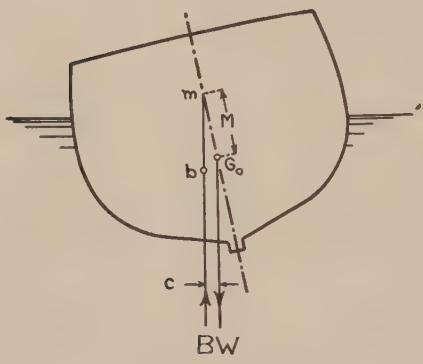

FIG. 19*a*

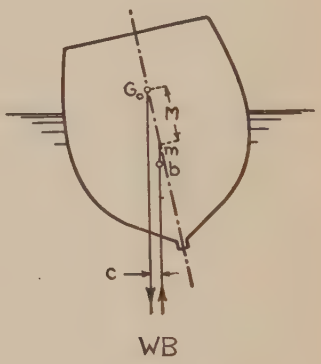

FIG. 19*b*

If, upon the removal of the couple which caused tilting, the couple Bc tends to restore the body to its original position of equilibrium, Bc is a righting couple; and the body is stable (see Fig. 19 *a*).

If, on the contrary, the couple Bc tends to rotate the body still farther from the original position of equilibrium, Bc is an upsetting couple; and the body is unstable (see Fig. 19 *b*).

67. The Metacenter. The point where the vertical through the center of buoyancy cuts an inclined axis of symmetry is called the ***metacenter,*** and is denoted by m.

68. The Metacentric Height. The distance measured on the axis of symmetry from the metacenter, for the position in which the body is floating, to the original center of gravity is called the ***metacentric height,*** and is denoted by M. For only a few cases is it constant; usually it varies with every change in position. The longitudinal and transverse metacentric heights are more frequently considered, but other values may be required.

69. Determination of the Metacentric Height *M*. If a floating body is in equilibrium with its axis inclined, the couple Fa which tends to cause it to heel must be equal to the righting couple Bc. Since $c = M \sin \theta$, θ being the angle of heel, we have

$$Fa = B \times M \sin \theta; \quad \text{and} \quad M = \frac{Fa}{B \sin \theta}.$$

For a vessel already constructed, M may be computed from the inclination caused by shifting loads (included in the weight W).

EXAMPLE. A weight of 100 tons (included in the displacement) is moved transversely 10 feet on a vessel of 10,000 tons displacement, causing it to heel through an angle of 10 minutes of arc. Compute the transverse metacentric height. We have $F = 100$; $a = 10$; $B = 10{,}000$. Therefore

$$M = \frac{100 \times 10}{10000 \times \sin 10'} = 34.5 \quad \text{feet.}$$

70. Stability shown by the Position of the Metacenter. Reference to Figs. 19 *a* and 19 *b* will show that, with the axis of symmetry inclined, if the metacenter m is above G, Bc will be a righting couple; if m is below G, Bc will be an upsetting couple.

If, when the axis of symmetry is vertical, m is below the center of gravity while the body may be in equilibrium, a slight push may produce complete upsetting.

71. Application of Principles of Flotation. Many complicated problems arise which require the application of the principles just considered, which are for the most part connected with the work of naval architects. The question of the stability of booms, rafts, dredges, gas holders, and similar structures of simple design belong, however, to the work of other engineers. The following example will illustrate the computations for determining the stability of flotation.

72. An Example of a Scow. Consider the transverse stability of a scow 20 feet wide, 12 feet deep, and 50 feet long, symmetrically constructed and loaded, having an average weight including all loads of 41.6 pounds per cubic foot, and floating in still water.

Let $ABCE$ in Figs. 20 *a* and 20 *b* represent the cross section of the scow (*a*) with axis of symmetry vertical, (*b*) with this axis tilted transversely. Let us use the following notation:

D = depth of flotation, or draught, in feet when axis of symmetry is vertical.

d = the distance in feet that the water line is raised above the water surface in tilting the vessel, measured on the face.

θ = angle of heel corresponding to d.

$2e$ = breadth of the vessel.

H = height of the vessel.

Stability with the axis of symmetry vertical. The center of gravity is seen to be on the axis of symmetry $Y-Y$, midway between

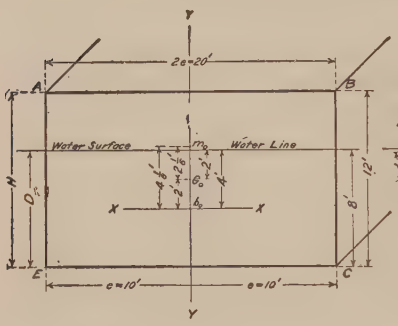

Fig. 20 *a*

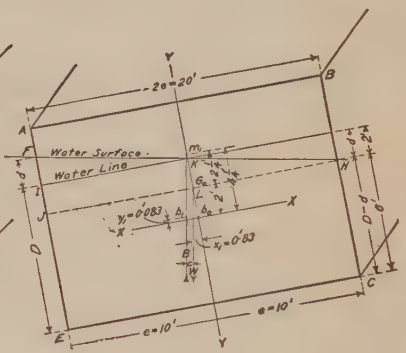

Fig. 20 *b*

the top and bottom of the scow; or 2 feet below the water surface. This is the original center of gravity, G_0. See Fig. 20 *a*. Since the weight of water displaced is

$$12 \times 20 \times 41.6 \times 50 = D \times 20 \times 62.4 \times 50,$$

we have

$$D = \frac{41.6}{62.4} \times 12 = 8 \quad \text{feet.}$$

The center of buoyancy is on the axis of symmetry 4 feet below the water surface. This is the original center of buoyancy, b_0.

Since the sum of the horizontal components and the sum of the vertical components of all forces are both zero (§§ 57 and 58), and the center of gravity and the center of buoyancy are in the same vertical line, the scow is in a position of complete equilibrium.

Stability with the axis of symmetry inclined. Consider the effect of tilting the scow as shown in Fig. 20 *b*. If the loading remains constant, the amount of water displaced must be constant for

any angle of heel. Therefore, *the volume of the wedge of water displaced on one side of the axis of symmetry equals the volume of the wedge released on the opposite side.*

In this position, the sum of the horizontal components and the sum of the vertical components of all forces are both zero.

It is necessary to find the relative positions of the new center of buoyancy b_1 and the original center of gravity G_0, and thus to determine whether the body is stable against overturning. This may be done by any convenient method with reference to any set of axes. Let the axis of symmetry be the $Y-Y$ axis, and a perpendicular to $Y-Y$ through b_0 be the $X-X$ axis.

To find the center of buoyancy. The center of buoyancy b_1 in the tilted position shown coincides with the center of gravity of the area $CEFH$, which is made up of four figures, rectangle $CEJH$, triangles HKL and FKI, and rectangle $LJIK$.

Let x_1 and y_1 be coordinates of b_1. Taking moments about the axis $Y-Y$, we have

$$2\,Dex_1 = ((D-d)2\,e \times 0) + \left(\frac{de}{2} \times \frac{e}{3}\right) - \left(\frac{de}{2} \times \frac{2\,e}{3}\right) - \left(de \times \frac{e}{2}\right);$$

$$(1) \qquad x_1 = -\frac{2\,de^2}{6\,De} = \frac{de}{3\,D}, \text{ measured to the left from } Y-Y;$$

Taking moments about the axis $X-X$, we have

$$2\,Dey_1 = -\left[(D-d)2\,e \times \frac{d}{2}\right] + \frac{de}{2}\left(\frac{D}{2} + \frac{d}{3}\right) + \frac{de}{2}\left(\frac{D}{2} - \frac{2\,d}{3}\right) + de\left(\frac{D}{2} - \frac{d}{2}\right);$$

$$(2) \qquad y_1 = +\frac{d^2e}{6\,De} = \frac{d^2}{6\,D}, \text{ measured upward from } X-X.$$

To find the angle of heel θ_1 for this position, we may write

$$(3) \qquad \tan\theta_1 = \frac{d}{e}; \quad \operatorname{ctn}\theta_1 = \frac{e}{d}.$$

To find the location of the metacenter (m_1) for this position. By definition, m_1 is on the axis $Y-Y$ and distant from b_0, b_0m_1; hence

$$(4) \qquad b_0m_1 = x_1 \operatorname{ctn}\theta_1 + y_1 = \frac{de}{3\,D} \times \frac{e}{d} + \frac{d^2}{6\,D} = \frac{e^2}{3\,D} + \frac{d^2}{6\,D}.$$

To find the metacentric height (M_1) for this position. By definition, the metacentric height is the distance

$$M_1 = G_0m_1 = b_0m_1 - b_0G_0. \tag{5}$$

To find the proper metacentric height (M_0). The equation (4) for finding the position of the metacenter

$$b_0m_1 = \frac{e^2}{3\,D} + \frac{d^2}{6\,D}$$

is a general one for a vessel of this shape, provided the top is not immersed; when

$$d = 0, \quad b_0m_0 = \frac{e^2}{3\,D}, \quad \text{and} \quad M_0 = G_0m_0 = \frac{e^2}{3\,D} - b_0G_0. \tag{6}$$

The locus of the center of buoyancy. As the vessel is heeled through a series of inclined positions, the path of the center of buoyancy is a parabola, of which $e^2/(3\,D)$ is a subnormal, and therefore a constant.

The magnitude of the couple Bc. We have

$$c = M_1 \sin\theta_1 = (b_0m_1 - b_0G_0)\sin\theta_1;$$

and

$$Bc = W(b_0m_1 - b_0G_0)\sin\theta_1. \tag{7}$$

The scow is in equilibrium. Because m_1 in the tilted position of the scow is clearly in such a location that Bc is a righting couple, and because the horizontal and vertical forces are in equilibrium, the scow is in complete equilibrium. Successive positions may be investigated in a similar manner.

Numerical results.

From (1), $x_1 = \dfrac{de}{3\,D} = \dfrac{2 \times 10}{3 \times 8} = 0.83$ feet.

From (2), $y_1 = \dfrac{d^2}{6\,D} = \dfrac{2^2}{6 \times 8} = 0.083$ feet.

From (3), $\tan\theta_1 = \dfrac{d}{e} = 0.2$; $\operatorname{ctn}\theta_1 = 5.0$; $\sin\theta_1 = 0.1960$

From (4) and (5)

$$M_1 = \frac{e^2}{3\,D} + \frac{d^2}{6\,D} - b_0G_0 = \frac{10^2}{3 \times 8} + \frac{2^2}{6 \times 8} - 2 = 2.250 \text{ feet.}$$

From (4) and (6),

$$M_0 = \frac{e^2}{3\,D} - b_0G_0 = \frac{10^2}{3 \times 8} - 2 = 2.17 \qquad \text{feet.}$$

From (7),

$$Bc = 20 \times 12 \times 50 \times 41.6 \times 2.250 \times 0.1960$$
$$= 220{,}100^+ \text{ pound-feet.}$$

In order to hold the scow in the tilted position, a couple $F_1a_1 = 220{,}100^+$ pound-feet must be exerted without altering the displacement. A load $F_1 = 22{,}010^+$ pounds already on board on the end of derrick boom, if swung athwart the scow a distance $a_1 = 10$ feet, would just tilt the scow into the position shown.[1]

73. General Formulas. The formulas (1) to (7), derived to compute x, y, θ, M, and Bc with reference to a transverse plane, are general formulas for any rectangular outline with reference to any plane, provided the deck is not submerged.

Other forms of solids may be computed by similar methods, although in some cases more intricate. Every case resolves itself into finding the position of the actual center of buoyancy relative to the original center of gravity (chosen for convenience) by the ordinary methods of mechanics. Moving a weight already on board changes the actual center of gravity, and if the new position of the center of gravity were determined it would be found, when equilibrium is established, to be on a vertical through the corresponding position of the actual center of buoyancy.

PROBLEMS

1. A rectangular solid of copper weighing 550 pounds per cubic foot is suspended in water by four wires at the corners, its upper surface being horizontal and 10 feet below the water surface. Its horizontal dimensions are 4 feet by 3 feet and its vertical thickness 1½ feet.

(*a*) Find the difference in vertical pressures on top and bottom surfaces.

(*b*) Show that its loss of weight is equal to the weight of a solid of water of the exact dimensions of the solid of copper.

(*c*) Find the tension in each wire.

(*d*) Show that loss of weight if entirely submerged is independent of the depth of submersion and is equal in weight to a volume of water of the same dimensions as the solid.

[1] The accuracy of this computation is dependent upon the accuracy of the assumed weight (41.6 pounds) per cubic foot and the accuracy of the measured distances.

2. A rectangular water-tight box is made of 1.5-inch boards, having outside dimensions: 10 feet long, 8 feet wide, and 3 feet deep; there is no cover. Assume that the wood weighs 36 pounds per cubic foot. How deep will it float in water?

3. A steel cylinder 25 feet long, 10 feet in diameter, made of $\frac{1}{2}$-inch metal, floats with its axis horizontal. How deep will it float? Also solve for axis vertical. If half full of water, how deep?

4. A vessel has 12,000 tons displacement. By filling alternately with 20 tons of water the boats suspended from each side, the vessel was caused to heel so that the bob of a pendulum 20 feet long moved through a total distance of 6 inches. The distance between the boats was 50 feet. What is the metacentric height of the vessel?

5. A dredge scow, with its bucket extended and empty, floats on an even keel. When the bucket has a load of 5 tons at a distance of 25 feet, from center line of scow, a pendulum 12 feet long swings 4 inches. Find the metacentric height. Weight of scow and load = 1,000 tons.

6. Given a scow 36 feet wide, 15 feet high, and 60 feet long. The draught is 9 feet. If its water line is tilted 6 feet above the water surface, find for this position: —

(*a*) Location of its center of buoyancy.
(*b*) Metacentric height.
(*c*) Righting couple.

CHAPTER VI

FUNDAMENTAL PRINCIPLES OF HYDROMECHANICS

74. Introduction. The fundamental laws of mechanics form the basis of hydraulic formulas. Such of the laws of mechanics [1] as are fundamental to hydraulics, briefly stated in the following paragraphs, will be further elaborated in their proper connection, when necessary.

75. The Velocity of Falling Bodies. The velocity V acquired in falling from rest through a height h under the action of gravity alone is

$$V = \sqrt{2gh} = 8.02\sqrt{h} \qquad \text{feet per second.}$$

If the body has an initial vertical downward or upward velocity V_0 feet per second,

$$V = \sqrt{2gh \pm V_0^2} \qquad \text{feet per second.}$$

76. The Space Traversed by Falling Bodies. A body falling from rest under the action of gravity alone will in t seconds fall a vertical distance

$$h = \tfrac{1}{2}gt^2 \qquad \text{feet.}$$

A body moving with an initial vertical downward or upward velocity V_0 will in t seconds travel a vertical distance

$$h = \tfrac{1}{2}gt^2 \pm V_0 t \qquad \text{feet.}$$

77. The Path of a Body when the Initial Velocity is not Vertical. If the initial velocity V_0 is not vertical, but has a direction $\theta°$, the vertical distance traversed is

$$h = \tfrac{1}{2}gt^2 \pm V_0 t \sin\theta \qquad \text{feet;}$$

and the horizontal distance is

$$x = V_0 t \cos\theta \qquad \text{feet.}$$

In these formulas h and x are the coordinates of a point in the path of a moving body, in hydraulics, the path of a jet of water, as from an orifice, a nozzle, or a weir.

[1] It is assumed that the student is familiar with the laws of mechanics; the formulas are here stated merely for convenience of reference.

78. Velocity Head. The vertical distance h_v which a body must fall under the action of gravity alone, in order to acquire a velocity of V feet per second, is

$$h_v = \frac{V^2}{2g} = 0.01555\ V^2 \qquad \text{feet.}$$

79. Kinetic Energy of Translation. The kinetic energy E contained in a body of weight, W pounds, moving with a linear velocity of V feet per second, is

$$E = \frac{WV^2}{2g} \qquad \text{foot-pounds.}$$

80. Kinetic Energy of Rotation. If a body of weight W pounds is rotating at the rate of n revolutions per second, and its radius of gyration is a feet, the linear velocity V of the point at the end of the radius of gyration is $V = 2\pi na$; and the energy of the body is

$$E = \frac{WV^2}{2g} = \frac{2W\pi^2a^2n^2}{g} \qquad \text{foot-pounds.}$$

81. Energy of Position. Potential Energy. Since a weight of W pounds, in falling from rest through a vertical distance h, will acquire a velocity $V = \sqrt{2gh}$, the energy which it is capable of acquiring, *i.e.* the potential energy of position, is $E = Wh$ foot-pounds. The distance h is called the ***head*** due to elevation and is denoted by h_e.

82. The Force Required to Produce Acceleration or Retardation. Acceleration or retardation is the rate of change of velocity. A constant force, F pounds, which, acting on a body weighing W pounds, will cause it to change its velocity dv feet per second in an interval of time dt, may be found as follows:

$$F = \frac{W}{g}\frac{dv}{dt}.$$

If the velocity changes uniformly with time, this may be written in the form

$$F = \frac{W}{g}\left(\frac{V_1 - V_0}{t}\right),$$

or, if t = one second,

$$F = \frac{W}{g}(V_1 - V_0),$$

where V_0 denotes the component of the original velocity parallel to the line of action of F, and V_1 denotes the component of the final velocity parallel to the line of action of F.

83. Centrifugal Force. The force required to restrain a body of weight W from moving out of a circular path of radius ρ, is called the centrifugal force, F_c. The centrifugal retardation, dv/dt, is equal to V^2/ρ. Therefore

$$F_c = \frac{WV^2}{g\rho} \quad \text{foot-pounds.}$$

84. The Principle of Work. A body can be transferred into a new position, or changed in form or size, or undergo a change in velocity, only by overcoming resistances which oppose the change. This process is called doing work. The amount of work done is equal to the energy expended, irrespective of whether it is usefully applied, and is measured by the total resistance. It is equal to the force exerted, F pounds, multiplied by the space, S feet, through which resistance is overcome. This may be written in the form

$$\text{Work done} = \text{energy exerted} = FS \quad \text{foot-pounds.}$$

The change of velocity from V_0 to V_1 feet per second, or equivalent changes in elevation or pressure, in a distance S, requires a force F, or an expenditure of energy of FS foot-pounds, equal to the change in energy of the weight W. Therefore

$$FS = W\frac{(V_1^2 - V_0^2)}{2g}.$$

85. Application of the Principle of Work to Falling Water. If in the preceding formula the initial velocity V_0 is zero, V_1 is the final velocity, S is equal to h, the distance through which water falls, and F is the force of gravity W, then

$$FS = Wh = \frac{WV_1^2}{2g}.$$

Even though the time required to fall through the distance, or to change the velocity or pressure, may require several seconds, if W pounds of water are changed every second in elevation, velocity, or pressure, the formula represents the work done every second.

86. Horsepower. The rate of expenditure of energy is usually stated primarily in foot-pounds per second, or in horsepower HP; one horsepower equals 550 foot-pounds per second. Hence

$$\frac{FS \text{ per second}}{550} = \text{horsepower.}$$

Take, as an example, a waterfall 31 feet high discharging 217 cubic feet per second. We shall have

$$\frac{62.4 \times 217 \times 31}{550} = 764\ HP.$$

87. Conservation of Energy. When there are no frictional resistances, the energy expended in doing work is not lost, but merely transformed. Energy exerted in a body changing its motion or position is represented by an exact equivalent amount of energy in some form in the body itself. Hence, if the body does not receive or part with energy, the total energy is a constant. This may be expressed as follows:

Total energy = kinetic energy + potential energy = constant.

This law appears in hydraulics as Torricelli's and Bernoulli's theorems.

88. Torricelli's Theorem. In 1643, Torricelli enunciated the theorem that "*the velocity of a fluid passing through an orifice in the side of a reservoir, is the same as that which is acquired by a body falling freely in vacuo from a vertical height, measured from the surface of the fluid in the reservoir to the center of the orifice.*"

89. Bernoulli's Theorem. In 1738, Bernoulli showed that the law of the conservation of energy is applicable to the flow of fluids. Bernoulli's theorem may be stated thus:

At every section of a continuous and steady stream of frictionless fluid, the total energy is constant; that whatever energy is lost as pressure is gained as velcoity. Therefore, in terms of head,

Total energy = velocity head + pressure head + head due to position = constant: or, in every stream section,

$$H_T = h_v + h_p + h_e = \frac{V^2}{2\,g} + \frac{p}{w} + h_e.$$

H_T is the total head.

h_e is the vertical distance from any reference plane to the point at which p is measured.

Note that p may be more or less than atmospheric pressure.

Consider a stream of frictionless fluid having a steady volume of flow of W pounds per second. Let A and B represent any two consecutive points in the stream.

At A the velocity is V_1 feet per second; the pressure head, h_{p_1} feet; and the elevation of A, h_{e_1} feet. At B, V_2 is the velocity, h_{p_2} is the pressure head, and h_{e_2} is the elevation.

At section A the total energy is

$$W\left(\frac{V_1^2}{2g} + h_{p_1} + h_{e_1}\right) = WH_T;$$

At section B the total energy is

$$W\left(\frac{V_2^2}{2g} + h_{p_2} + h_{e_2}\right) = WH_T.$$

Therefore

$$\frac{V_1^2}{2g} + h_{p_1} + h_{e_1} = \frac{V_2^2}{2g} + h_{p_2} + h_{e_2} = H_T.$$

This formula, properly modified to include the effect of frictional resistances, is the basis of all theoretical formulas for the flow of water.

90. Frictional Resistances Modifying Theoretical Formulas. To the movement of water are opposed frictional resistances, so called, of which neither the character nor the laws governing their action have been precisely defined. Many experiments, some highly trustworthy, others less reliable, have been made for the purpose of determining the laws governing fluid friction. So far, these investigations have made available a large mass of invaluable information, but have produced laws of only limited applicability. From these researches, however, hydraulic coefficients covering a wide range of conditions have been derived. Methods of reducing these resistances have also been learned, as well from practice as from experimental research. Theoretical formulas expressing the fundamental principles of hydromechanics are the basis of practical formulas, serving not only to determine the character of problems, but also the limiting value of results. Practical formulas are, therefore, theoretical formulas modified by numerical coefficients derived from practice and research, — or, in other words, empirical formulas.

91. The Hydraulic Grade Line. If, on the profile of a channel, vertical lines be drawn to represent graphically the pressure head at every section, a line connecting the upper ends of these is called the hydraulic grade line, or hydraulic gradient.

The elevation of the hydraulic grade line above a channel at any point is the measure of the amount of pressure at that point. In an open channel the hydraulic grade line is the water surface.

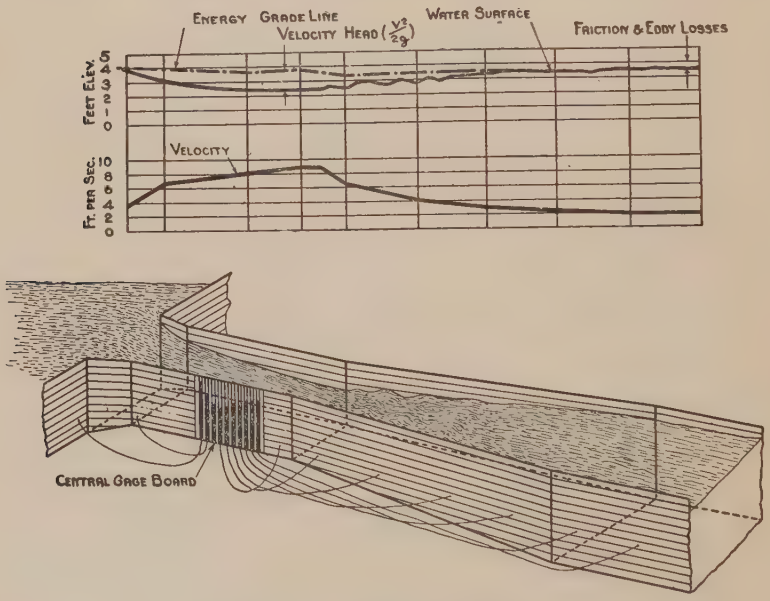

Fig. 21. — Venturi Flume

In the case of a pipe, at some point on the line, the hydraulic grade line may pass below the pipe, in which case the pressure at that point is less than atmospheric. Under such conditions siphon action takes place. Should the grade line pass more than some 34 feet below the pipe, the siphon would break.

It is always desirable to keep a pipe below the hydraulic grade line, but it sometimes is necessary to extend the pipe above it. The most common example of this is the siphon, where the high point is well above both the ends. In large siphons it is necessary to make provision for removing the entrapped air which collects at the high point.

A Venturi flume, with flaring sides and bottom, is shown to illustrate Bernoulli's Theorem.[1] Water is supplied through a bell-mouthed entrance to a uniform section. The velocity gradually increases until it reaches a maximum at the lower end of the uniform section. In order to obtain this velocity the original static head is drawn on, with a resulting lowering of the water surface. In the case shown the maximum velocity was 9.2 feet per second, and the water surface stood 1.55 feet below the original level. The water, moving at this high velocity, leaves the end of the uniform section and enters one whose walls and floor flare outward with a 1 in 10 slope. The velocity gradually decreases with a consequent raising of the water surface as the velocity head is converted back into pressure head. In the case shown 1.10 feet of the original draw down were regained; the remaining 0.45 foot being lost in friction and eddies.

Pressure head can always be converted into velocity with very little loss, but the reverse is not true; both careful design and skillful execution are necessary to convert velocity into pressure head efficiently.

PROBLEMS

1. The water tank of a locomotive has its intake 9 feet above the water in the feed trough placed in the middle of the track. What is the least speed in miles per hour which will project water through a scoop into the tank (*a*) neglecting frictional resistances; (*b*) assuming that frictional resistances absorb enough energy to raise water 7 feet?

2. Show by a sketch and assumed values of your own the application of Bernoulli's theorem to the flow of a frictionless fluid in a closed channel (pipe under pressure).

3. A jet of water flows from a nozzle with a velocity of 30 ft. per sec., at an angle of 50° from horizontal. Assuming no air resistance, determine its path.

4. If 100 pounds of water per second discharge from a nozzle at a velocity of 40 feet per second, what gross horsepower is available?

5. If 2000 pounds of water per second are reduced in pressure in flowing through a turbine from 40 pounds per square inch above to 10 pounds per square inch below atmospheric pressure, compute the energy expended and the gross horsepower.

6. What force must be exerted to change the velocity of 1000 pounds of water from 20 feet per second to 5 feet per second in one second?

7. A stream of water, 3000 cubic feet per second, falling through 40 feet, will make available how much gross horsepower?

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[1] This is a simplified sketch of a flume built by the Proprietors of the Locks and Canals, at Lowell, in 1897, for experiments on velocity regain.

CHAPTER VII

THE FLOW OF WATER

92. Flow of Water. The flow of water is caused either by the action of gravity or by differences in pressure at succeeding points in the stream. There are four forms of flow: flow in a free jet, in an open channel, in a closed channel, and capillary flow.

93. Free-Jet Flow. Free-jet flow occurs when a stream of water is surrounded on all sides by a gas. The simplest form of this is a nozzle discharging into the open air. If such jet flow, instead of being into the open air, or a gas, takes place under water, so that the jet is surrounded by water on all sides, the flow is said to be *submerged.*

94. Open-Channel Flow. Flow in an open channel is the commonest kind of flow met with in nature. It occurs when water flows in a surrounding channel or stream bed with its surface free and exposed only to atmospheric pressure. All stream flow and flow in open canals comes under this head.

95. Closed-Channel Flow. Flow in a closed channel occurs when the stream is surrounded by bounding surfaces on all sides and has no free surface exposed to the atmosphere. The commonest form of this is flow in pipes under pressure.

96. Capillary Flow. Capillary flow is essentially flow in minute closed channels at an extremely low velocity. Its most common example is the flow, or percolation, of water through beds of sand or gravel. This is met with in nature in the case of an underlying bed of gravel or other pervious material through which a stream of water slowly flows. A stream sometimes has a bed of pervious material through which a portion of the flow of the stream makes its way by percolation.

97. Steady and Unsteady Flow. For any period of time during which the area of a stream cross section, the pressure, and the direction and the mean velocity of flow in that section are all constant, the flow is said to be steady; that is, the discharge is constant for successive equal intervals of time. If the discharge varies with time, it is unsteady. True steady flow is almost

never obtained; it is merely more or less closely approximated. Conditions of steady flow, although often lasting but a short time, are usually assumed in computations of discharge.

98. Stream-Line Flow. Under ideal conditions water moves in stream-line flow. In this form of flow each particle of water follows, and is followed by, another particle, all moving along a definite path at a definite velocity. The whole stream can be divided up into an infinite number of lines of flow, along each of which the above condition holds true.

99. Critical Velocity. Unfortunately stream-line flow is seldom met with in practice, occurring only when the velocity is very low. If a stream of water, starting from rest, is gradually put into motion, stream-line flow will result. If the velocity of the stream is gradually accelerated this condition of stream-line flow will continue up until a certain velocity is reached, when the flow will suddenly break and become turbulent, continuing so at any higher velocity. The velocity at which this change in flow occurs is called the *critical velocity*. If a stream of water in turbulent flow is gradually slowed down, a point will be reached where the flow again becomes stream-line. This critical velocity is considerably lower than the former for a given stream. It is known as *the lower critical velocity*, as opposed to the first mentioned, or *higher critical velocity*.

These critical velocities were first determined by Osborne Reynolds,[1] who injected a thin stream of dye into water flowing through glass tubes. He derived an expression for the critical velocity in terms of the tube diameter, but his experiments were on a small scale with tubes not over two inches in diameter. Later experiments have shown that, at best, the critical velocity is rather indeterminate except by actual experiment.

Flow at velocities less than the critical is seldom met with in practice. Flow through sand is, of course, flow at a point below the critical velocity, but except for this practically every case met with in applied hydraulics consists of dealing with non-stream-line turbulent flow.

100. Turbulent Flow. Practically speaking, outside of the laboratory, uniform stream-line flow is never met with. Instead

[1] Osborne Reynolds, PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY, 1883.

the motion is found to be turbulent and disturbed, surging backwards and forwards, and from side to side. To the eye the flow may appear to be uniform and steady, but an examination will show that the motion consists of a series of surges and pulsations (Fig. 22). The lines of flow are sinuous, weaving back and forth over and through each other.

One might think that under such conditions of turbulent flow it would be impossible to measure the flow with any degree of

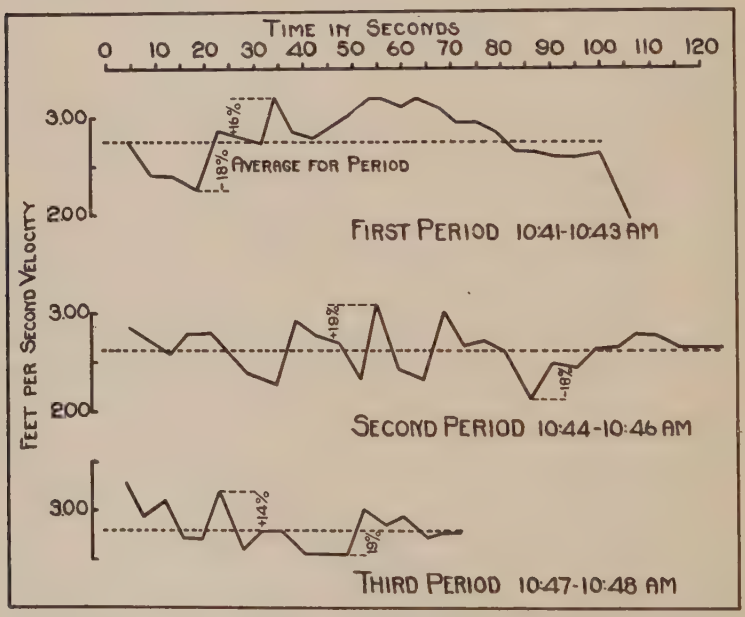

FIG. 22. — CURVES SHOWING VARIATION IN VELOCITY IN A LARGE POWER CANAL. THE CURRENT METER WAS HELD AT A DEFINITE POINT DURING EACH PERIOD

accuracy. If the flow is steady, and this condition is usually assumed in computations of discharge, it will be constant for successive equal intervals of time. These intervals of time cannot be short, say a few seconds, because due to turbulence the flow may vary greatly during so short a period. If, however, a considerably longer period is taken, the variations due to turbulence will balance themselves, and the observation will be the average of all the variations and a very close approximation of the true average flow. It is desirable, therefore, that hydraulic

observations should extend over as long a period of time as is compatible with the limitations of the measurement, and the requirement that the flow remain essentially steady.

101. Resistance to Flow. Water in motion must overcome the friction against the walls of the channel through which it flows, as well as its own internal friction. Since water is a viscous fluid there necessarily must be internal friction even under conditions of stream-line flow. The velocity along the various stream lines of flow will vary across the cross section of the stream and friction will result between neighboring particles, varying with the difference in velocity between the two particles. Under conditions of the turbulent flow the internal friction is much increased and energy is dissipated in surging, boiling and eddying.

Bernoulli's theorem has shown that the velocity head of water in motion can be converted back to pressure head, but the energy dissipated in internal friction goes into heat and never can be regained for use.

Resistance to flow is independent of the pressure exerted by the water on the wetted perimeter. It is approximately proportional to the total area of rubbing surface; that is, the length of the channel multiplied by its wetted perimeter.

Resistance increases with increased roughness of channel; as some power of the velocity, first or higher; with an increase in the suspended matter in the water; with abrupt changes in the cross-sectional area of the channel; at bends or at junctions with other channels.

Resistance decreases with a rise in temperature.

102. Flow Phenomena. Under theoretical conditions velocity head and pressure head are interchangeable. A frictionless fluid issuing vertically from an orifice would shoot up to the level of the liquid in the container from which it issued. Actually, of course, this is not true, due to the viscosity of the liquid and the friction of the air, but, generally speaking, velocity head and pressure head are freely interchangeable, and with but little loss

If a body of water in motion is suddenly stopped, the velocity head in the water will cause it to pile up at the point of stoppage, but in addition to this the water behind acts through its momentum, and causes this piling up action to increase very materially. Action of this sort occurs in canals and waterways, when a gate

is suddenly closed. It is more commonly met with in pipes and conduits under pressure. Here instead of the water surface being raised, the pressure increases. This sudden pressure rise, which is often accompanied by a dull pound, is known as water hammer. In large power penstocks it becomes of great importance, and provision must be made for pressure relief or considerable damage will result, either in bursting the penstock or destroying the power house.

103. The Hydraulic Jump. A body of water moving in an open channel at a high velocity and small depth may, under the proper conditions, suddenly slow down in velocity and increase correspondingly in depth. This may occur in a very short distance and have the appearance of a standing wave. It can often be noted below a spillway dam where the sheet of falling water strikes the dead water on the downstream side of the dam. It can be produced artificially in a channel of uniform size and slope by placing an obstruction on the bottom. There must be a certain relation between the depths before and after the jump, and the velocities. There are for any given channel and quantity of water flowing in it, two alternative depths at which the water may flow. It is the transition from the stage of low depth and high velocity to that of higher depth and lower velocity that results in the hydraulic jump.

104. The Vortex. A mass of water revolving about an axis in the water itself is known as a vortex. This often forms over the outlet through which water is being drawn from a receptacle. Turbulent conditions of flow often cause vortices in water flowing in open channels, but these usually last only for a short time, while the first case mentioned will persist as long as the water continues to flow through the orifice.

If the axis of the vortex curves and returns upon itself, a vortex ring results. This is exactly similar to smoke rings blown out into the air.

In a vortex the particles of water approach the axis in a spiral direction in such a way that the velocity of a particle of water multiplied by its distance from the axis is a constant. The vortex is chiefly of importance through its connection with the water wheel and centrifugal pump design, where flow of this sort occurs.

When water flows by an obstruction, such as a bridge pier, or around a sharp corner, eddies are formed. They may often take

the form of small vortices, accompanied by surging and boiling. Such vortices will not long persist, but the generally disturbed condition will continue for some distance beyond the obstruction. Eddies can largely be avoided by making obstructions stream-line in shape and eliminating all sharp corners.

105. The Measurement of Flowing Water. While at first sight the measurement of water seems to be very simple, it actually is one of the most difficult tasks met with in engineering. The flow of water is not governed by any exactly determinate laws such as those that govern the flow of electricity, nor is it susceptible to an exact mathematical determination. The various methods of water measurement used today are the result of experiment and gradual development, and all, except the bulk and weight methods, are almost entirely empirical.

106. Measurement by Bulk. The easiest, simplest and also the most exact method of measuring water is by bulk. The flowing water is directed into some sort of receiving basin of known dimensions. It is very seldom that this method can be used outside of the laboratory, particularly when a large quantity of water must be handled. In the case of a reservoir or large forebay this method can sometimes be used. A very exact contour map of the basin is made, and the volume between successive elevations is determined. A known rise or fall in the water level can easily be converted into volume of water added or subtracted.

107. Measurement by Weight. Measurement by weight is necessarily restricted to the laboratory. The water is directed into a receptacle placed on the platform of a set of scales. The amount of water flowing in a given time is weighed directly and this weight is converted into volume. For small quantities of water this method is quite as accurate and often more convenient than the volumetric.

108. Measurement by Orifice. Water can be measured by passing it through an opening of known size and shape and noting the head required. The simplest form is a round hole in a flat plate, discharging into the open air. In this case the head is measured to the center of the opening.

A fire nozzle is merely another form of orifice. It can be exactly calibrated and makes an excellent water meter. The nozzle of a Pelton wheel is identical in principle with a fire nozzle.

A weir is an opening in a partition wall through which water flows, touching the edges of the opening on its sides and bottom, but having its upper surface free and coincident with the level from which the water is discharging. The weir is probably the most nearly standard method of measuring water. It is, however, too often used in a slipshod and unscientific manner. It must be remembered that the use of the weir is based entirely upon the results of experiment, and that, in order to secure exact results, the conditions of one of these experiments must be duplicated. Used correctly under correct conditions the weir is probably one of the most exact methods of measuring water. Expense and physical conditions do not, unfortunately, allow the weir to be much used outside of the laboratory, except for small volumes of flow.

A reaction water wheel is nothing but a number of orifices discharging under a common head. They are usually submerged on the downstream side, but that does not alter the principle. A water wheel can be calibrated by a number of measurements at different heads and conditions of gate opening, and it then becomes a very exact water meter.

109. Velocity and Cross Section. This method is based upon measuring the cross-sectional area of the flowing water on a plane perpendicular to the direction of flow and of determining the mean velocity of flow over this area. Beautifully simple in principle, this method becomes much more complicated in practice since neither the area nor the mean velocity can be determined exactly, but must be approximated to a greater or lesser degree. In this method the area of the section below the water surface is determined as exactly as possible. This is then divided up into a number of arbitrary sections, and the average velocity over each of these sections is measured by one of several methods. The average of the velocities at these sections is taken as the mean velocity and this, multiplied by the area of the section, gives the quantity of water flowing. The velocities in the different parts of the total cross section may be determined by any of several alternative methods, but, except for the actual method of velocity determination used, the principle for all is the same. This method of measurement is that most used in practice, and, while not as exact as most of the other methods, when correctly used it is capable of a precision sufficient for most requirements.

Floats placed in moving water and timed while traversing a given distance can be used to measure the velocity. The floats may be surface, subsurface, or rod floats, almost as long as the depth of the water, and weighted so that they float vertically. The use of floats requires a long straight section of channel of uniform cross section.

A current meter is a small wheel shaped either like a screw propeller or a cup anemometer, suspended on bearings in a frame with some sort of mechanism recording the revolutions of the wheel. In use the current meter is placed at the point where the velocity is desired to be measured, and the revolutions in a given period of time recorded. There is, for every meter, some definite relation between the revolutions over a period of time and the velocity of the water causing the meter to rotate. This relation must be determined by calibrating, or "rating," the meter. In most meters it is practically constant, varying slightly with the wear on the bearings. By applying the results of the rating to the revolutions per second of the meter, the velocity of the water is obtained.

The Pitot tube, first used by the scientist of that name in 1730, consists, in its simplest form, of a small tube open at both ends, and with one of its ends bent at right angles to the main portion. If such a tube is immersed vertically in water, with the bent part under water and pointing directly upstream into the current, the water will rise in the main portion of the tube until it stands at an elevation above the outside water surface equal to the velocity head of the current.

A Pitot tube can be used to measure velocities in a canal in the same way that a current meter is used, but its use is generally restricted to pipes and conduits under pressure, where its simplicity and small size are particularly favorable.

A traveling screen may sometimes be used in a stretch of small canal of very uniform section. A small car, running on a track parallel to the canal, carries a light canvas screen which fits the cross section of the canal with a minimum of clearance. The velocity of the car agrees very closely with the mean velocity of the water, but a slight correction must be made for leakage between the edges of the screen and the canal walls. This method, while said to be very accurate, is restricted to a very few special uses.

Color gaging may often be used with very successful results. A small portion of dye injected into the water may be observed visibly and its velocity recorded. One must be careful, however, to avoid recording the maximum velocity, rather than the mean, and for this reason this and the method next to be described are mostly restricted to use in pipes.

Studies of the velocity distribution in pipes almost always show that the velocity is highest at the center, falling off toward the sides. One would think that if a chemical were injected into the pipe that it would be carried along by the highest velocity at the center, while the outer parts trailed behind. Experiments disprove this, and show that the portion of water containing the chemical remains in a more or less solid slug, and does not extend lengthwise for any great distance. This can only be accounted for by the supposition that sufficient turbulence is present in the pipe to cause the flow to surge from side to side so that all the particles are carried along at a mean velocity rather than those at the center being carried at the maximum velocity, while the outer particles move much more slowly.

Salt velocity gaging is similar in principle to the color velocity method. Each of two pairs of electrodes located at different points along a pipe or penstock is connected to a battery and sensitive galvanometer. A solution of ordinary salt passing down the pipe will cause the galvanometers to deflect successively, and the time of transit can be readily noted. This method has the advantage over color gaging that less personal error is introduced, and the exact time of transit is more easily determined. Moreover it can be used in places where the appearance of coloring matter could not be observed, as in pipes with a submerged outlet.

Salt-solution gaging. A salt solution of known strength is added to the water to be measured, and samples of the water are taken at a point some distance downstream. Analysis of the amount of salt in the samples will show the amount of water necessary to secure the resulting dilution, which is the quantity of water flowing. This method of gaging is particularly applicable to turbulent waters. It is very complicated and expensive, but seems to give excellent results.[1]

[1] See paper by B. F. Groat, *Chemi-hydrometry*, TRANSACTIONS A. S. C. E., 1916.

CHAPTER VIII

THE PITOT TUBE—PIEZOMETERS—THE PITOMETER

110. The Pitot Tube. If a tube of glass or smooth metal, open at both ends and bent through 90°, is held in running water with one leg horizontal, parallel to flow and opposed to the current, and the other leg vertical and open at the top (position *A*, Fig. 23), the impact of the moving water will cause a column of water

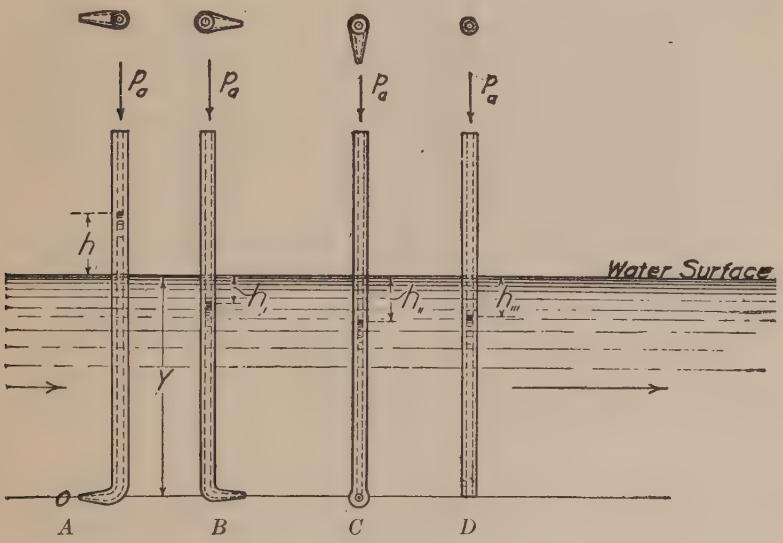

FIG. 23

to stand in the vertical leg at some height h above the surrounding water surface. This height h depends upon, and should theoretically be proportional to the square of, the velocity at the point O; the height of the water column $Y + h$ is therefore a measure of the pressure head plus the velocity head at the point O.

Experiments [1] have proved that in tubes having small points so shaped as not to disturb the flow, with openings of cylindrical

[1] Darcy, ANNALES DES PONTS ET CHAUSSÉES, 1858, 1er semestre; Bazin, ANNALES DES PONTS ET CHAUSSÉES, 1890, 1er semestre; White, JOURNAL ASSOCIATION OF ENGINEERING SOCIETIES, 1901; Gregory, TRANSACTIONS A. S. M. E., December, 1903; Williams, Hubbell, and Fenkell, TRANSACTIONS A. S. C. E., 1902.

or converging or diverging cone form, h very nearly equals $V^2/(2\,g)$. Therefore with such forms $V = (2\,gh)^{1/2}$. If h can be measured, the velocity at any point may be calculated.

In 1730 Pitot [1] was the first to use such a device to determine the velocity of flow; hence the name. His apparatus consisted of a bent cylindrical tube with one leg horizontal and its orifice opposed to the current, and a straight vertical cylindrical tube, set with its bottom opening at the same level as the orifice of the bent tube (tubes A and D, Fig. 23). Pitot assumed that V was equal to $(2\,gh)^{1/2}$, where h denotes the difference in level in the two vertical tubes.

This apparatus did not give a true measure of the velocity head h because the vertical tube is not a true piezometer and did not accurately measure the pressure head Y.[2] Pitot's tube was in fact little more than a scientific toy until about 1850, when Darcy made practical Pitot tubes which he and Bazin used in their experiments upon the flow of water.

111. Darcy's Experiments. Darcy determined by experiment that the three chief difficulties in the way of the practical use of Pitot's tube were [3] as follows:

(1) The oscillations of the water surfaces in the tubes and the disturbances in the stream due to the tubes make an accurate measurement of the head h difficult. If the velocity is feeble, any differences in elevation may entirely disappear.

To reduce oscillations and disturbances as much as possible, Darcy made the orifices much smaller than the tubes, the diameter of the orifices being .0015 meter and of the tubes .01 meter.

(2) An observer looking at the tubes from above cannot easily measure with his eye the differences of water levels, especially if they are slight.

To facilitate reading, Darcy designed an apparatus as shown in Fig. 24. Two vertical glass tubes, one connected to the impact tube, the other to the pressure tube, are set parallel to each other; at G the tubes unite in one; and at this point is a valve. At H and I are two valves both on one stem. Between the tubes is placed a graduated scale. To operate, suction is applied above

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[1] Mémoires de l'Académie, November, 1732.

[2] See Darcy, Annales des Ponts et Chaussées, 1858, 1er semestre, p. 353.

[3] Annales des Ponts et Chaussées, 1858, 1er semestre, pp. 351–359.

G which rarefies the air in both vertical legs; G is then closed, and simultaneously H and I. The tubes are then lifted out of the water and read.

(3) A tube projecting into a stream in any direction does not give a true measure of the pressure head Y (Fig. 23). Hence to determine the relation between the measured difference in water levels and the velocity, a coefficient for the tube must be determined by calibration.

The instrument shown in Fig. 24 was rated by three different methods (comparing with floats, moving through still water and measuring a known flow) and was found to have a coefficient very close to unity.

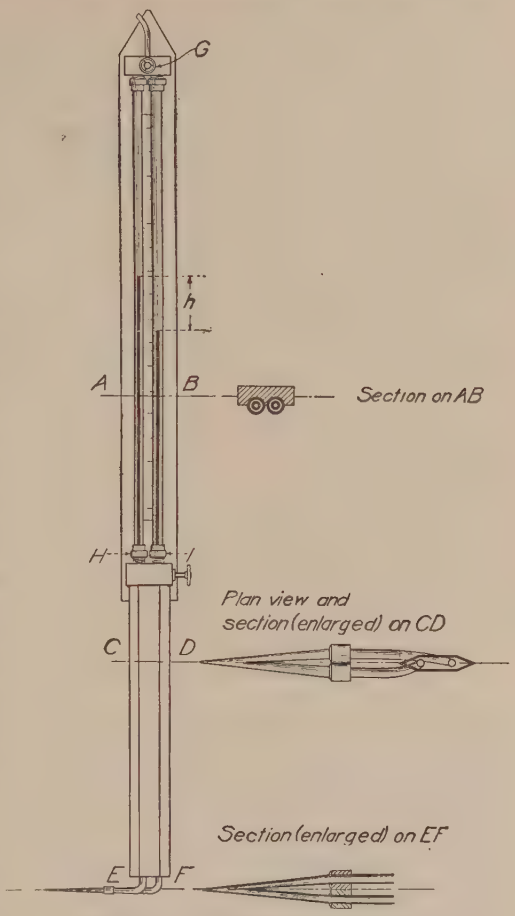


FIG. 24. — DARCY'S PITOT TUBE

112. Bazin's Tube. Bazin, in his weir experiments,[1] used a Pitot-Darcy tube having an impact orifice normal to the current and a lateral orifice opening without any projection in a surface by which the stream flowed tangentially. The tubes were inset on a thin board beveled to a sharp top edge, which Bazin stated was similar to Darcy's apparatus. It had a coefficient K equal to 1. See Fig. 25.

[1] ANNALES DES PONTS ET CHAUSSÉES, 1890, 1er semestre, p. 54.

113. Mills's Experiments upon Piezometers. Hiram F. Mills [1] published in 1878 the results of some six thousand observations made with extraordinary accuracy to determine the proper form of piezometer orifice. With twenty-two openings varied in shape and direction and a range of velocities from 0.6 to 8.9 feet per second, he found that with *an orifice whose edges are in the plane of the side of the channel and passage normal thereto, the piezometer column will indicate the true height of the water surface in an open*

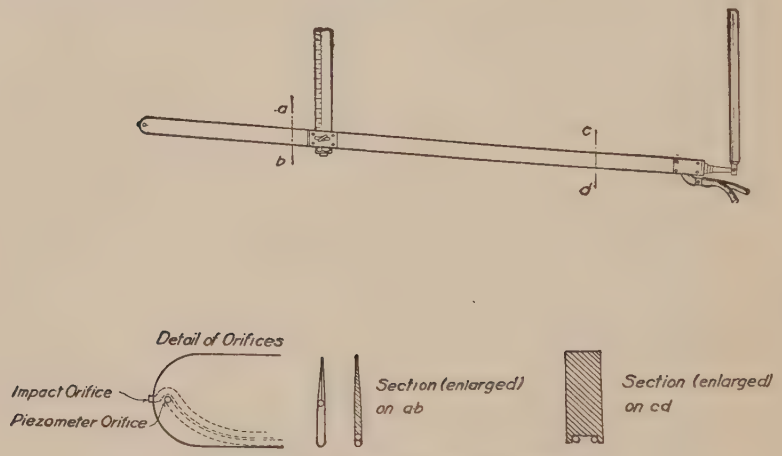


FIG. 25. — BAZIN'S PITOT TUBE

channel or the pressure in a closed channel. But if the passage inclines either upstream or downstream, or the edges of the orifice project beyond the plane of the side, the true height of the surface or the true pressure will not be indicated.

114. Conclusions from Various Experiments. The conclusions to be drawn from the experiments mentioned above, and from later American experiments of White,[2] Gregory,[3] Williams,[4] and others, may be stated as follows:

(1) That with almost any form of small circular impact orifice and a pressure orifice at right angles to and flush with a surface parallel to the stream flow, the difference in level in two vertical

[1] Chief Engineer, Essex Co., Lawrence. TRANSACTIONS AMERICAN ACADEMY OF SCIENCE, 1878.

[2] JOURNAL ASSOCIATION OF ENGINEERING SOCIETIES, 1901.

[3] TRANSACTIONS A. S. M. E., December, 1903.

[4] TRANSACTIONS A. S. C. E., 1902.

tubes connected to these orifices is very nearly the head due to velocity.

(2) That the coefficient K of a given tube is constant for all velocities.

(3) That relatively slight inaccuracies in pointing the piezometer orifice may result in considerable errors; for this reason there is, under certain conditions of use, difficulty or uncertainty in measuring the pressure head. A bent tube with its orifice pointing directly *downstream* (position B, Fig. 23) may often be used to advantage instead of a true piezometer; because, although the height of the column of water which will rise in such a tube will be less than the true pressure head Y, it will have a regular relation to the velocity head; and because the apparatus is not very sensitive to deviations in pointing. Such a tube combined with an impact tube is called a ***pitometer.*** The coefficient K of a pitometer is considerably less than 1.

(4) That in measuring velocities in a jet of water, only an impact tube is required; and its coefficient may be taken as 1.

(5) That, especially in measuring low velocities, some form of differential gage is required in order to magnify the difference of water levels in the two tubes so that it can be read (§ 41).

(6) That all forms of tubes must be rated to determine the coefficient K, and to test the accuracy of construction, and periodically rerated to detect effects of wear.

115. Rating Pitot Tubes in Still Water. Tubes have been successfully rated by moving them in still water at a known speed, the velocity V, at which the tube is moved through the water in feet per second, being assumed to have an effect in producing head h equivalent to an equal velocity of running water. Therefore $K = V/(2\,gh)^{1/2}$.

116. Rating in Running Water. Present practice, however, favors a rating in moving water under conditions as nearly as possible like those in which the measurement of flow is to be made. Readings of head h are taken at the center of subdivisions of equal area so spaced as to give a complete traverse of the channel, and from these are derived the values of velocities due to the heads. As K is independent of velocity, the uncorrected mean velocity V_m may be taken as the arithmetic mean of all these velocities.

The actual discharge Q is measured during the period of observation by volume, weight, or by another calibrated measuring device. The theoretical discharge Q_t equals the mean velocity V_m multiplied by the cross-sectional area A of the stream or jet; hence

$$Q_t = V_m A.$$

Therefore the actual discharge Q is

$$Q = KV_m A; \quad \text{and} \quad K = \frac{Q}{V_m A}.$$

117. Determination of Discharge. The discharge is determined as follows: (1) all the apparatus is tested to determine its accuracy; (2) calibrations are made to determine the area of the cross section of the stream, which should be subdivided into parts of equal or convenient area, in pipes most readily by circles; (3) the apparatus should be set at the desired point in the stream (for closed channels insertion is made through a stuffing box set in a corporation cock set in the pipe); (4) the tubes are connected to the gages either by stiff rubber or metal pipes; (5) the difference h between the water level in the two tubes is observed; (6) the velocity is determined for each observation as equal to $(2\,gh)^{1/2}$, and the mean velocity V_m in the stream is taken as the arithmetic mean of all the computations; (7) the discharge is computed by multiplying the cross-sectional area of the stream by mean velocity V_m, corrected by K. Therefore

$$Q = KV_m A.$$

118. The Pitometer. A commercial form of the Pitot tube is called the pitometer.[1] See Fig. 26. It consists of a rod meter, and a differential gage with suitable connections to which may be added either a revolving drum with paper for manually recording observations, or a photographic apparatus for automatic continuous records.

The rod meter consists of a brass sheath of flat oval cross section containing two $\frac{1}{4}$-inch brass tubes, each terminating in a curved phosphor-bronze orifice with a cutwater. See Fig. 27. When in use, one orifice points against the current, the other with the current, both tubes parallel with the current. For

[1] Developed by John A. Cole and Edward S. Cole, and owned by the Pitometer Co., New York.

insertion through the one inch corporation cock the tubes are set in collars which allow them to be rotated through 180°.

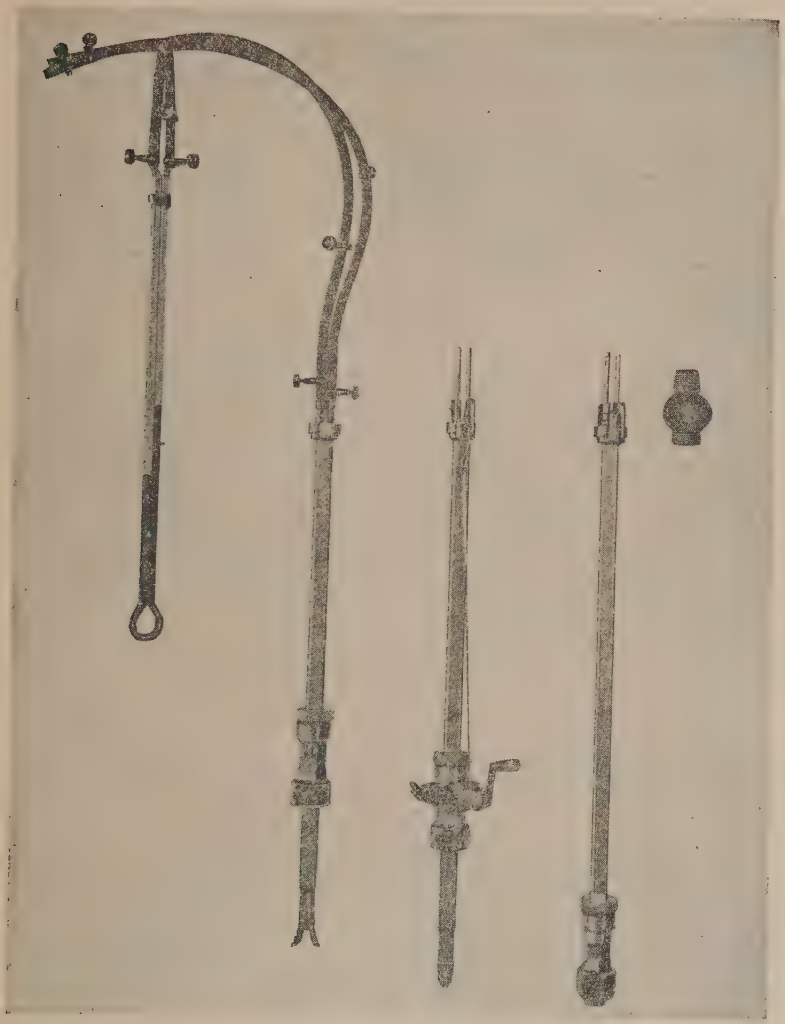


FIG. 26. — PITOMETER
(Courtesy of the Pitometer Co.)

The differential gage used for measuring the difference in pressure in the two tubes is a glass U-tube half filled with a mixture usually of carbon tetrachloride and gasoline having a specific

gravity usually of 1.25 to 1.50, and colored dark red. When in use water fills the remaining spaces. Thus the difference in pressure in the two tubes is magnified according to the specific gravity of the red liquid. Velocities as low as 0.5 foot per second may be measured. (See differential gages, Chapter III, § 40.)

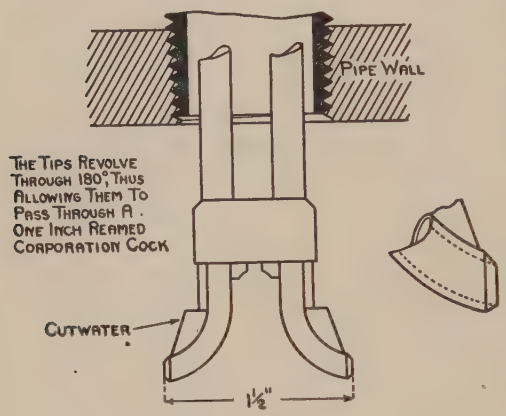

PIPE WALL
THE TIPS REVOLVE THROUGH 180°, THUS ALLOWING THEM TO PASS THROUGH A ONE INCH REAMED CORPORATION COCK
CUTWATER
1½"

FIG. 27

The coefficient K of this instrument has been found to average about 0.87.[1] The formula for velocity therefore becomes

$$V = K[2\,g(s' - 1)d]^{1/2},$$

where s' denotes the specific gravity of the heavier liquid.

If used under varying temperatures, the specific gravity of the heavier liquid as well as that of the water will change, and must be corrected to secure high precision.

If $K = 0.87$, and $s' = 1.25$,

$$V = 0.87\left(\frac{2\,gd}{4}\right)^{1/2} = 3.49\,(d)^{1/2},$$

where d denotes deflection of the heavier liquid, or the difference in elevation in feet between the tops of the two columns of the heavier liquid.

The photopitometer consists of a portable box in which a drum carrying photographic paper revolves before a fine vertical slit just in front of which is locked one leg of the U-tube. The rays of light are partly intercepted; and as the liquid rises and falls a line or band of shades is recorded on the sensitive film. The record of the pressure head in the pipe is also superimposed on the film by means of a movable finger operated by a gage having an independent piezometer connection with the pipe; or the height of the heavy liquid in both tubes may be recorded. By scaling the ordinates the velocity may be computed at any

[1] The old model pitometer with funnel-reamed tips had a K of 0.84.

time; and furthermore, the chart gives a graphic record of all changes in rate, which may at times be more useful than the absolute determination of discharge.

119. Gaging Flow in a Pipe by a Pitometer. The Traverse. As the pitometer measures only deflections due to the velocities at the point where the orifices are placed, either a number of point measurements must be made, or a ratio must be established between the velocity at the point measured, generally the center of the pipe, and the mean velocity of the entire cross section. With a constant rate of flow the velocity will vary from point to point in the stream, gradually increasing from the walls of the pipe toward the center. The mean velocity is obtained by dividing the cross-sectional area into a number of concentric rings and one central circle. The velocity in each of these sub-areas is determined by one or more observations. Each sub-area is multiplied by its velocity, and the resulting products added together give the total flow through the pipe. This, divided by the area of the section, gives the mean velocity.

120. The Velocity Curve. The deflections may be plotted as ordinates during observations on a paper stretched on the drum, or afterwards on cross-section paper and a mean curve drawn through these points. From the deflections, the velocities may be computed and a velocity curve plotted, and from this, the mean velocity may be scaled for any desired ring.

The velocity curve across a pipe is often not uniform in different planes due to the effect of curves and irregularities. If accurate results are desired, traverses should be made along at least two diameters perpendicular to each other.

121. The Pipe Coefficient. The mean velocity for the whole pipe section is the total of these ring volumes divided by the area of the pipe as shown on the diagram, Fig. 28. The mean velocity divided by the center velocity, as read from the curve, is the pipe coefficient, or "decimal." This coefficient must be obtained either with a constant center velocity [1] or from many previous observations on the same pipe; once determined for a given pipe and given conditions of lining, it may be taken as practically constant for all velocities.

[1] In determining the coefficient an auxiliary pitometer is frequently placed in tandem to indicate the center velocity, while the traverse is being made.

122. Discharge. The mean velocity of each ring in feet per second multiplied by the area of the ring in square feet gives the ring volume in cubic feet per second. The total discharge is the sum of the ring volumes of flow.

Example. The following diagram (Fig. 28) and computations illustrate the usual method of computing discharge, in this case for a 24-inch pipe.

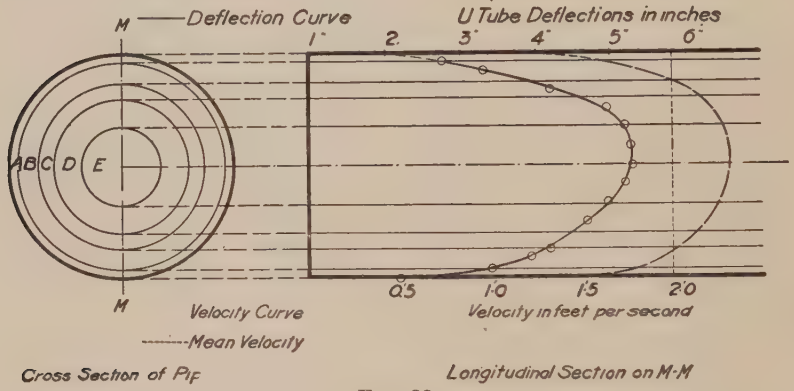


Fig. 28

Ring	Ring Areas Sq. Ft.	Ring Velocity Ft. per Sec.	Ring Volumes of Flow Cu. Ft. per Sec.	
A	0.502	1.57	0.788	
B	0.873	1.87	1.632	
C	0.698	2.05	1.431	
D	0.720	2.18	1.570	
E	0.349	2.24	0.781	
	3.142		6.202	Total discharge of pipe

$$\text{Mean Velocity} = \frac{\text{Discharge of Pipe}}{\text{Area of Pipe}} = \frac{6.202}{3.142} = 1.97$$

$$\text{Pipe Coefficient} = \frac{\text{Mean Velocity}}{\text{Center Velocity}} = \frac{V}{V_c} = \frac{1.97}{2.24} = 0.879$$

123. Deflection and Velocity Curves for Several Pipes taken from observations are shown in Fig. 29; all the pipes here shown had been in service, some for many years at the time of measurement.

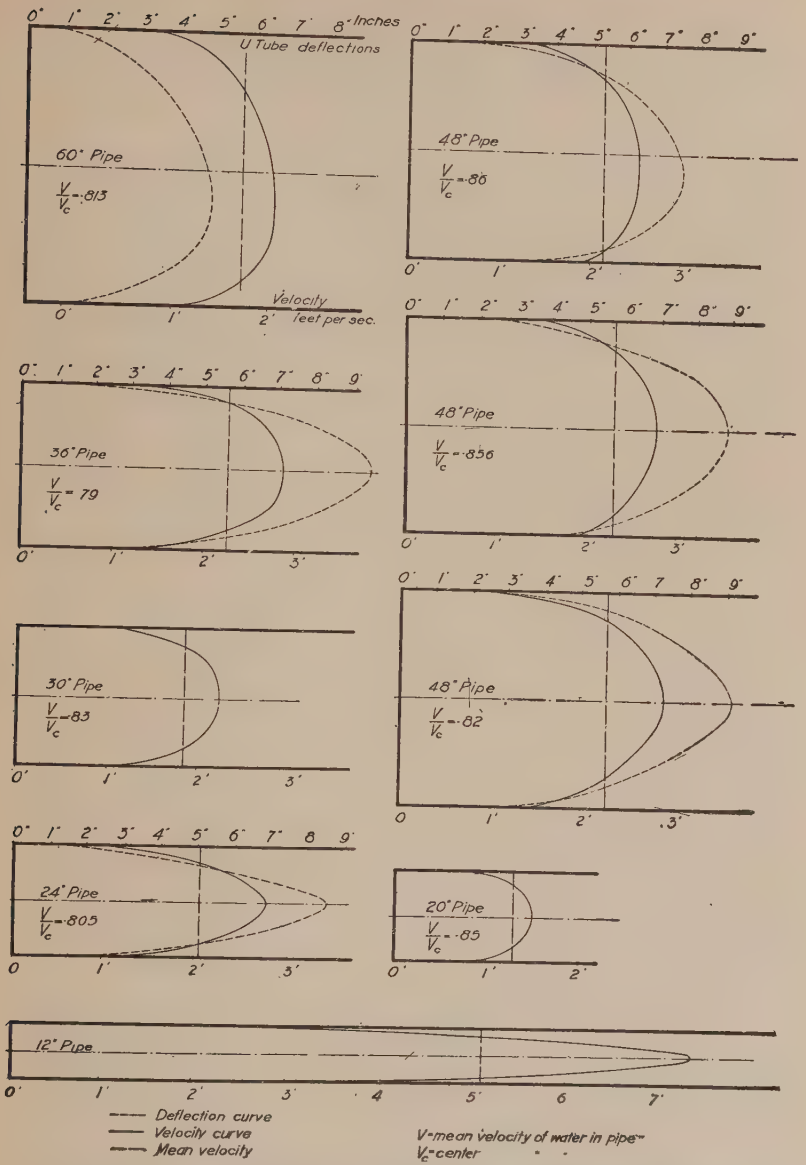

0' 1' 2' 3' 4' 5' 6' 7' 8' Inches
U Tube deflections
60" Pipe
V/Vc = .813
Velocity
0' 1' 2' feet per sec.
0' 1' 2' 3' 4' 5' 6' 7' 8' 9'
48" Pipe
V/Vc = .86
0' 1' 2' 3'
0' 1' 2' 3' 4' 5' 6' 7' 8' 9'
48" Pipe
V/Vc = .856
0' 1' 2' 3'
0' 1' 2' 3' 4' 5' 6' 7' 8' 9'
36" Pipe
V/Vc = .79
0' 1' 2' 3'
0' 1' 2' 3' 4' 5' 6' 7 8' 9'
48" Pipe
V/Vc = .82
0 1' 2' 3'
30" Pipe
V/Vc = .83
0' 1' 2' 3'
0' 1' 2' 3' 4' 5' 6' 7' 8 9'
24" Pipe
V/Vc = .805
0 1' 2' 3'
20" Pipe
V/Vc = .85
0' 1' 2'
12" Pipe
0' 1' 2' 3' 4 5' 6 7'
Deflection curve
Velocity curve
Mean velocity
V = mean velocity of water in pipe
Vc = center

FIG. 29. — DIAGRAMS OF ACTUAL PITOMETER TRAVERSES IN OLD PIPES

124. Continuous Measurements of Discharge. Having determined the "pipe coefficient," the tubes may be fixed with the orifices at the center of the pipe, and the observed center velocity, at any rate of flow, may be reduced to mean velocity by this pipe coefficient.

125. Applications. A pitometer or other form of Pitot tube may be used to measure the velocity and hence the volume of flow either in open or closed channels without interfering with the use of the channel, with relatively small expense for installation and operation. It has proved its value in detecting waste or misuse of water in street mains and other channels and where changes in rates of flow are significant, and in measuring the slip of pumping engines. In using these meters in old pipes, ample allowance for reductions of area due to incrustations must be made; and the certain change in the distribution of velocities due to incrustations and roughness must be provided for. Where possible, a number of point measurements of the velocity will produce a higher degree of accuracy than the single observation at the center and the use of the pipe coefficient. This latter method can be considered only an approximation at best.

Pitot tube measurements are the same in principle as float and current-meter measurements, and they are subject to the same inaccuracies, due principally to the turbulent flow almost always met with in pipes. A large number of observations covering a considerable period of time will usually give a reasonably accurate result. It is desirable, when possible, to make three complete traverses of the pipe and to use the average of these.

A Pitot tube may be used sometimes to gage flow in open channels, particularly when they are small. The size of the Pitot tube is such that it may be used very close to the walls and bottom of the channel and in channels too small for a current meter. Under such conditions the observation must be made with very great care.

NOTE. One must not get the impression from examining these velocity curves that flow in a pipe consists of a number of concentric ring-shaped streams, each moving smoothly along, and each inner ring moving at a slightly higher velocity than the one which surrounds it. Such flow does occur at velocities below the critical velocity, but practically all cases met with in practice are above the critical velocity, and the flow is turbulent. A given particle of water will one instant be near the wall of the pipe and moving at the slowest velocity; the next instant it will be at the center moving at the fastest. The particles will surge back and forth across the section so that all the particles will move along at an approximate mean velocity, rather than those at the center remaining there and drawing ahead of those on the sides.

PROBLEMS

1. If the specific gravity of carbon tetrachloride in a differential gage is 1.25, compute for a pitometer the theoretical velocities in feet per second corresponding to the following deflections in inches:

0.5 1.0 4.0 8.5

Also for same deflections when specific gravity is 1.5.

2. A pitometer is set with its orifices at the center of a pipe. The water level in the impact tube is at elevation 123.7 feet, in the pressure tube at 123 feet, both measured from same datum plane. Calculate the mean velocity of flow in the pipe. K is 0.87 and the pipe coefficient is 0.85.

3. Traverses of a 10-inch pipe made with a pitometer at two different times gave the following deflections in inches of tetrachloride (if specific gravity is 1.50):

	Distance from center								
	4.5″	4″	3″	2″	0	2″	3″	4″	4.5″
(*a*)	0.8	1.19	1.55	1.74	1.90	1.84	1.67	1.40	1.10
(*b*)	2.64	3.92	5.10	5.57	5.93	5.89	5.40	4.50	3.70

Draw deflection and velocity curves, compute the discharge, and determine the pipe coefficient for both sets of observations. K is 0.87.

CHAPTER IX

THE VENTURI METER

126. The Venturi Meter. The Venturi meter is a practical application of Bernoulli's theorem to the measurement of water flowing in pipes under pressure. Two truncated hollow cones are inserted, as shown in Fig. 30, in a line of pipe having the same internal diameter as the base of the cones.

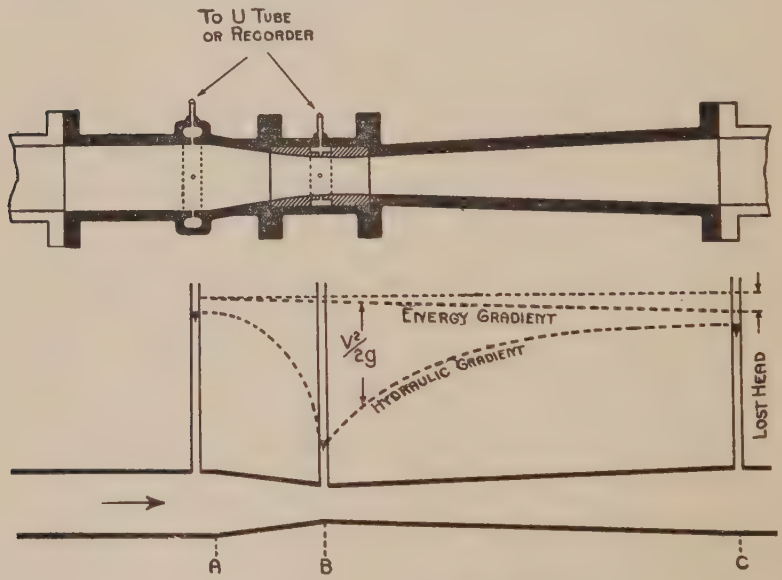


Fig. 30. — Venturi Meter

127. Notation. Let the diameters of the pipe in feet at A be D_a, at B be D_b, at C be D_c; the corresponding areas be A_a, A_b, A_c, in square feet; the velocities in feet per second at A be V_a, at B be V_b, and at C be V_c; the pressure heads in feet measured from the center of the pipe at A be h_a, at B be h_b, and at C be h_c; the friction loss of head to the throat, h'_λ, and the friction loss of head for the whole meter, h_λ.

The upstream section at A is called the *inlet*, the section at B is called the *throat*. The section at C is the same area as at

A. The pipe is filled with water flowing in the direction of the arrows, that is, from A to C.

Since the same quantity of water is passing through both the inlet and the throat, and since the area at B is smaller, the velocity at B is greater than at A; but as they have a constant relation for any single meter, depending upon the ratio of the two areas, a simultaneous observation of the pressure heads at the inlet and throat provides a method of determining the velocity at the throat (and also at the inlet), and from the velocity and areas the discharge.

128. Derivation of Formula. The formula for the discharge through a Venturi meter is derived as follows. Let Q denote actual discharge in cubic feet per second. If the meter is set level we have, by Bernoulli's theorem,

$$(1) \qquad \frac{p_a}{w} + h_a + \frac{V_a^2}{2g} = \frac{p_a}{w} + h_b + \frac{V_b^2}{2g} + h_\lambda'.$$

But

$$(2) \qquad Q = A_a V_a = A_b V_b = A_c V_c.$$

Then

$$(3) \qquad A_a^2 V_a^2 = A_b^2 V_b^2; \quad \text{and} \quad V_a^2 = \frac{A_b^2 V_b^2}{A_a^2} = \frac{D_b^4 V_b^2}{D_a^4}.$$

Substituting the value of V_a^2 from (3) in (1) and transposing, we have

$$(4) \qquad \frac{V_b^2}{2g}\left(\frac{D_a^4 - D_b^4}{D_a^4}\right) = h_a - h_b - h_\lambda'.$$

From (4), we have

$$(5) \qquad V_b = \frac{D_a^2}{(D_a^4 - D_b^4)^{1/2}} [2g(h_a - h_b - h_\lambda')]^{1/2},$$

and

$$(6) \qquad Q = A_b V_b = \frac{\pi D_b^2 D_a^2}{4(D_a^4 - D_b^4)^{1/2}} [2g(h_a - h_b - h_\lambda')]^{1/2}.$$

Since in the ordinary use of the meter h_λ' cannot be measured, its effect may be and usually is included by the use of a coefficient of discharge C. Let $h_a - h_b = H$. Then

$$(7) \qquad Q = C \frac{\pi D_b^2 D_a^2}{4(D_a^4 - D_b^4)^{1/2}} (2gH)^{1/2}.$$

NOTE. If h_b is less than atmospheric pressure, a not infrequent occurrence,

$$h_a - (-h_b) = h_a + h_b.$$

Formula (7) may be simplified for use with meters which have a given ratio of throat and inlet diameters. Let us call this ratio R, so that $R = D_a/D_b$, and let

$$(8) \qquad K = \frac{\pi R^2}{4} \left(\frac{2g}{R^4 - 1} \right)^{1/2}.$$

Then

$$(9) \qquad Q = CKD_b^2 H^{1/2}.$$

Values of R	$\frac{3}{1}$	$\frac{2\frac{1}{2}}{1}$	$\frac{2}{1}$
Values of K	6.34	6.38	6.50

129. The Value of *C*. The coefficient C varies with the velocity at the throat, the ratio of the diameters at inlet and outlet, and the actual dimensions of the meter. As meters are ordinarily constructed, the coefficient C is between 0.97 and 1.0.

The total loss of head in a Venturi meter, that is, the lost head between A and C, is relatively small as compared with other types of water meters.

130. Invention by Clemens Herschel. In 1886–1888 [1] Clemens Herschel invented this meter, established its scientific and commercial value, and named it after an Italian hydraulician who had discovered its principle.

When first used the pressures were observed by piezometers or manometers. The meter could therefore be operated only when continued observations were possible, and is now frequently so used, especially in laboratory work. In 1892–1894 the engineers of the makers [2] of the meter devised a calculating machine or register driven by weights to indicate the total flow on a direct reading dial, and record the rate of flow on a chart.

[1] TRANSACTIONS A. S. C. E., 1887, vol. 17, p. 228 *et seq.*

[2] Builders Iron Foundry, Providence, R. I.

The tube is usually made of cast iron with bronze-lined throat. Meters have been built of concrete with metal linings, and of wood stave pipe with a metal throat. Pressure-chambers

TABLE II

EXPERIMENTAL VALUES OF C FOR CERTAIN VENTURI METERS. EXPERIMENTAL DETERMINATIONS OF LOST HEAD (h_λ) IN CERTAIN VENTURI METERS

	V at throat in ft./sec.	5	10	15	20	25	30	35	40	45	50	55
A	C			1.09	1.02	0.990	0.982	0.975	0.968	0.960	0.955	0.954
	h_λ		0.29	0.89	1.5	2.1	2.8	3.5	4.4	5.5	7.1	12.3
B	C	Furnished by makers of instruments.										
	h_λ	0.05	0.23	0.53	0.94							
C	C	0.934	0.982	0.988	0.989	0.991	0.992	0.993	0.995	0.997	0.999	
	h_λ	0.06	0.27	0.57	0.98	1.47	2.11	2.88	3.80	5.06	8.20	
D	C	0.985	0.980	0.975	0.969	0.966	0.959	0.954				
	h_λ	0.04	0.19	0.39	0.70	1.08	1.53	2.13				
E	C	0.998	0.996	0.991	0.984	0.980						
	h_λ	0.04	0.16	0.34	0.62	1.00						

A — Experiments of Coker & Strickland, TRANSACTIONS CAN. S. C. E., 1902.
Area of the throat, 0.000787 square feet.
Ratio of throat area to inlet area, 1 to 18.3.
These values have been interpolated from curve plotted from results of above experiments.

B — Experiments of Marx, Wing, and Hoskins, TRANSACTIONS A. S. C. E., 1898.
Ogden Venturi on 54-inch pipe.
Area of the throat, 3.547 square feet.
Ratio of throat area to inlet area, 1 to 4.48.
These values are the mean of losses shown by two meters, interpolated from curves plotted from results of above experiments.

C — Experiments of Clemens Herschel, TRANSACTIONS A. S. C. E., 1887.
Venturi on a 12-inch pipe.
Area of the throat, 0.08634 square feet.
Ratio of throat area to inlet area, 1 to 9 (practically).
Values of C and h_λ taken from diagram accompanying above paper.

D — Experiments of Clemens Herschel, TRANSACTIONS A. S. C. E., 1887.
Venturi on a 108-inch pipe.
Area of the throat, 7.074 square feet.
Ratio of throat area to inlet area, 1 to 8 (practically).
Values of C and h_λ taken from diagram accompanying above paper.

E — Experiments of Clemens Herschel, TRANSACTIONS A. S. C. E., 1887.
Venturi on a 48-inch pipe.
Area of the throat not stated.
Ratio of throat area to inlet area, about 1 to 4.5.
These values taken from curve: Plate III, 115 *Experiments*, Herschel, 1897, p. 43.

(piezometer rings) surround the upstream end and throat, and pressure pipes lead therefrom to the manometer or register.

131. Ratio of Inlet and Throat. In Herschel's first meter the ratio of inlet and throat diameters was 3 to 1, the areas being consequently 9 to 1. With this ratio the pressure at the throat is likely to be less than atmospheric, thus producing a positive column of water at the inlet and a suction at the throat. This requires certain additional corrections in computation, and a perfectly air-tight connection from the throat. Such arrangements are easy to make in laboratory work, but are somewhat troublesome in practical installations. These matters should be, and are usually, designed to make both h_a and h_b greater than atmospheric pressure. This generally requires a smaller ratio of diameters than 3 to 1, depending, however, upon the diameter of the pipe, the pressure in the pipe, and the rate of flow.

132. Setting up and Reading the Venturi Meter Manometer. A pressure pipe not less than $\frac{3}{4}$ inch inside diameter is run from valve A to the "upstream" or inlet pressure-chamber of meter tube, also a similar pipe from valve B to the throat pressure-chamber. These pipes should be preferably of lead, brass, or lead-lined or tin-lined iron, and should be connected at the sides of the pressure-chambers, not at the top or bottom. A valve or corporation cock should be placed in each pipe close to the meter tube, so that if an accident occurs to the pipe the water can be shut off. The pipes should be laid on a slight grade, which may be either upward or downward, but there must be no summits where air can collect and affect the pressures. It is very necessary that there be unbroken columns of water from the meter tube to the mercury in the manometer. These pressure pipes should be as short and as direct as possible. Figure 31 shows a manometer.

The manometer should be filled about half full of mercury by unscrewing one of the air chambers and inserting a small funnel.

The air valves C and D are for the purposes of drawing off any air that might collect in the air chambers. The by-pass cock is for the purpose of equalizing the mercury levels and thus determining whether there is any air in either pressure pipe. It may also be opened to cause a circulation of water through the

two pressure pipes for the purpose of cleaning them. At other times this by-pass should be kept closed.

The drain valves E and F are for the purpose of draining off any mercury that may accidentally be blown over by an excessive pressure in one of the pressure pipes. The glass tubes are $\frac{5}{8}$ inch outside diameter and 36 inches long, set in rubber gaskets.

The sliding graduated scale is set with its zero at the top of the lower column of mercury and the reading gives the difference in pressure in inches of mercury. This difference multiplied by the constant ratio 1.133 for reducing inches of mercury to feet of water gives H in feet of water, from which, by formula (7), § 128, the rate of discharge at the instant of observation may be computed.

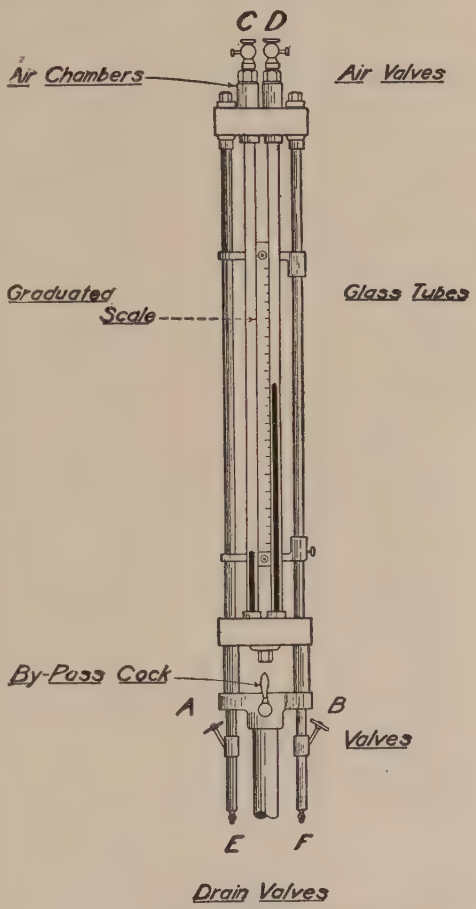


FIG. 31. — MERCURY MANOMETER OR PRESSURE GAGE

133. Application of the Venturi Meter. The meter can be made for pipes of any diameter, and the precision of results obtained is independent of the size. The makers list the following sizes, designated by the inside diameter of the two ends in inches: 2, 3, 4, 5, 6, and by two-inch increments up to 60.

The capacity in U. S. gallons per 24 hours varies from 4000 for the 2-inch to 117,000,000 for the 60-inch. The 2-inch meter

tube weighs about 50 pounds, and the 60-inch about 58,000 pounds. Smaller sizes than 2-inch are in use, and much larger than 60-inch up to 108 or 120 inches. The common sizes are 6 to 60 inches.

Experience has shown that, with the register, continuous measurements have been made with an error not exceeding 2 per cent. In laboratory experiments with mercury columns to measure the head at inlet and throat, rates of flow may be determined with an error frequently not exceeding 1 per cent. The total head lost between the points A and C (Fig. 30) is so little that the insertion of a meter in the mains of a water supply system is rarely objectionable. It may be used in high-pressure power plants. It is frequently utilized in water and sewage filters to regulate the rate of filtration. Its chief use is in street mains, in measuring liquids heavier or lighter than water, and in filter control.

PROBLEMS

1. The following results were obtained by experiments on a Venturi meter; diameter of inlet = 3.92 inches, diameter of throat = 1.32 inches. (*a*) Run of 15 minutes: 52,042 pounds of water discharged; pressure at inlet, + 55 pounds per square inch, and at throat − 10.3 pounds per square inch. (*b*) Run of ten minutes: weight of water = 22,057 pounds; inlet pressure = + 14 pounds per square inch, and throat pressure = − 12.86 pounds per square inch. Gage at inlet in both experiments was 16 inches above the center line of the pipe. Determine C.

2. In a Venturi water meter, of which the coefficient of discharge is 0.96, the larger diameter is $2\frac{1}{2}$ feet and the smaller 1 foot, the heads are + 132 feet and + 11 feet. What will be the discharge in gallons per minute?

3. The ratio of the areas of the throat and inlet areas of a Venturi water meter are as 1 to 9. The larger pipe is 18 inches in diameter. What is its smaller diameter, and its discharge in cubic feet per second, if the coefficient of discharge is 0.98, and the head on larger end is 66 feet, on smaller end is − 15 feet?

4. The discharge through a Venturi meter is 810 gallons per minute; diameter of pipe is 12 inches. Ratio of diameters, 2 to 1. Pressure head at entrance is 21.4 feet. Find velocity at inlet and at throat, and pressure head at throat. C is 0.99.

CHAPTER X

ORIFICES

134. Orifices. An *orifice* is an opening; in hydraulics, an opening of some regular geometric form used in measuring the flow of water. The use of the term *orifice* is commonly limited to openings pierced in relatively thin, flat, stiff walls of metal, wood, or masonry; but actually it includes mouthpieces, short tubes, nozzles, gates, sluices, and other openings, even those in which the distance from inlet to outlet is considerable.

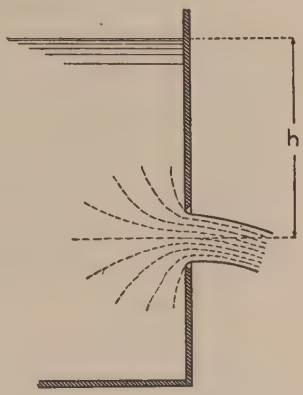


FIG. 32. — A VERTICAL CIRCULAR ORIFICE

A vertical orifice is one set in a vertical plane. Figure 32 represents a vertical circular orifice.

A horizontal orifice is one set in a horizontal plane.

135. Notation. The following notation will be used in discussing orifices of all kinds.

L = breadth in feet of a rectangular orifice.
O = height in feet of a rectangular orifice.
d = the diameter of a circular orifice in feet.
a = area of an orifice in square feet.
A = area of the cross section of channel of approach in square feet.
Q = the actual discharge in cubic feet per second.
Q_t = the theoretical discharge in cubic feet per second.
V = the actual mean velocity in feet per second through an orifice, that is, at the "vena contracta."
V_t = theoretical mean velocity in feet per second through an orifice.
V_A = the mean velocity in feet per second of the water flowing through the tank or channel of approach to the opening, called the velocity of approach.

p_u = the intensity of pressure on the upper surface of the water, due to external forces.

p_c = the intensity of pressure on the center of the orifice, due to external forces.

h_c = difference in elevation in feet between the water surface and the center of the orifice.

h_u = difference in elevation in feet between the water surface and the upper edge of the orifice.

h_b = difference in elevation in feet between the water surface and the bottom edge of the orifice.

$H_c = h_c + \left(\frac{p_u - p_c}{w}\right)$ = head in feet on center corrected for difference in pressure on water surface and on the orifice.

W = the volume of discharge through the opening in pounds of water per second.

g = 32.16, the acceleration due to gravity in feet per second per second.

C = coefficient of discharge = $C_v C_c$.

C_v = coefficient of velocity.

C_c = coefficient of contraction.

136. Theory of the Orifice. The determination of the volume of flow through an orifice is based on Torricelli's theorem, viz. that the velocity of water passing through an orifice is the same as that which would be acquired by a body falling freely in vacuo from the surface of the water to the center of the orifice; hence the theoretical velocity should be

$$V_t = (2\,gh_c)^{1/2}, \tag{1}$$

and the theoretical discharge should be

$$Q_t = aV_t = a(2\,gh_c)^{1/2}. \tag{2}$$

The actual mean velocity is slightly less than given by equation (1) and the actual discharge somewhat less than given by equation (2).

This difference is allowed for by applying a coefficient of correction C_v to the theoretical velocity. The full area of the orifice is not effective in discharging the flow, since the phenomenon of contraction takes place. This is corrected for by an additional coefficient C_c. Equation (2) then reads:

$$Q = C_v C_c a(2gh_c)^{1/2}.$$

In practice the two coefficients are usually combined into one which is called *the coefficient of discharge* for the orifice:

$$C_v \times C_c = C.$$

The water surface above the orifice may be subjected to some pressure other than atmospheric, and the orifice may discharge against some similar pressure. Under such conditions the effective head on the orifice is determined by adding algebraically to the head on the center of the orifice the algebraic difference between the pressure on the upper surface and that on the discharge side of the orifice reduced to equivalent feet of water.

137. Form of Jet. The jet of water issuing from an orifice usually contracts to an area smaller than the opening itself, again enlarges more or less irregularly, and does not maintain the cross-sectional shape of the opening itself except for circular orifices.

138. Vena Contracta. The section of minimum area, called the ***vena contracta,*** is usually distant from the inner plane of the orifice about 0.50 to 0.80 of the least dimension of the opening.

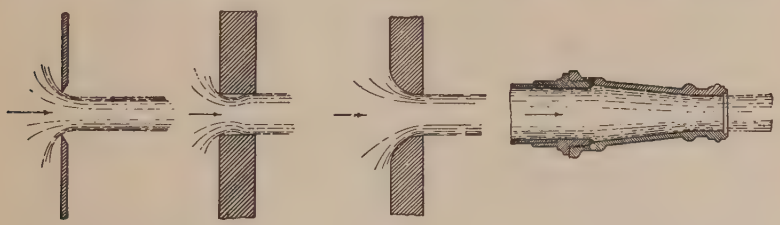


FIG. 33

In determining velocity and discharge, this is the critical section, because many experiments have demonstrated that the velocity at this point is nearly equal to $(2\,gh_c)^{1/2}$, and the *actual discharge through the orifice equals very nearly the area at the vena contracta multiplied by* $(2\,gh_c)^{1/2}$.

In well-designed orifices, friction and eddy disturbances have little effect. Experience has shown that the actual loss of energy is slight.

In Fig. 33, *A* is an orifice in a thin wall; *B* is a sharp-edged orifice in a thick wall; *C* is a bell-mouthed, or rounded, orifice; *D* is a nozzle.

139. The Coefficient of Contraction. The coefficient C_c mentioned in § 136 is called the coefficient of contraction. It is the ratio of the area of the jet at the vena contracta and the area of the orifice, that is,

$$C_c = \frac{\text{area at vena contracta}}{\text{area of orifice}}, \text{ also } = \frac{C}{C_v}.$$

Its value varies from 0.60 to 1.00, with a mean value of about 0.60 for orifices in a thin wall.

The coefficient of contraction (C_c) may be found by direct measurements of the diameter of the jet at the contracted section,

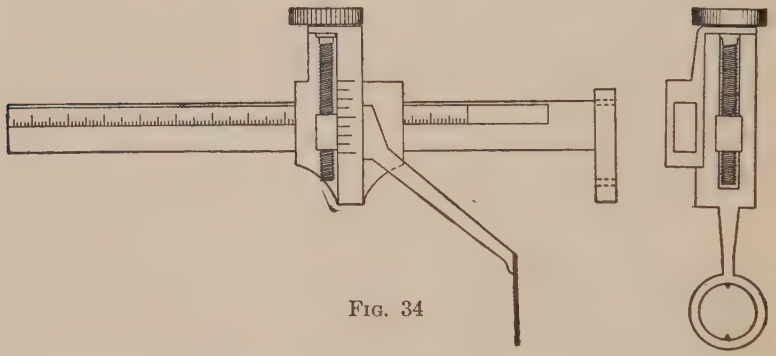


Fig. 34

by such an apparatus as shown in Fig. 34, or it may be computed from experimental determinations of C and C_v.

140. The Coefficient of Velocity. The coefficient of velocity C_v mentioned in § 136 is the ratio of the actual mean velocity of the jet at the vena contracta and the velocity due to the whole head on the orifice, that is,

$$C_v = \frac{V}{(V_A^2 + 2gH_c)^{1/2}}.$$

Its value is usually very close to unity, sometimes being as high as 0.999.

The coefficient of velocity may best be determined by measurements with a Pitot tube. Formerly the trajectory curve of the jet was measured and the velocity figured from this. This method, however, does not take the friction of the surrounding air into account, and the result is not accurate. For effect of velocity of approach V_A see § 151.

141. The Coefficient of Discharge. The coefficient of discharge C is the ratio between the actual discharge Q and the theoretical discharge Q_t, that is,

$$C = \frac{Q}{a(V_A^2 + 2gH_c)^{1/2}}.$$

The coefficient of discharge may be computed from volumetric or weight determinations of discharge with a high degree of precision.

142. Coefficient Determinations. The methods of determining directly C_v and C_c are both instructive and interesting, but the results obtained are not wholly satisfactory or as precise as an experimental determination of the coefficient of discharge. Even with jets from a circular orifice, a close measurement of the contracted section is difficult, owing to the vibrations of the jet. The jets from a rectangular orifice can rarely be measured with any reasonable degree of accuracy. The actual velocity of a jet has been determined by a Pitot tube with a high degree of precision.

143. Orifices in Thin Walls. If an orifice cut in a true plane presents as nearly as may be only a line of contact to the emerging jet, the orifice is designated an *orifice in a thin wall, square-edged* or *sharp-edged*. To secure this condition, it is essential only that the thickness of the wall be such that the jet will not, after leaving the inner face, again touch on the side of the opening.

144. Standard Orifice. If an orifice in a thin wall is set far enough from the side of the vessel or channel to secure *full contraction of the jet, is round or square, and has no dimensions greater than one foot* (for which shapes and dimensions reliable coefficients are available), it is called a standard orifice.

145. Complete Contraction. It is difficult to say exactly under what conditions full contraction is secured. The edges of the orifice must be sharp, so that the jet breaks away clear and touches along one line only. Generally speaking, if the orifice is in a plane with the bounding walls distant from it on all sides at least three times the least dimension of the orifice, the contraction may be considered complete. Exceptions to this are the cases of small orifices and orifices under low heads. These will be discussed later.

146. Orifices in Thick Walls. If the opening be in a plate of such thickness, even though square-edged, that the stream again touches the sides of the opening after leaving the inner face (see Fig. 33, *B*), or if the opening be of gradually diminishing area such that the jet clings to the sides (see Fig. 33, *C* and *D*), the contraction will be partly suppressed, and for a given head the discharge will be more than for the orifice of the same area in a thin plate.

The limit at which an orifice becomes a tube, nozzle, or mouthpiece is not well defined. Orifices in a thick wall (excepting nozzles) are not as accurate for measuring water as orifices in a thin wall. This is because, not only are few reliable coefficients available, but also they are more difficult to secure, owing to great varieties of shape and dimensions possible, and the difficulties of duplicating any shape to conform to those from which the few experimental coefficients available were obtained.

147. Comparison of Orifices as to Contraction and Discharge. Weisbach made a great number of experiments to determine the law according to which the contraction changes with the angle α.

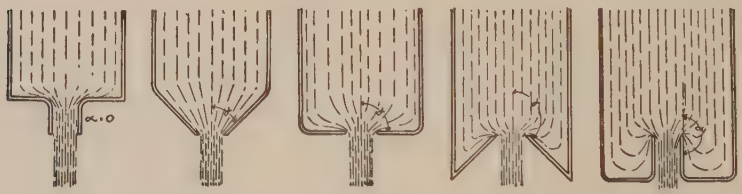


FIG. 35

Figure 35, *A* to *E*, shows the range from the greatest contraction to practically none, the discharge increasing as the contraction decreases. By comparing the following coefficients of discharge with the figures, the difference between various kinds of orifices is obvious. (See §§ 159–164.)

COMPARISON OF ORIFICES

Diameter = 0.066 foot. Heads 1 to 10 feet

	A		*B*		*C*	*D*					*E*
α	180°	157½°	135°	112½°	90°	67½°	45°	22½°	11¼°	5¾°	0°
C	0.54	0.55	0.58	0.61	0.63	0.68	0.75	0.88	0.92	0.95	0.97

148. Precision Measurements. For measuring water with a high degree of precision by orifices, only orifices in a thin wall should be employed, using an orifice for which the greatest number of reliable coefficients are available.

For discharging a maximum volume through an opening of a given diameter, orifices with wide and well-rounded lips should be used.

149. Methods of Procedure. The procedure in measuring the flow of water with an orifice comprises the following steps.

(1) Testing and setting gages for measuring head or pressure.
(2) Calibrating the opening and the channel of approach by direct measurements and computing therefrom the areas a and A.
(3) Determining the difference in elevation between the water surface and the orifice; and the pressures p_u and p_c, if other than atmospheric pressure.

The head is measured from the water surface to the center of the plane of the opening. To be mathematically accurate the head should be measured from the center of the vena contracta, especially in horizontal orifices, because the full velocity does not develop before reaching this point. The head may be measured by means of a hook gage, by means of a piezometer tube, by means of a mercury column, or by means of a calibrated steam gage. The head should be measured sufficiently distant from the plane of the orifice to avoid inaccuracies due to dropping of the water surface toward the opening.

(4) Selecting the coefficient of discharge suited to the shape and dimensions of the opening and the head.
(5) Computing the discharge.

150. Horizontal Orifices. Comparatively few experiments have been made on orifices set in a horizontal plane. There is, however, no reason to expect that such a setting should change the coefficient of discharge, and coefficients for vertical orifices can be used for horizontal ones. A 0.656′ circular orifice tested by Bazin gave a coefficient of 0.598 when set vertically, and 0.603 when set horizontally.

151. The Effect of the Velocity of Approach. The discharge through an orifice is increased by the velocity of approach. The actual pressure head acting on the orifice is increased by the

velocity head of the velocity of approach, and this total head is the net head acting on the orifice. Since

$$H = h_c + \frac{V_a^2}{2g},$$

we have

$$Q = Ca\sqrt{2gh_c + \frac{V_a^2}{2g}}.$$

If $C_c a/A$ is the ratio between the areas of the vena contracta and of the channel of approach, we may write, neglecting C_v

$$V_A = V\frac{C_c a}{A},$$

$$V^2 = 2gh + V^2\frac{C_c^2 a^2}{A^2} = \frac{2gh}{1 - \frac{C_c^2 a^2}{A^2}}.$$

In other words, the head is increased in the ratio

$$\frac{1}{1 - \frac{C_c^2 a^2}{A^2}},$$

and the discharge in the ratio

$$\frac{A}{\sqrt{A^2 - C_c^2 a^2}}.$$

In most cases met with, unless the ratio A/a is small, the effect of the velocity of approach on the discharge is slight. The following table will give some idea of its effect in increasing the discharge.

$\frac{A}{a}$	1.5	2.0	3.0	5	10	15	25
Per cent increase in discharge	32.1	14.7	5.8	2.0	0.5	0.2	0.1

In order to secure complete contraction of the jet, the ratio A/a must be at least 40 or 50, and the velocity of approach has a negligible effect, even in measurements requiring a high degree of precision. If the contraction is not complete, an exact coefficient cannot be chosen, and, unless the ratio is less than say 5, the error resulting from neglecting the velocity of approach is within the limits of the measurement.

152. The Effect of Suppressing Contraction. The suppression of contraction may be made partial or complete by widening or

rounding the lips of the orifice, by reducing the distance between the edges of the opening and the sides or bottom of the channel, by reducing the head below the height required for complete contraction, or by placing two or more orifices side by side in the same wall.

For complete suppression, the coefficient of discharge is practically unity. In an orifice in a thin wall suppression cannot be complete. *For partial suppression*, other conditions being the same, the coefficient of discharge is less than unity, more than the coefficient for the same orifice with full contraction; and slight changes in form may produce wide variations in the coefficient. The published experiments are insufficient to warrant the framing of a satisfactory law for computing the results of suppressing contraction; but they indicate that the discharge is not increased in direct proportion to the percentage of perimeter on which the contraction is suppressed.

The following experiments are useful in estimating the results of suppression. The data available are very meager and of limited application. The effect of suppression in various forms of orifices will be considered again in Chapter XI.

They were carried out by Lesbros in 1834 on contraction with an orifice 0.656 feet square under heads up to 5.6 feet.[1] He varied the amount of contraction by cutting down the channel of approach by partition walls practically coincident with one or more sides of the orifice. The results are given in the following table.

Amount of Contraction	Coefficient
Complete contraction	0.60
Bottom contraction suppressed	0.62
One side contraction suppressed	0.61
Two sides contraction suppressed	0.64
Bottom and one side suppressed	0.64
Bottom and two sides suppressed	0.67–0.68

No allowance was made for the effect of the velocity of approach, but even in the case where contraction was suppressed on three sides, the area of the approach channel was sufficient to allow its effect to be disregarded without suffering much loss in accuracy.

[1] *Expériences Hydrauliques*, Paris, 1851.

153. Submerged Orifices. When the opening is entirely covered by water on the downstream face, the orifice is said to be submerged; and the head h_e becomes the difference in the elevation between the water surfaces above and below the orifices. See Fig. 36.

The discharge for submerged orifices is slightly less than for orifices of similar size and form with free discharge under the

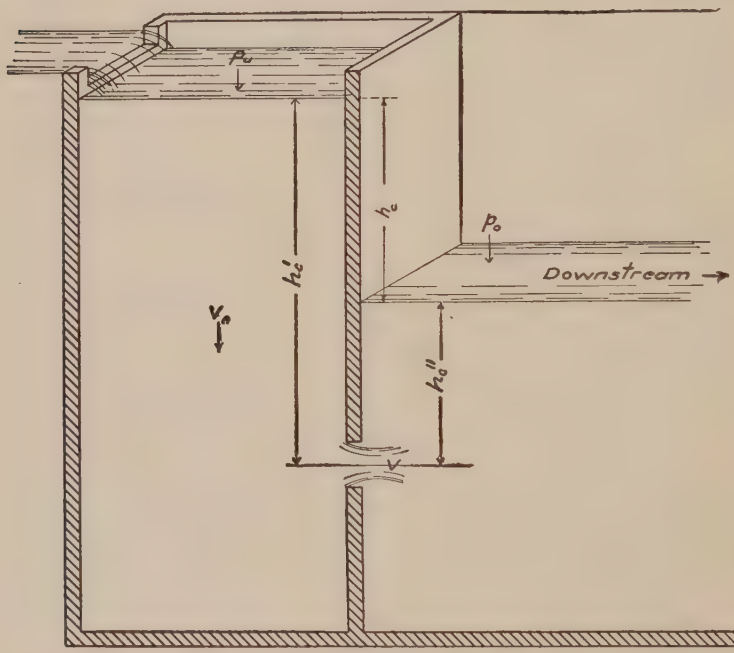


Fig. 36. — Submerged Orifice

same head. The difference according to Francis is probably as much as 3 per cent for very low heads, but for a head of 10 feet and over the difference is probably negligible, decreasing also with increase in the size of the opening. For orifices more than one square foot in area, and heads greater than three times the least dimensions, the available coefficients indicate that no correction is necessary.

154. Partially Submerged Orifices. For such openings the discharge cannot be closely estimated except by special calibration for each case.

155. Discharge under a Dropping or Rising Head. The time required either to empty or to fill a vessel through an orifice under a continuously varying head may be computed as follows. See Fig. 37. Let A denote the cross-sectional area in square feet of a prismatic vessel; the total volume above the center of the

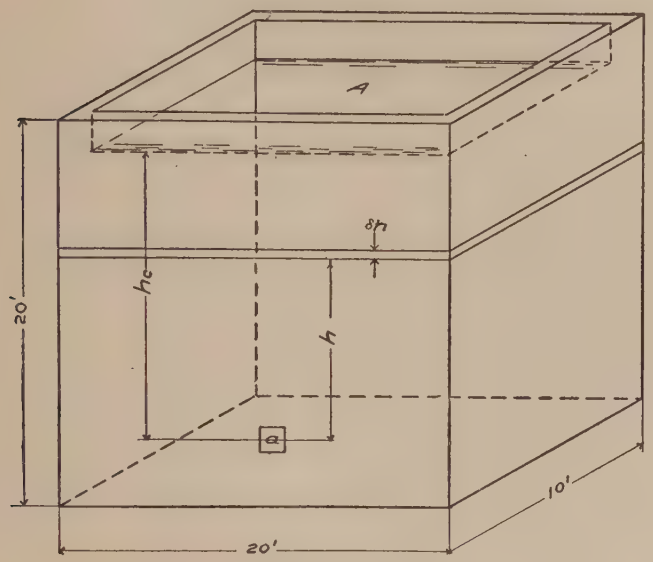


FIG. 37

orifice is then Ah_c. Let h denote head at any instant. If, in any time dt, the head is lowered dh feet, then the volume discharged equals Adh, and the rate of discharge is $Ca(2\,gh)^{1/2}$. Therefore

$$dt = \frac{-Adh}{Ca(2\,gh)^{1/2}} = \frac{-A}{Ca(2\,g)^{1/2}} \times \frac{dh}{(h)^{1/2}}.$$

Integrating within the limits of h, corresponding to $t = 0$ and $t = T$, viz., h_c and h_1,

$$\int_0^T dt = \frac{A}{Ca(2\,g)^{1/2}} \times \int_{h_1}^{h_c} \frac{dh}{(h)^{1/2}}.$$

Therefore

$$T = \frac{2\,A}{Ca(2\,g)^{1/2}} (h_c^{1/2} - h_1^{1/2}).$$

For emptying the vessel down to the center of the orifice, that is, when $h_1 = 0$, we have

$$T = \frac{2\,A}{Ca(2\,g)^{1/2}}\,(h_c)^{1/2}.$$

Obviously this is just twice the time required to discharge the same volume of water under a constant head h_c.

156. Selection of Coefficients for Discharge under a Dropping Head. It is usually sufficient to take the mean of the coefficients for the two limiting heads. If great accuracy is required, the intervals of time required to lower the head by short drops in succession may be separately computed. Their sum will be the total time of emptying.

157. The Miner's Inch. The miner's inch is the volume of flow of water that will be discharged through an opening one inch square under a head, not necessarily the same in every locality, but usually about 0.55 foot. The term, which was originally used somewhat loosely to measure water for mining purposes in the western part of the United States, has been in late years defined by law in equivalent cubic feet per second, ranging in value in different states from about 0.02 to nearly 0.03 cubic foot per second. It is an awkward term, but its use is often necessary on account of custom and law.

158. The Coefficient of Discharge. For convenience in use the coefficients C_v and C_c are combined into one which is known as the coefficient of discharge, $C_v \times C_c = C$, as stated in § 136.

A great many experiments have been made to determine the coefficient of discharge for an orifice with full contraction. A majority have either been on a very small scale or have lacked the necessary care and precision, but several notable experiments stand out as of special value.

In 1827–28, Captains Poncelet and Lesbros of the French Army conducted a series of experiments at Metz.[1] They used several different orifices, but the one in which we are most interested was an orifice 0.656 foot square with full contraction, under heads varying from 0.4 to 5.7 feet. The water discharged by the orifice was measured volumetrically, and the measurements were

[1] *Expériences Hydrauliques*, Mémoires de l'Academie des Sciences, Paris, 1832.

carried out with a high degree of precision. They found an average value of C for this orifice of 0.600.

Hamilton Smith [1] carried out some experiments in California in 1874. He tested several circular orifices as well as rectangular ones. The water was measured by volume.

CIRCULAR ORIFICES

DIAMETER	HEAD	C
0.25′	1.33′	.608
0.42	1.20	.603
0.66	1.09	.591
1.01	1.05	.587 (vortex formed)

In 1884–5 Smith made some additional experiments on small square and circular orifices at Holyoke, Mass.

SIZE	HEAD	C
0.05′ round	0.28′	.646
	4.6	.605
0.10 round	0.13	.631
	1.7	.604
	4.5	.601
0.05 square	0.31	.641
	4.6	.610
0.10 square	0.18	.629
	1.7	.608
	4.6	.606

General T. G. Ellis made some experiments on large sized orifices at Holyoke in 1874.[2] The results are not so trustworthy as might be desired, since the water was measured by a weir. He found a coefficient of discharge for a 2-foot circular orifice of 0.605 and for a 2-foot square orifice of 0.603. Probably the contraction on the bottom side of the orifice was not quite complete.

Henri Bazin made some experiments on orifices in 1890–92 at Dijon. The water was measured volumetrically. The head was about three feet. A 0.656 foot square orifice gave a coefficient of 0.607, and a 0.656 foot circular one gave 0.598 when set vertically, and 0.603 when set horizontally.[3]

[1] Hamilton Smith, *Hydraulics*, New York, 1886.

[2] T. G. Ellis, *Hydraulic Experiments with Large Apertures*. TRANSACTIONS A. S. C. E., 1876.

[3] H. Bazin, *Experiments on the Contraction of the Liquid Vein*, New York, 1896.

Judd and King conducted some experiments on small circular orifices in 1905–6 at the Engineering laboratory of the Ohio State University.[1] The orifices were from $\frac{3}{4}$ to $2\frac{1}{2}$ inches in diameter, and the heads ran from 4 to 93 feet. The discharge was measured in a calibrated basin and all the details of the tests were carefully carried out. It was found that a head variation of from 4 to 93 feet made practically no difference in the coefficient.

Diameter	Average Coefficient
$\frac{3}{4}''$	.611
$1''$	.610
$1\frac{1}{2}''$	.608
$2''$	.608
$2\frac{1}{2}''$	.596

H. J. I. Bilton experimented with very small orifices, from $\frac{1}{40}''$ to $\frac{3}{4}''$, and also drew a number of conclusions regarding orifices from a study of his own results and those of other experimenters.[2] His work was very exact. Some of his conclusions are as follows.

That, in order to obtain complete and perfect contraction, a certain minimum diameter and head are required. These appear to be approximately $2\frac{1}{2}$ inches and 17 inches, respectively.

That orifices of $2\frac{1}{2}$ inches diameter and over, under heads of 17 inches and over, have a common coefficient of discharge, lying between 0.59 and 0.60, but which is probably about 0.598 (subject to the head being not less than two or three diameters).

That in the cases of orifices smaller than $2\frac{1}{2}$ inches diameter, contraction is never perfect and complete under any head, but is suppressed more and more as the diameter decreases, each size of orifice having its own constant or normal coefficient of discharge and its own critical head.

That as the diameter decreases, the normal coefficient increases, as does also the critical head.

That the discharge of a circular orifice under any given head is the same, whether the jet be horizontal, vertical, or at any intermediate angle.

[1] *Some experiments on the frictionless orifice.* Engineering News, Sept. 27, 1906.

[2] Engineering News, July 9, 1908.

BILTON'S TABLE OF COEFFICIENTS

Circular orifices of diameters in inches as shown in column headings

HEAD IN FEET	1/4	1/2	3/4	1	1½	2	2½
0.17	.683	.663					
0.25	.680	.657	.646	.640			
0.5	.669	.643	.632	.626	.618	.612	.610
0.75	.660	.637	.623	.619	.612	.606	.604
1.0	.653	.630	.618	.612	.606	.601	.600
1.42	.645	.624	.614	.608	.603	.599	.598
1.5	.643	.623	.613	.608	.603	.599	.598
1.83	.638	.621	.613	.608	.603	.599	.598
3.75 and over	.628	.621	.613	.608	.603	.599	.598

A careful examination of most of the accurately made orifice experiments, substantiates Bilton's reasoning, and his conclusions can be taken as the general laws for an orifice with full contraction.

In 1912 the Proprietors of the Locks and Canals at Lowell carried out some experiments. Several orifices and weirs were tested, the discharge being measured by bulk in an old navigation lock. A 12-inch circular orifice under 2.6 ft. head, and a triangular one, 2.5 × 2.5 × 3.5 ft. under 1.5 ft. head, both gave a coefficient of discharge of 0.604.

The coefficient of discharge of any fully contracted orifice, round or square, with a diameter or side of two inches or more, and a head of one foot or over, can be taken as 0.60 *with a resulting accuracy well within the limits of hydraulic work.*

If great precision is desired, the orifice must be calibrated, regardless of its shape or size.

For orifices smaller than two inches, the values in Bilton's table should be used, unless the orifice can be calibrated directly.

It must be remembered that, unless the head on the orifice is over two or three diameters, the contraction is not complete.

Rectangular orifices have a somewhat higher coefficient than square or circular ones. This is due to the contraction being somewhat suppressed.

For orifices in which the length is more than twice the depth, a coefficient of 0.62 is very nearly correct. It is, however, always

preferable to use a round or square orifice when possible, since much more is known about their behavior.

TABLE III

MAIR'S COEFFICIENTS OF DISCHARGE FOR VERTICAL CIRCULAR ORIFICES[1]

Square-edged clean-cut holes bored in thin gun-metal plates

Head from Center of Orifice	Approximate Diameter of Orifice in Inches								
	1	1¼	1½	1¾	2	2¼	2½	2¾	3
	Absolute Area in Square Feet								
	.00546	.00852	.012281	.016749	.021806	.027576	.033898	.040933	.049139
Feet									
0.75	.616	.614	.616	.610	.616	.612	.607	.607	.609
1.00	.613	.612	.612	.611	.612	.611	.604	.608	.609
1.25	.613	.614	.610	.608	.612	.608	.605	.605	.606
1.50	.610	.612	.611	.606	.610	.607	.603	.607	.605
1.75	.612	.611	.611	.605	.611	.605	.604	.607	.605
2.00	.609	.613	.609	.606	.609	.606	.604	.604	.605

TABLE IV

CIRCULAR ORIFICES UNDER HIGH HEADS

Experiments made by Prof. F. W. Greve, Purdue University (ENGINEERING NEWS, Jan. 6, 1921). Water measured by weight.

1-Inch Orifice		1.5-Inch Orifice	
Head	Coefficient	Head	Coefficient
47.48	.600	47.48	.599
59.03	.596	70.58	.599
62.98	.600	93.68	602
70.58	.598	116.78	.595
93.68	.594	139.88	.592
105.23	.595	162.98	.598
116.78	.597	168.08	.598
128.33	.599	209.18	.598
139.88	.597	232.28	.599
157.43	.599		
174.53	.601		
186.08	.600		
197.63	.598		
209.18	.599		
220.73	.600		
232.28	.597		

[1] J. G. Mair, PROCEEDINGS INSTITUTE OF CIVIL ENGINEERS, vol. 84, p. 427.

TABLE V

DRUMMOND'S COEFFICIENTS OF DISCHARGE FOR VERTICAL RECTANGULAR ORIFICES CUT WITH SQUARE EDGES IN THIN BRASS PLATES

Length in feet L	2″) .167	2″) .167	½″) .042	2″) .167	1″) .083	4″) .333	½″) .042	1″) .083
Height in feet O	2″) .167	½″) .042	2″) .167	1″) .083	2″) .167	½″) .042	4″) .333	1″) .083
Ratio $\frac{L}{O}$	1 to 1	4 to 1	1 to 4	2 to 1	1 to 2	8 to 1	1 to 8	1 to 1
Authority	Thomas Drummond [1]							
Head								
6″ = 0.500′	.614	.634	.634	.621	.619	.637	.638	.621
7″ = 0.583′	.613	.634	.633	.619	.618	.637	.636	.619
8″ = 0.667′	.612	.632	.632	.618	.619	.634	.635	.618
10″ = 0.833′	.611	.628	.628	.616	.618	.634	.635	.617
12″ = 1.000′	.610	.627	.626	.615	.617	.631	.633	.615

PROBLEMS

1. A 3-inch circular orifice cut in a thin plate is fastened across the end of a 24-inch pipe. The head above the center of the orifice is three feet. Determine the discharge, (*a*) neglecting the velocity of approach, (*b*) correcting for the velocity of approach.

2. The same orifice is fastened across a 6-inch pipe. Determine the discharge as in **1**, assuming, for the sake of simplicity, that the contraction remains complete.

3. A smooth conical nozzle, 6 inches long and one inch in diameter, is used on a 2½-inch hose. The observed pressure at the base of the nozzle is 32 pounds per square inch. What is the discharge, assuming that $C = 0.98$?

4. A 6-inch square orifice, cut in a thin plate, is placed vertically in the end of a flume 4 feet square in cross section so that two of the sides are exactly flush with the bottom and one wall of the flume. The water surface stands 3 inches below the top of the flume. Determine the discharge.

5. A plate steel box, 4 feet square by 6 feet deep, has four 6-inch circular orifices cut in its sides. The centers of these orifices are one foot above the bottom of the box. The box is placed in a large tank of water so that the level outside of the box is six inches below the top. How long will it take to fill the box to a point 3 inches below the level of the outside water? Assume that the coefficient of discharge of the orifices is the same when submerged, as when discharging freely.

[1] TRANSACTIONS CANADIAN S. C. E., vol. 14 (1900), p. 131.

CHAPTER XI

SPECIAL FORMS OF ORIFICES

159. Mouthpieces. An orifice extended either inwardly or outwardly in a direction nearly parallel to the axis of the jet in a manner that will modify its flow is usually called a mouthpiece, a short tube, a conical tube, or a nozzle, according to the shape and proportions of the ajutage. Most of the types here mentioned, though illustrating important principles, have little value as measuring devices on account of the lack of precise coefficients, and because simpler forms of measuring apparatus can be used. Nozzles, however, are useful as water-measuring devices, as well as for directing jets of water, and they will be discussed separately in Chapter XII.

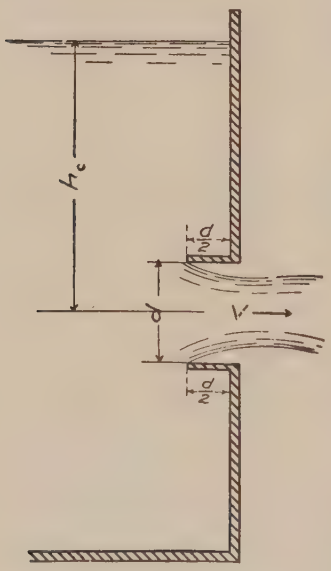


FIG. 38.—BORDA'S MOUTHPIECE

160. Borda's Mouthpiece. A short cylindrical tube projecting inwardly of such dimensions that the jet touches the wall of the tube at its inner end only is usually called *Borda's mouthpiece.* The inner edge of the tube must be relatively thin, and its length should be about equal to the radius of the opening. See Fig. 38.

The coefficient of velocity for such a tube is probably nearly 0.99 or higher; but the contraction of the jet is even more pronounced than for an orifice in a thin wall with full contraction; hence its coefficient of discharge is less.

There have been, however, but few experimental determinations of the coefficient of discharge; its mean value from the experiments of Borda, Bidoni, and Weisbach is 0.53. (See § 147.)

161. The Reentrant Short Tube. If a cylindrical tube set as in Fig. 38 has a length of from 2 to 3 diameters instead of $\frac{1}{2}$ a

diameter, the jet, after contracting, will again enlarge as in the standard short tube (§ 163) and fill the bore; and the discharge will be greater than for Borda's mouthpiece.

162. Coefficients of Discharge. The mean value of the coefficient of discharge (C) for reentrant tubes is usually given as about 0.72.

Experiments by H. J. I. Bilton [1] give the following values of C for heads from 0.5 foot upwards; higher limits not stated. Length of all tubes $2\frac{1}{2}$ diameters.

Diameter of tube in inches,	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
Coefficient of discharge,	.91	.87	.85	.83	.81	.79	.77	.76	.75

163. Standard Short Tubes. A cylindrical tube with the entrance end set flush in a flat wall, having a length of from 2 to 3 diameters, is commonly called a standard short tube. Figure 39 shows such a tube.

Let V = velocity at EF;
V_c = velocity at the contracted section (CD);
p_0 = intensity of pressure in the tube at CD;
p_a = atmospheric pressure on water surface, and on EF.

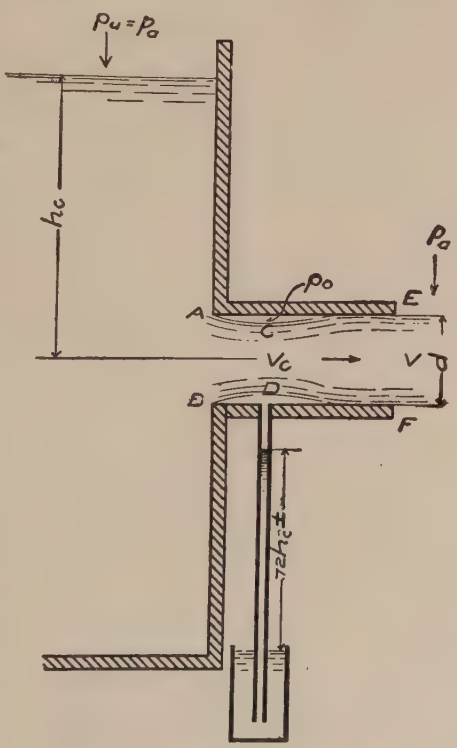


FIG. 39. — STANDARD SHORT TUBE

At CD the stream is contracted to about the same relative dimensions as if it were discharging freely into air from an orifice in a thin wall; but if the tube is of sufficient length, that is, about 2 to 3 diameters, the jet will,

[1] ENGINEERING NEWS, July 9, 1908.

after contracting, expand and again fill the tube at the discharge end EF, thus sealing from the atmosphere the space around CD. The air imprisoned in this space is therefore rarefied by the flowing water, which acts as an air pump. Thus, since the intensity of pressure p_0 at this point is less than atmospheric pressure, the velocity V_c will be in excess of the velocity due to the measured head h_c. V_c may be computed as follows. By Bernoulli's theorem, we have

$$(1) \qquad h_c + \frac{p_a}{w} = \frac{V_c^2}{2g} + \frac{p_0}{w} + \frac{V_c^2}{2g}\left(\frac{1}{C_v^2} - 1\right) = \frac{p_0}{w} + \frac{V_c^2}{2gC_v^2}.$$

Hence

$$(2) \qquad V_c = C_v\left[2g\left(h_c + \frac{p_a}{w} - \frac{p_0}{w}\right)\right]^{1/2},$$

and

$$(3 \qquad Q = C_c a C_v\left[2g\left(h_c + \frac{p_a}{w} - \frac{p_0}{w}\right)\right]^{1/2} = C_c a V_c.$$

Since formula (3) does not include losses due to sudden enlargement between CD and EF, and since C and p_0 are difficult to determine, the simple orifice formula is commonly used, that is,

$$(4) \qquad Q = CaV = Ca(2gh_c)^{1/2},$$

where C denotes the coefficient of discharge for the tube as a whole.

Coefficients of discharge for short tubes have not been thoroughly determined. Weisbach [1] gave values of C for tubes 3 diameters in length, under heads 0.75 to 1.97 feet as follows:

Diameters, feet,	0.033	0.066	0.098	0.131
C,	0.84	0.83	0.82	0.81

For a mean value of C, Weisbach gave 0.815. The value generally used is 0.82. Hence

$$(5) \qquad Q = 0.82\frac{\pi d^2}{4}(2gh_c)^{1/2}.$$

The head lost h_λ in a short tube, computed as for an orifice, is

$$(6) \qquad h_\lambda = \frac{V^2}{2g}\left(\frac{1}{0.82^2} - 1\right) = 0.487\frac{V^2}{2g} = 0.5\frac{V^2}{2g} \text{ (nearly).}$$

[1] *Mechanics*, Coxe's Translation, p. 808.

Since the stream, on leaving the tube at EF, has no contraction, the coefficient of discharge is taken to be identical with the coefficient of velocity at EF, and is used in computing lost head.

This apparatus does not increase the actual energy; it merely makes available a part of the energy which would otherwise be unused because of the contraction of the jet. The intensity of pressure p_0 can never be such as will cause the discharge through the contracted section to exceed

$$\frac{\pi d^2}{4}(2gh_c)^{1/2}.$$

Actually, p_0 is always less, on account of losses due to friction and sudden change of section. Moreover, p_0 cannot be less than 14.7 pounds per square inch below atmospheric pressure.

There is a definite theoretical relation between h_c and p_0 which may be deduced as follows. By Bernoulli's theorem, we have

$$(7) \qquad h_c + \frac{p_a}{w} = \frac{V^2}{2g} + \frac{p_a}{w} + \frac{V^2}{2g}\left(\frac{1}{C^2} - 1\right) = \frac{V^2}{2gC^2} + \frac{p_a}{w};$$

hence

$$(8) \qquad \frac{V^2}{2gC^2} = h_c.$$

From (3) and (4), we have

$$Q = C_c a V_c = aV = Ca(2gh_c)^{1/2}.$$

Then

$$(9) \qquad V_c = \frac{C}{C_c}(2gh_c)^{1/2}; \quad \text{and} \quad \frac{V_c^2}{2g} = \left(\frac{C}{C_c}\right)^2 h_c.$$

From (1) and (7), we find

$$(10) \qquad \frac{p_0}{w} + \frac{V_c^2}{2gC_v^2} = \frac{V^2}{2gC^2} + \frac{p_a}{w}.$$

Substituting from (8) and (9), we obtain

$$(11) \qquad \frac{p_0}{w} = \frac{p_a}{w} + h_c - \left(\frac{C}{C_c C_v}\right)^2 h_c.$$

If $C = 0.815$, $C_c = 0.62$, and $C_v = 0.995$ (C_c and C_v apply to the area at CD), equation (11) becomes

$$\frac{p_0}{w} = 33.9 - 0.72\, h_c.$$

If a water barometer were inserted at CD, the water in its tube should rise to $0.72\ h_c$. If the tube were shorter than $0.72\ h_c$, the water would be sucked up through the tube and discharged at the outlet. In experiments by Venturi and Weisbach, the water rose from 0.7 to 0.8 h_c. A short tube has little practical value as a water-measuring device.

Some experiments on the discharge of short tubes were made at the University of Michigan in 1916–17 by O. W. Boston.[1] The diameters of the tubes were from one to eight inches, and their lengths were three times their diameters. The one-inch and four-inch tubes showed results considerably different from the others, and it is thought that some constant error, possibly in the measurement of their diameter, might have occurred. The eight-inch tube acted as a sharp-edged orifice, giving a coefficient of 0.598.

Values of C for Short Tubes

Head in Feet	Diameter of Tube in Inches						
	1.03	2.05	3.22	4.06	5.06	6.06	6.98
6.0	.791	.814	.814	.783	.814	.814	.814
5.0	.791	.814	.814	.783	.814	.814	.813
4.0	.791	.814	.814	.783	.814	.813	.813
3.0	.790	.814	.814	.783	.813	.813	.812
2.0	.789	.813	.813	.781	.811	.810	.810
1.0	.788	.810	.809	.775	.805	.804	.803
0.4	.784	.803	.800	.766	.796	.794	.793
0.2	.778	.800	.796				

164. Long Tubes, or Short Pipes. A similar contraction takes place at the entrance of tubes of greater length than 3 diameters. The amount of the contraction will depend upon the form of the entrance. In general, for want of very precise information, the assumption must be made that the jet inside any pipe will form as if issuing into the air from an orifice shaped like the pipe entrance. From what is known of jets from orifices, we may assume a suitable coefficient and compute the diameter of the contracted section, and therefrom deduce the lost head in entrance and enlargement. If the pipe is longer than 3 diameters, the friction head lost in the pipe must also be added to the head lost in the tube.

[1] Transactions A. S. M. E., Dec., 1923.

165. Bell-Mouthed Orifices. If the entrance to an orifice is gradually rounded, the contraction of the jet will be almost entirely suppressed, that is, C_c will be nearly unity. In any case, the discharge will be materially increased.

The area of a bell-mouthed orifice is measured at its smaller or outlet end. Weisbach tested the orifice shown in Fig. 40, which was made to conform approximately to the shape of a jet from an orifice in a thin wall.

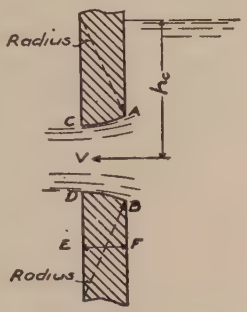


Fig. 40. — Bell-Mouthed Orifice

$AB = 0.131$ foot $\quad CD = 0.103$ foot
$EF = 0.06$ foot $\quad$ radius $= 0.112$ foot

The ratio between the area at CD and AB is 0.62. The coefficient of discharge under heads of from 0.4–1.9 feet was 0.96. With a somewhat more rounded orifice, Weisbach secured a still higher coefficient of discharge.

The orifice shown in Fig. 41 is Ellis's[1] horizontal square submerged orifice with a wooden mouthpiece attached. The section of the opening was a quarter ellipse of which the semidiameters were 0.5 foot and 0.33 foot.

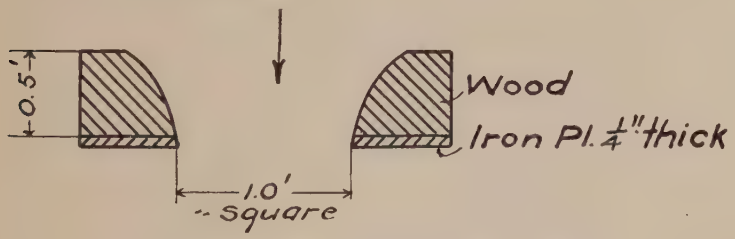


Fig. 41. — Ellis's Bell-Mouthed Orifice

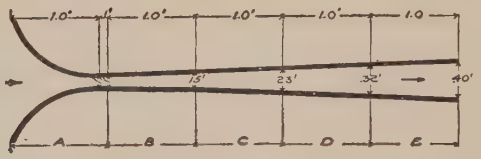
The coefficient for this orifice under a head of from 2 to 18 feet averaged 0.94. The same orifice without the wooden mouthpiece gave a coefficient of 0.60.

The tube shown in Fig. 42 is from experiments of J. B. Francis.[2] The smallest diameter was 0.1 foot.

Flow through an orifice may be increased by the application of a diverging tube. This tube acts as an exhausting pump and

[1] Transactions A. S. C. E., vol. 5 (1876), p. 61.
[2] *Lowell Hydraulic Experiments*, 4th ed., pp. 212 *et seq.*

the result is the same as if the orifice were discharging into a partial vacuum. Consider a bell-mouthed entrance, such as that



Parts	Head, Ft.	C
A	.033 to 1.51	0.81–0.94
A & *B*	.02 " 1.47	1.15–1.59
A, *B*, & *C*	.014 " 1.31	1.42–2.16
A, *B*, *C*, & *D*	.014 " 1.36	1.38–2.43
Whole tube	.014 " 1.41	1.44–2.42

FIG. 42. — FRANCIS'S COMPOUND TUBE

shown in Fig. 42, to which a diverging tube has been added. Assume for the sake of simplicity an entire absence of friction. Let

a = area at minimum section
A = area at final outlet
v = velocity at minimum section
V = velocity at discharge from tube
p_a = atmospheric pressure
p_0 = pressure at minimum section.

By Bernoulli's theorem, we may write

$$h_c + \frac{p_a}{w} = \frac{v^2}{2g} + \frac{p_0}{w} = \frac{V^2}{2g} + \frac{p_a}{w}.$$

Since the tube is flaring, we have $v > V$; hence $p_0 < p_a$. If we set $p_a = p_0 + B$, we shall have

$$h_c + \frac{p_0 + B}{w} = \frac{v^2}{2g} + \frac{p_0}{w},$$

$$h_c = \frac{v^2}{2g} - \frac{B}{w},$$

$$h_c + \frac{B}{w} = \frac{v^2}{2g}.$$

In other words, the total head producing the discharge at a is greater than the actual head on the orifice, and the coefficient of discharge as applied to a is greater than unity.

The draft tube as applied to the discharge side of a water wheel acts in the same manner. A portion of the velocity head of the discharged water is converted into a negative pressure at the discharge side of the wheel, thus increasing the effective head acting on the wheel.

When a compound tube is formed by adding a diverging tube to an orifice, the amount of increase in the coefficient of discharge of the orifice is determined by the amount of flare of the diverging tube. The coefficient may run as high as 5.00, approximately.

166. Sluice Gates. The coefficient of discharge of a sluice gate may vary from 0.60 to 0.95 or more, depending upon the type and setting of the gate. In most large gates contraction is largely suppressed, on the bottom, at least, if not on the sides. In the case of smaller gates, especially if they discharge freely into the air, the contraction is often more complete than it is with the larger gates. Some of the high head sluice gates in the dams built by the Reclamation Service are shaped like the nozzles of a Pelton wheel and have a coefficient of about 0.95.

In analyzing a sluice gate, in order to arrive at an approximation of its coefficient, there are three main factors determining the discharge which must be considered. They are the velocity of approach, the amount which the contraction is suppressed (it is very seldom that complete contraction is secured, or is desirable, in an actual gate), and the short tube effect, if any, caused by continuing the walls beyond the plane of the gate. A careful analysis of a sluice gate made with due regard to these factors, combined with a study of the coefficients given in § 152 and Table VI, will usually result in a coefficient which is reasonably correct. *The true coefficient of discharge can be obtained only by actual calibration under operating conditions.*

A head or sluice gate may often be calibrated and used to determine the amount of water drawn. When the gate discharges submerged, the effect of backwater on the discharge must not be neglected.

Some experiments on submerged 4-foot tubes of different lengths and with different forms of entrances were made by C. B. Stewart at the University of Wisconsin. A study of the results will be of considerable help in determining the probable coefficient of discharge of a sluice gate or head gate.

TABLE VI

SUBMERGED SLUICE. VALUE OF COEFFICIENTS[1] OF DISCHARGE FOR FLOW THROUGH HORIZONTAL SUBMERGED TUBE, 4 FEET SQUARE, FOR VARIOUS LENGTHS, LOSSES OF HEAD, AND FORMS OF ENTRANCE AND OUTLET

LOSS OF HEAD IN FEET	FORMS OF ENTRANCE AND OUTLET	LENGTH OF TUBE, IN FEET						
		0.31	0.62	1.25	2.50	5.00	10.0	14.0
.05	*A*	.631	.650	.672	.769	.807	.824	.838
	a	.672			.742	.810		.848
	b	.740			.769	.832		.862
	c	.834			.769	.875		.890
	c'							.875
	d	.948			.943	.940	.927	.931
.10	*A*	.611	.631	.647	.718	.763	.780	.795
	a	.636			.698	.771		.801
	b	.685			.718	.791		.813
	c	.772			.718	.828		.841
	c'							.830
	d	.932			.911	.899	.892	.893
.15	*A*	.609	.628	.644	.708	.758	.779	.794
	a	.630			.689	.767		.803
	b	.677			.708	.787		.814
	c	.765			.708	.828		.839
	c'							.829
	d	.936			.910	.899	.893	.894
.20	*A*	.609	.630	.647	.711	.768	.794	.809
	a	.632			.694	.777		.819
	b	.678			.711	.796		.833
	c	.771			.711	.838		.856
	c'							.846
	d	.948			.923	.911	.906	.905
.25	*A*	.610	.634	.652	.720	.782	.812	.828
	a	.634			.705	.790		
	b	.683			.720	.809		
	c	.779			.720	.854		
	c'							
	d	.965			.938	.928		
.30	*A*	.614	.639	.660	.731	.796	.832	.850
	a	.639						
	b	.689						
	c	.788						
	c'							
	d	.980						

Results of experiments made at the hydraulic laboratory of the University of Wisconsin by C. B. Stewart, on the discharge through orifices and tubes four feet square and of various thicknesses or lengths and with various conditions of contraction. In computing *C*, velocity of approach was neglected, as it was low.

[1] ENGINEERING NEWS, Jan. 9, 1909.

A. Entrance; all corners 90 degrees.
Outlet; tube projecting into water on downstream side of bulkhead.

a. Entrance; contraction suppressed on bottom.
Outlet; tube projecting into water on downstream side of bulkhead.

b. Entrance; contraction suppressed on bottom and one side.
Outlet; tube projecting into water on downstream side of bulkhead.

c. Entrance; contraction suppressed on bottom and two sides.
Outlet; tube projecting into water on downstream side of bulkhead.

c'. Entrance; contraction suppressed on bottom and two sides.
Outlet; square corners with bulkhead to sides of channel preventing the return current along the sides of the tube.

d. Entrance; contraction suppressed on bottom, two sides, and top.
Outlet; tube projecting into water on downstream side of bulkhead.

167. Contraction. For purposes of measurement of flow it is desirable to have complete contraction. For practically all other purposes, however, it is desirable that the contraction be suppressed as much as possible, not only to minimize hydraulic losses by smooth transition, but also to allow a more economic design of the conduit. Not only should all abrupt corners be avoided in hydraulic design, but any changes in velocity should be made gradually by means of a converging or diverging chamber. The basic principle of all hydraulic design is that the water should be led at the minimum velocity through channels of maximum smoothness, and all bends should be smooth and easy.

PROBLEMS

1. Compute the discharge through a Borda's mouthpiece 2 inches in diameter under a head of 8 feet.

2. Compute the discharge through a reentrant short tube $1\frac{1}{2}$ inches in diameter under a head of 15 feet.

3. Compute the discharge through a short tube 2 inches in diameter under a head of 2 feet. Compute the intensity of pressure inside the tube around the contracted area.

4. If the coefficient of contraction C_c of the contracted section of the vein discharging through a short tube is 0.68, and C_v is 0.98, find negative pressure head in terms of h_c.

5. Compute the discharge through the bell-mouthed horizontal orifice 1 foot square under a head of 10 feet; also compute for an orifice in a thin wall of the same dimensions, and under the same head.

6. A tube shaped somewhat like Francis's compound tube (Fig. 42) has the following dimensions: diameter of inlet, 1.0 foot; diameter of throat, 0.2 foot; diameter at outlet, 0.8 foot. The length from inlet to throat is 3.0 feet, and from throat to outlet 9.0 feet. Determine the mean velocity in the throat, the intensity of pressure in the throat, and the probable discharge through the tube, if the head is 16 feet.

CHAPTER XII

NOZZLES AND FIRE HOSE

168. Nozzles. A nozzle is a conical converging tube attached to the outlet end of a pipe having the same internal diameter as the base of the nozzle. It is merely an orifice with the contraction suppressed. and usually has a very high velocity of

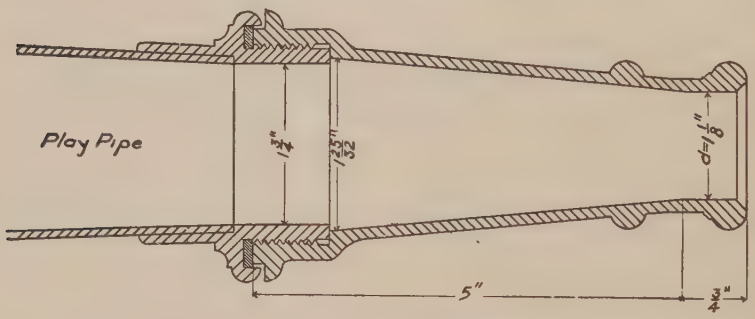


Fig. 43. — Smooth Nozzle

approach. Orifice formulas apply, therefore, to nozzles; but must include the effect of the head due to velocity of approach, which is usually a considerable part of the total head.

Smooth nozzles. Nozzles are commonly classified as smooth and ring nozzles. Smooth nozzles are gradually tapering bores

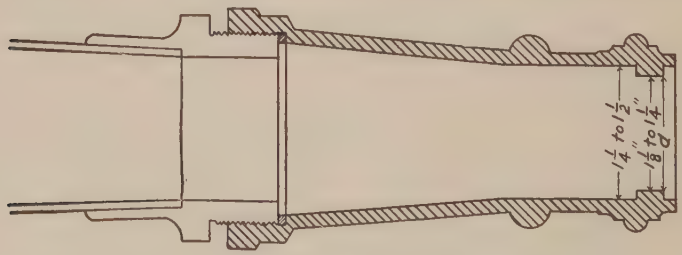


Fig. 44. — Square Ring Nozzle

having no interior constrictions to cause contraction of the jet. Figure 43 shows a smooth nozzle of the best type; its coefficient C is about 0.977.

Ring nozzles. Ring nozzles are gradually tapering bores into which are fitted orifices or short tubes which are not only of smaller diameter than the smaller end of the bores, but which, on account of their form, will cause marked contraction of the jets as they leave the nozzles. The coefficients of discharge

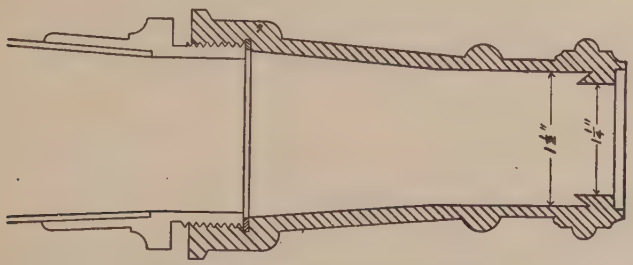


FIG. 45. — UNDERCUT RING NOZZLE

depend upon the contraction of the jet, which depends upon the form of the edge of the orifice, and the ratio of its area to that of the nozzle just behind the ring.

Figure 44 illustrates a square ring nozzle, for which $C = 0.74$.

Figure 45 illustrates an ordinary knife-edge or undercut ring nozzle, for which $C = 0.713$.

Figure 46 illustrates an extreme form of knife-edge or undercut ring nozzle; it is practically a Borda's mouthpiece fitted into a ring nozzle. The coefficient of this particular nozzle is 0.582.

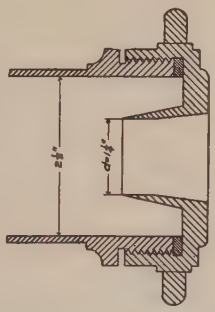


FIG. 46. — EXTREME FORM OF UNDERCUT RING NOZZLE

169. Discharge Formulas. Figure 47 shows a nozzle arranged for measuring discharge. Formula for discharge in terms of the pressure head and the diameters at the base and orifice of the nozzle may be derived from Bernoulli's theorem as follows.

Let D = the diameter in feet of the pipe at the section where the pressure head is measured; viz. just upstream from the base (entrance) of the nozzle or of the play pipe if one is used.

A = the area in square feet corresponding to diameter D.

d = the diameter in feet of the nozzle orifice.

a = the area in square feet corresponding to diameter d.

V = the actual velocity in feet per second at the smallest section of the jet.

V_t = the theoretical velocity in feet per second corresponding to the total head.

V_A = the actual velocity in feet per second in the section where the pressure head is measured; *velocity of approach*. On Fig. 47 written V_a.

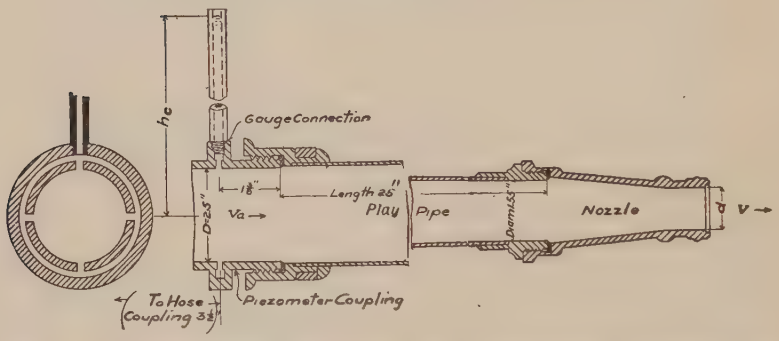


FIG. 47. — NOZZLE WITH PLAY PIPE AND PIEZOMETER ARRANGED FOR MEASURING DISCHARGE

C = coefficient of discharge = $C_v C_c$.

C_v = coefficient of velocity.

C_c = coefficient of contraction.

h_c = piezometer reading, in feet of water, at the base of the nozzle, reduced to the elevation of the center of the nozzle orifice.

p_c = piezometer reading in pounds per square inch corresponding to h_c.

H_c = piezometer reading (h_c) + velocity head $\left(\frac{V_A^2}{2g}\right)$; the total hydrostatic head (in feet).

$p = \left(h_c + \frac{V_A^2}{2g}\right) 0.433$ = total head in pounds per square inch.

Q = the actual discharge in cubic feet per second.

By Bernoulli's theorem, first neglecting irrecoverable losses of energy, we have

$$(1) \qquad \frac{p_a}{0.433} + h_c + \frac{V_A^2}{2\,g} = \frac{p_a}{0.433} + \frac{V_t^2}{2\,g} = \frac{p_a}{0.433} + H_c.$$

But, if the discharge is steady, we may write

$$(2) \qquad Q = AV_A = C_c a C_v V_t = CaV_t.$$

Therefore

$$(3) \qquad V_A = \frac{CaV_t}{A} = \frac{Cd^2V_t}{D^2}.$$

Substituting in (1) the value of V_A from (3), we find

$$(4) \qquad h_c + \frac{C^2d^4V_t^2}{D^4\,2\,g} = \frac{V_t^2}{2\,g}.$$

Transposing, we have

$$(5) \qquad \frac{V_t^2}{2\,g} = \frac{h_c}{1 - C^2\frac{d^4}{D^4}} = H_c, \qquad V_t = \left(\frac{2\,gh_c}{1 - C^2\frac{d^4}{D^4}}\right)^{1/2} = (2\,gH_c)^{1/2}.$$

Therefore, the actual velocity V is

$$(6) \qquad V = C_v\left(\frac{2\,gh_c}{1 - C^2\frac{d^4}{D^4}}\right)^{1/2} = C_v(2\,gH_c)^{1/2},$$

and the actual discharge Q is

$$(7) \qquad Q = 8.02\,C\,\frac{\pi d^2}{4}\left(\frac{h_c}{1 - C^2\frac{d^4}{D^4}}\right)^{1/2} = 6.3\,C\,d^2\left(\frac{h_c}{1 - C^2\frac{d^4}{D^4}}\right)^{1/2}$$
$$= 6.3\,C\,d^2(H_c)^{1/2}.$$

170. Freeman's Experiments.[1] Experimental coefficients of discharge have been determined by Hamilton Smith, Jr., Ellis, Weston, Freeman, and others. The experiments previous to Freeman's were limited to a few of the many types of nozzles in use. In 1888 John R. Freeman conducted at Lawrence, Mass., an extensive set of experiments[2] upon the discharge of water through fire hose, and through nozzles $\frac{7}{8}$ to $1\frac{1}{2}$ inches in diameter, of many types. His observations were made with a high degree

[1] TRANSACTIONS A. S. C. E., vol. 21, pp. 303–482.
[2] For the Associated Factory Mutual Insurance Companies.

of precision, and settled many important questions concerning fire-stream discharge. The chief subjects investigated were:

Comparative efficiency of different kinds of nozzles.
Coefficients of discharge of nozzles.
Loss of pressure due to friction in various kinds of fire hose.
Loss of pressure due to curves in fire hose.
Loss of pressure due to reduction of waterway at couplings.
The effects of area and form of nozzle and of variations in pressure, upon the height and distance reached by jets.
Distribution of velocities in jets.

In 1890 [1] Freeman made a second set of experiments upon smooth cone nozzles $1\frac{3}{4}$, 2, and $2\frac{1}{2}$ inches in diameter, to determine the coefficients of discharge, and to demonstrate the value of nozzles as water meters.

171. Freeman's Apparatus. The arrangement of the apparatus used in both series by Freeman was essentially as shown in

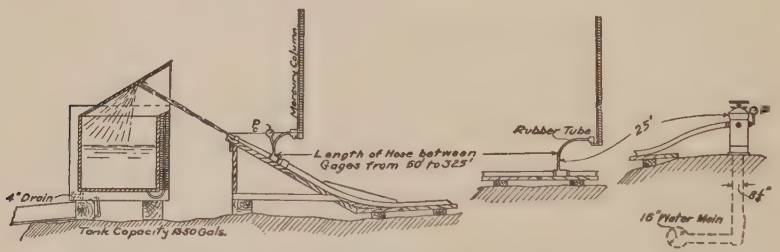


FIG. 48. — LAYOUT OF APPARATUS IN FREEMAN'S EXPERIMENTS

Fig. 48. This was modified somewhat in different experiments (Fig. 49).

In some observations, a screen of wire netting was inserted at the point B, Fig. 49; in others, it was absent. Three longitudinal diaphragms or rifle-blades about 16 inches long were inserted in some experiments, but not in others. Again the pipe BC was for some observations removed and a piezometer coupling only about three inches long with six equally spaced holes was attached directly to the siamese coupling. None of these changes affected the discharge coefficients more than about one per cent, and usually much less.

[1] TRANSACTIONS A. S. C. E., vol. 24, pp. 492–527.

172. Freeman's Measurements of Quantity. The quantities discharged were measured by volume in a rigid wooden tank lined with sheet zinc. The error in measurements of quantity probably did not exceed one-third of one per cent.

The time of each experiment was determined by a stop watch in the first series, and by an electric chronograph in the second set. The duration of each experiment was controlled by deflecting the

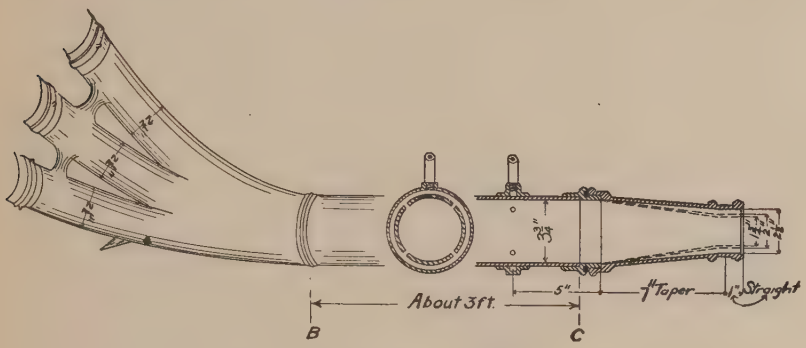


FIG. 49. — FREEMAN'S NOZZLE OF 1890

nozzle by hand. The error in time of filling the tank probably did not exceed one-fifth of one per cent.

The discharges in gallons per minute in the first set of experiments were probably measured with an error not exceeding one-half of one per cent, and the average error was less. In the second set, the limit of error was about one-tenth of one per cent.

173. Freeman's Measurements of Pressure or Head. The gages used for measuring intensities of pressure were open mercury columns in glass tubes 20 feet in height and $\frac{3}{16}$-inch internal diameter. Measurements were usually checked by Bourdon gages, but for some rougher determinations they were used alone. Figure 50 shows one of the mercury gages.

A gage of this sort is very useful if high pressures must be dealt with, and if a greater accuracy is desired than usually can be obtained with a Bourdon gage. One point must be noted regarding the use of a gage of this sort. As the mercury is forced up into the tube, the level in the mercury pot falls, and the zero of the gage changes.

All things considered, it would seem reasonable to expect these mercury gages, including corrections applied, to measure a pressure correctly within one-tenth of one per cent.

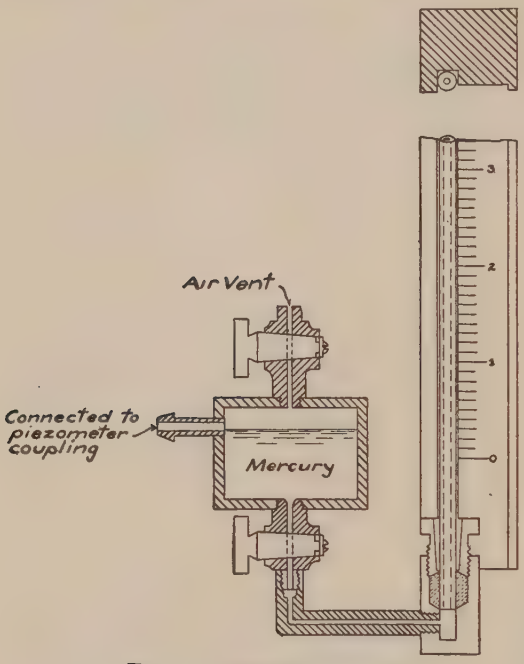


Fig. 50. — Mercury Gage

Piezometer couplings were used whenever the internal pressure in a pipe was to be measured. Such a coupling, frequently called a piezometer ring, should be used if precise determinations of pressure are desired. It can be connected to any form of gage. Figure 51 shows the coupling used in the first set of experiments, and Fig. 49 shows those used in the second set. Still another was used similar to that in Fig. 49, but shorter, and having six holes instead of four.

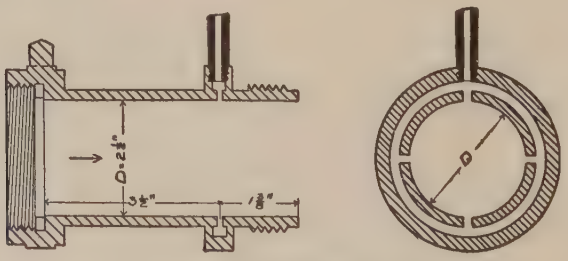


Fig. 51. — A Piezometer Coupling

174. Freeman's Measurements of Loss of Pressure. The fire hose was laid on a straight smooth plank walk in lengths of 50 to 325 feet, usually 150 to 300. One piezometer coupling with

gage was located about 25 feet from the hydrant, another at the base of the play pipe. (See Fig. 48.) The difference in pressure between the two gages was observed for various quantities of water flowing through the hose as measured in the tank. This difference in pressure, for any given flow, when corrected for differences in elevation between gages and hose, is the head or pressure lost; here expressed in pounds per square inch. The total difference divided by the length of hose between piezometers is the head or pressure lost per linear foot of hose, which included the loss in couplings.

175. Comparative Efficiency of Various Nozzles. For throwing streams and for measuring flow, the smooth cone nozzle with a simple play pipe, as shown in Fig. 47, was found superior to any other type.

176. Determination of Coefficients of Discharge for Smooth Nozzles. The coefficients of discharge were found by dividing the actual discharge in gallons per minute by the theoretical discharge.

Even though the nozzles were of different interior form and length, the coefficients are nearly uniform. The nozzles used in the experiments of 1888 were selected at random from stock. Those used in 1890 were especially made, but were of about the same quality. Figure 52 is a logarithmic diagram showing the relation between indicated pressure and discharge for various cone nozzles.

For smooth cone nozzles of various sizes, the coefficients given in Table VII were determined. The coefficients are applicable with or without a play pipe if the pipe and the nozzle base are of the same diameter and free from restrictions or enlargements.

177. Coefficients for Ring Nozzles. For ring nozzles, the contraction varied; therefore the coefficient changed with every variation in the ratio of nozzle orifice area to the area of the nozzle just behind the ring. The coefficients derived from Freeman's experiments on ring nozzles were graphically shown by a curve from which the values in Table VIII were read, which, until more complete experiments are presented, may be found of use in indicating the probable value of coefficients for similar cases. Freeman's experiments included many other forms of nozzles, but the results are too voluminous to be given.

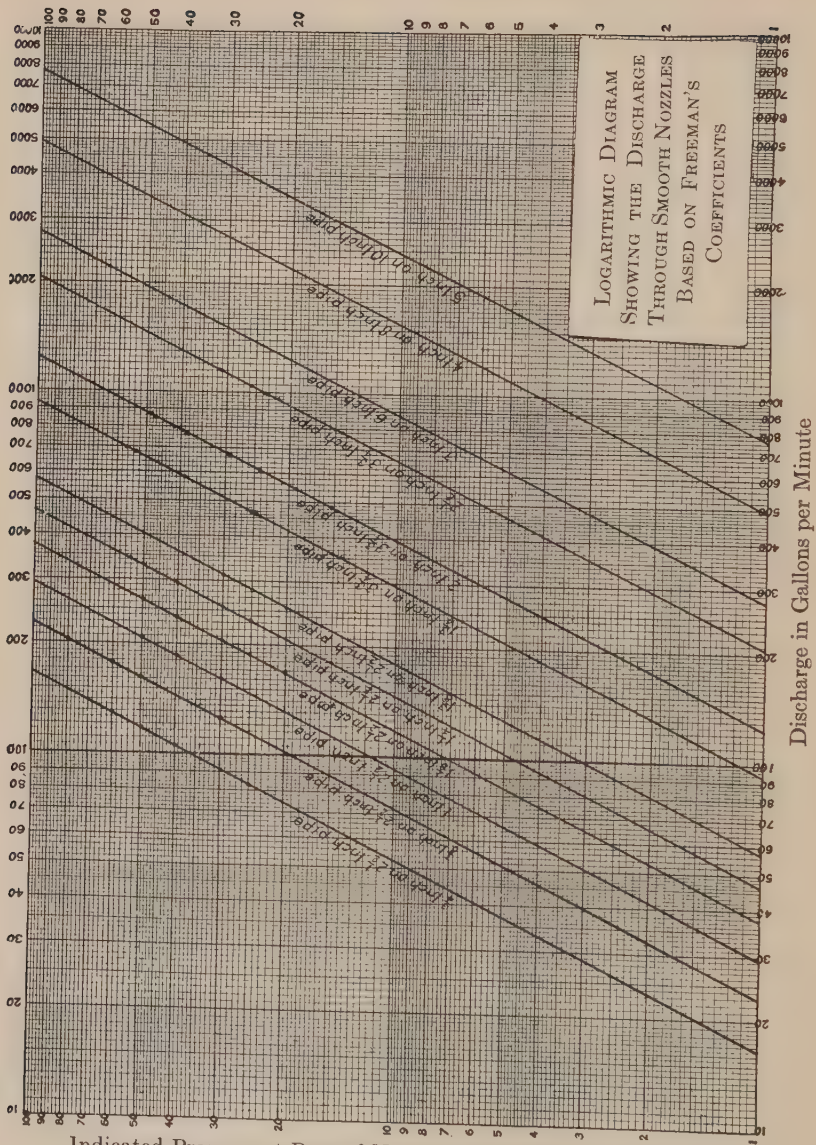

Logarithmic Diagram
Showing the Discharge
Through Smooth Nozzles
Based on Freeman's
Coefficients
Discharge in Gallons per Minute
Indicated Pressure at Base of Nozzle, in Pounds per Square Inch

Fig. 52

Coefficients for square ring nozzles are given in Table VIII. Values of C printed in italics are below the limits of Freeman's experiments.

TABLE VII

COEFFICIENTS OF DISCHARGE FOR SMOOTH NOZZLES

Nominal Diameter, Inches	Actual Diameter, Inches	Number of Observations	Observed Pressures, Pounds Sq. In. at Base of Play Pipe	Discharge Measured in Tank, Gals. per Minute	Coefficients of Discharge (Mean Value)	Remarks on Arrangement of Nozzles and Connections
			Experiments of 1888			
0.75	0.751	4	25 to 60	83.4 to 129.3	.983	As shown in figures 47 and 48
0.875	0.880	7	30 to 80	125.0 to 205.2	.982	As shown in figures 47 and 48
1.000	1.001	3	30 to 50	161.0 to 208.5	.972	As shown in figures 47 and 48
1.125	1.126	15	20 to 50	169.0 to 266.5	.976	As shown in figures 47 and 48
1.25	1.250	18	15 to 54	180.4 to 344.2	.971	As shown in figures 47 and 48
1.375	1.375	12	20 to 30	252.0 to 310.6	.959[1] (.977)	As shown in figures 47 and 48 (If larger play pipe were used)
	1.55	3	12 to 20	261.0 to 338.5	.976	Play pipe without nozzle of size shown in figure 47
			Experiments of 1890			
1.75	1.748	13	14.16 to 56.42	350.4 to 699.6	.999	Connected to pipe or siamese by piezometer coupling (figure 49) in the various ways described in § 174
2.00	1.998	17	14.88 to 48.62	474.2 to 864.8	.996	
2.50	2.4995	13	13.80 to 31.25	763.6 to 1156.0	.997	
1.75	1.748	5	20.14 to 57.48	406.1 to 689.3	.995	Connected to cast-iron cylinder 15 inches in diameter by a flaring tin cone
2.00	1.998	9	5.03 to 51.45	265.5 to 850.	.995	
2.50	2.4995	5	19.91 to 37.86	820.7 to 1132.0	.987	
For large sizes up to 6″; heads more than 10′; and angle of convergence 10° to 15°					.995	Suggested by Freeman

TABLE VIII

$$\text{Ratio} = \frac{\text{Diameter of outlet of nozzle}}{\text{Diameter of nozzle area just behind the ring}}$$

Ratio	.1	.2	.3	.4	.5	.6	.7	.8	.85	.9	.95	1.0
C	*.60*	*.61*	*.61*	*.62*	.63	.65	.68	.71	.73	.77	.87	.975

[1] Throat of play pipe was too small for nozzle of this size; hence low coefficient.

178. Loss of Pressure due to Friction in Hose. The character of the interior surface of the hose as to smoothness is of the utmost importance in determining friction losses; in unlined linen hose the loss per foot was found to be about 2.5 times that for smooth rubber hose of the same diameter in which the friction loss was least.

The precise diameter of the hose must be used in calculating velocities in hose (or pipes), as Freeman found considerable difference between the nominal and actual diameters of hose.

The friction loss varied nearly, but not precisely, as the square of the velocity of flow.

The following Table IX summarizes the head lost in friction with different quantities of discharge for various kinds of hose; also gives coefficients C_z for use in the Chezy formula for flow of water in pipes. C_z is used here only to distinguish it from the nozzle coefficient. The letters in Table IX refer to the following kinds of hose:[1]

A, solid rubber, very smooth.
C, woven cotton, rubber-lined, a heavy regular fire hose, interior surface almost free from ridges.
D, nearly the same as C, but of lighter weight.
E, knit cotton, rubber-lined, medium smooth.
I, like E, but rougher.
M, same as D, but of smaller diameter.
K, woven cotton, rubber-lined, cheap, with medium thin lining, when under pressure interior is full of small ridges.
L, unlined linen hose, good quality.

179. Loss of Pressure due to Curves in Fire Hose. For fire hose laid in its ordinary smooth curves, but not cramped or kinked, the friction loss will be about 6 per cent greater than in perfectly straight hose. Table X summarizes the effect in increasing the loss of head when hose was laid in curves of 2, 3, or 4 feet radii.

180. Loss of Head at Couplings. The loss of head due to reduction of the area of the stream at the couplings was found to be about equal to the sum of the loss in sudden contraction on entering the bushing, and the loss in sudden enlargement in

[1] See photographs of casts of interior surface and a more complete description of each kind, on pages 347 to 353 in Freeman's original article.

TABLE IX

Friction Head in Fire Hose

Kind of Hose	Nominal Diameter	Actual Average Diameter, Inches	Diameter of Couplings	Mean Velocity in Hose, Ft. per Sec.	Discharge, Gallons per Minute	Lost Head per 100 Ft. Length Corrected for Expansion, Lbs. per Sq. In.	Lost Head per Foot of Length in Feet	C_2 in Chezy Formula $V = C_2\sqrt{RS}$	Mean Value f = Lost Head per 100 Ft., $G = 240$ $\Delta = 2\frac{1}{2}$
	Inches	Δ	Inches	V	G		S		Lbs. per S. In.
A	2.5	2.65	2.52	12.50	214.9	8.06	.1863	123.3	13.4
				13.96	240.0	10.00	.2311	123.6	
				17.00	292.2	14.73	.3404	124.0	
				20.00	343.8	20.40	.4714	124.0	
C	2.5	2.47	2.53	13.40	200.1	10.66	.2464	119.1	14.1
				16.07	240.0	15.00	.3467	120.4	
				20.00	298.7	22.80	.5269	121.5	
D	2.5	2.49	2.47	13.20	200.3	10.50	.2427	117.7	14.2
				15.81	240.0	14.50	.3351	120.0	
				19.00	288.4	20.48	.4733	121.3	
				21.00	318.7	24.70	.5708	122.1	
E	2.5	2.68	2.50	7.50	131.9	3.50	.0809	111.6	16.0
				10.00	175.8	6.23	.1440	111.6	
				13.65	240.0	11.30	.2611	112.3	
				17.00	298.9	17.01	.3931	114.8	
I	2.5	2.69	2.51	11.50	203.7	10.20	.2357	100.1	19.4
				13.55	240.0	13.50	.3120	101.7	
				16.00	283.4	18.96	.4382	102.2	
				18.00	318.9	22.35	.5165	105.8	
M	2.0	2.12	2.07	14.00	154.0	14.92	.3448	113.4	14.6
				17.00	187.0	21.19	.4897	114.8	
				21.81	240.0	33.20	.7673	118.4	
K	2.5	2.53	2.48	3.50	54.8	1.13	.0261	94.3	25.5
				5.00	78.4	2.52	.0582	90.3	
				7.50	117.5	5.99	.1384	87.8	
				10.00	156.7	10.60	.2450	88.0	
				12.00	188.0	14.82	.3425	89.3	
				15.31	240.0	24.10	.5570	89.3	
				17.00	266.4	29.20	.6748	90.1	
				19.00	297.7	35.76	.8264	91.0	
L	2.5	2.60	2.50	3.50	57.9	1.79	.0414	73.9	33.2
				5.00	82.7	3.63	.0839	74.1	
				7.50	124.1	7.99	.1846	74.9	
				10.00	165.5	13.50	.3120	76.9	
				14.50	240.0	27.20	.6286	78.5	$f = \frac{204600}{C_2^2}$
				17.00	281.3	37.03	.8558	78.9	
				19.00	331.0	50.30	1.1624	79.6	

TABLE X

Loss of Head Due to Curves

Length of Hose, 57.2 Feet

Kind of Hose	Actual Diameter, Inches	Mean Flow, G. Gallons per Min.	Loss in Head for Flow in Col. 3, Pounds per Sq. In.	Increased Loss in Head Due to Curvatures	Increased Loss in Head Divided by Velocity Head	Degree of Curvatures	Alignment, Except as Noted, Straight
Sample D	2.49	258	10.04	0.47	0.24	180°	4 curves, 2 feet radius, each
Sample D	2.49	293	12.66	0.44	0.17	180°	45°, each 2 feet straight
Sample D	2.49	323	15.18	0.61	0.20	180°	between each curve
Sample D	2.49	258	10.13	0.56	0.29	360°	1 circle, 2 feet radius to
Sample D	2.49	293	12.90	0.68	0.27	360°	center of hose
Sample D	2.49	323	15.40	0.83	0.27	360°	
Sample D	2.49	258	10.23	0.66	0.34	360°	4 curves, 2 feet radius, 90°
Sample D	2.49	293	12.96	0.74	0.29	360°	each, 2 feet straight be-
Sample D	2.49	323	15.33	0.76	0.25	360°	tween each curve (X)
Sample D	2.49	258	10.33	0.76	0.39	360°	4 curves, 3 feet radius, 90°
Sample D	2.49	293	12.95	0.73	0.29	360°	each, 3 feet straight be-
Sample D	2.49	323	15.62	1.05	0.24	360°	tween each curve (Y)
Sample D	2.49	258	10.43	0.86	0.44	360°	4 curves, 4 feet radius, 90°
Sample D	2.49	293	13.19	0.97	0.39	360°	each, 4 feet straight be-
Sample D	2.49	323	15.73	1.15	0.38	360°	teen each curve (Z)
Sample A	2.64	304	9.04	0.49	0.23	360°	Like X
Sample A	2.64	304	9.28	0.73	0.35	360°	Like Y
Sample A	2.64	304	9.32	0.77	0.37	360°	Like Z

leaving the bushing (see §§ 301, 303). For example, for a $2\frac{1}{4}$-inch hose coupling on a $2\frac{1}{2}$-inch hose for a discharge of 240 gallons per minute, the loss would be about 0.25 pound per bushing; or, since hose is usually in 50-foot lengths, 0.50 pound per 100 feet. For washers smaller than the hose the loss is about 1.25 times the loss of head, due to sudden contraction of the stream, assuming the coefficients of contraction as for ring nozzles (see § 304).

181. Vertical and Horizontal Distance attained by Jets from Nozzles. Theoretically, the height attained should be equal to the pressure head plus the velocity head; actually, it is less.

For nozzles up to $1\frac{1}{2}$ inches, with the same head, the larger nozzles will give a higher jet than the smaller ones. For vertical and horizontal distances, see Table XI.

The height J of the extreme drops of vertical jets in still air from nozzles $\frac{3}{4}$ to $1\frac{3}{8}$ inches diameter may be computed by the following formula from Freeman:

$$J = H_c - 0.00135 \frac{H_c^2}{C_c^{1/2} d},$$

where d is the nozzle diameter measured in inches.

The height of a thoroughly first-class fire stream will be

When J in feet equals	50	75	100	125	150
Height in per cent of J	82%	79%	73%	67%	63%

182. Distribution of Velocity in Jets from a Nozzle. By experiments with a Pitot tube, the distribution of velocity was found to be uniform throughout the jet from the best nozzles except near the periphery of the jet. The actual velocity on the central portion of the jet was equal to the full theoretical velocity due to the head H_c.

183. Freeman's Formulas. In the following formulas derived by Freeman for use in computing fire-stream discharge, the symbols used were as follows. All diameters of pipes and orifices are given in inches. Other symbols are identical with those given in § 169.

P = intensity of pressure at the top of the hydrant in pounds per square inch.

If the nozzle is higher than the top of the hydrant, subtract from the hydrant pressure the difference in elevation in feet multiplied by 0.433; if the nozzle is lower, add the difference in pressure.

O = coefficient of discharge of hydrant nipple: for sharp square corner, 0.82; for well-rounded entrance, 1.0; for ordinary hydrant the value determined was 0.71.

L = length of the hose in feet.

d = diameter of the nozzle orifice in inches.

D = diameter of channel where pressure is measured in inches.

D_h = diameter of hydrant nipple in inches.

Δ = actual diameter of hose in inches.

G = discharge in gallons of 231 cubic inches per minute.

F = the loss of head in pounds per square inch per 100 feet of hose with 240 gallons per minute flowing for any given kind of hose; depends upon diameter, roughness, obstructions, and curves.

For unlined linen hose $2\frac{1}{2}$ inches diameter, $F = 30$.

For inferior rubber-lined hose $2\frac{1}{2}$ inches diameter, $F = 26$.

For smoothest rubber-lined hose $2\frac{1}{2}$ inches diameter, $F = 13$.

The pressure lost per 100 feet in sinuosities is about one pound per square inch, when $G = 240$.

The pressure lost per 100 feet for ordinary obstructions is about 0.5 pound per square inch, when $G = 240$.

Given d, D, and h_c, to find Q, we have

$$Q = 0.04374\, Cd^2\left[\frac{h_c}{1 - C^2\left(\frac{d}{D}\right)^4}\right]^{1/2}. \tag{8}$$

Example 1. Given, a $1\frac{1}{4}$-inch smooth nozzle attached to $2\frac{1}{2}$-inch pipe, and the observed head in a piezometer at the base of the nozzle, 100 feet. Compute the discharge in cubic feet per second.

$$Q = 0.04374 \times 0.971 \times 1.25^2\left[\frac{100}{1 - 0.971^2\left(\frac{1.25}{2.50}\right)^4}\right]^{1/2} = 0.684$$

For discharge from hose with open butts, that is, hose without nozzle or play-pipe, the formulas are

$$P = G^2\left(\frac{0.001123}{D_h{}^4O^2} + 0.0000001736\, LF\right),$$

$$G = \left(\frac{P}{\frac{0.001123}{D_h{}^4O^2} + 0.0000001736\, LF}\right)^{1/2}.$$

Example 2. Compute the discharge through 400 feet of unlined $2\frac{1}{2}$-inch linen hose with open butt, which is attached to a fire hydrant in which the pressure is 100 pounds per square inch. Let $D_h = 2\frac{1}{2}$ and $O = 0.82$.

$$G = \left(\frac{100}{\frac{0.001123}{2.5^4 \times 0.82^2} + 0.0000001736 \times 400 \times 30}\right)^{1/2} = 217.$$

184. Application. Nozzles may be used satisfactorily to measure the discharge from pipes under pressure wherever a pipe can be installed of sufficient length to establish uniform flow before

the water reaches the nozzle. Nozzles are especially useful in measuring discharge through pipes or fire hose, or in testing the performance of pumps or impulse water wheels. Such a test involves in many cases the loss of all the water measured; and in all cases, except in the case of impulse wheels in service, a waste of the energy of the jet. With nozzles to which Freeman's coefficients are applicable, discharge measurements can be made with an error not exceeding 2 per cent. The apparatus necessary to make measurements of this kind is easily transported, and the expense of making them is slight.

Of late, a special form of Pitot tube connected directly to a small calibrated Bourdon gage has come into use for measuring the discharge from nozzles. The gage is held in the hand, and the opening of the Pitot tube is placed near the center of the jet. The discharge is determined by converting the recorded pressure into velocity, applying a nozzle discharge coefficient, and multiplying by the area of the nozzle discharge. This method of measurement is found more accurate on the whole than the method in which the hydrant pressure is used, since it eliminates the question of the condition of the hose and also simplifies the measurement and allows one gage to be used for a number of streams.

185. Fire-Stream Table. Table XI, which for the most part is taken from a similar table by Freeman, will give results sufficiently accurate for fire stream computations. The values of F used are given in § 183. Values of G vary as the square root of p_c or p; values in other columns vary directly as p_c or p.

Examples. (*a*) Given $p_c = 80$, for a $1\frac{1}{8}$-inch smooth nozzle attached to 400 feet of best quality rubber-lined hose. What pressure P is required at the hydrant, assuming the nozzle orifice and hydrant top to be at the same elevation?

From the table, the lost head for first 100 feet = 112; for each additional 100 feet, 25.5. Therefore $P = 112 + 3 \times 25.5 = 188$ pounds per square inch.

(*b*) Given a 1-inch smooth nozzle with 300 feet of the best quality hose, $p_c = 50$, what P will be required at the hydrant?

By direct interpolation between 40 and 60 pounds the loss for the first 100 feet as shown by the table is 62.5; for each additional 100 feet the loss is 9.8; hence

$$P = 62.5 + 2 \times 9.8 = 82.1 \quad \text{pounds per square inch.}$$

TABLE XI

Fire Stream Table

This table was computed by Freeman's formulas. Nozzles are assumed to be attached to 2½-inch play pipe, and hose 2½ inches diameter.

p_c	p	Vertical Jet		Hor. Dist. Jet Elevated 30° to 45°		G	Pressure Required at Hydrant (P) to Maintain Pressure (p) at Nozzle					
							For First 100 Ft.			Each Additional 100 Ft.		
Indicated (Gage) Pressure at Base Play Pipe	Effective (Static) Pressure at Base Play Pipe	Av. Highest Drops, Still Air	Height Good Fire Stream Modified by Wind	Av. Extreme Drops Level of Nozzle Still Air	Limit of Good Effective Fire Stream, Still Air	Gallons per Minute Discharged	Unlined Linen Hose	Inferior Rubber-lined Cotton Hose Rough Inside	Best Quality Rubber-lined Cotton Hose Smooth Inside	Unlined Linen Hose	Inferior Rubber-lined Cotton Hose Rough Inside	Best Quality Rubber-lined Cotton Hose Smooth Inside
Lbs. per Sq. In.		Ft.	Ft.	Ft.	Ft.		Lbs. per Square Inch.					
$C = 0.974$ ¾-inch Smooth Nozzle or ⅞-Inch Ring Nozzle												
20	20.2	40	33	72	29	73	23	23	22	2.8	2.4	1.2
40	40.3	78	60	112	40	104	46	46	43	5.6	4.9	2.4
60	60.5	104	72	136	54	127	69	68	65	8.4	7.3	3.6
80	80.7	123	79	153	62	147	93	91	86	11.2	9.7	4.9
100	100.8	134	83	167	68	164	116	114	108	14.0	12.1	6.0
$C = 0.974$ ⅞-Inch Smooth Nozzle or 1-Inch Ring Nozzle												
20	20.3	41	34	74	33	100	26	25	23	5.2	4.5	2.3
40	40.6	81	62	124	49	142	52	50	46	10.6	9.2	4.5
60	60.9	112	77	153	61	174	78	75	69	15.9	13.7	6.9
80	81.2	132	85	172	70	201	103	101	91	21.2	18.3	9.1
100	101.5	144	90	186	76	224	129	126	114	26.3	22.7	11.3
$C = 0.974$ 1-Inch Smooth Nozzle												
20	20.5	43	35	77	37	132	30	29	25	9.1	7.9	3.9
40	41.0	83	64	133	55	186	60	58	50	18.0	15.7	7.8
60	61.5	117	79	167	67	228	90	87	75	27.0	23.4	11.7
80	82.1	140	89	189	76	263	120	115	100	36.0	31.3	15.6
100	102.6	152	96	205	83	295	150	144	125	45.3	39.2	19.6
$C = 0.974$ 1⅛-Inch Smooth Nozzle												
20	20.8	43	36	80	38	168	36	34	28	14.7	12.8	6.4
40	41.7	84	65	142	59	238	73	69	56	29.5	25.6	12.8
60	62.5	122	83	178	72	291	109	103	84	44.1	38.3	19.1
80	83.3	146	92	203	81	336	145	138	112	59.0	51.1	25.5
100	104.1	158	99	224	89	376	182	172	140	73.6	64.0	32.0

FIRE STREAM TABLE — *(Continued)*

Lbs. per Sq. In.	Ft.	Ft.	Ft.	Ft.		Lbs. per Square Inch					
					$C = 0.974$ 1½-INCH SMOOTH NOZZLE						
20	21.2	44	37	83	40	209	45	42	32	22.7	19.8	9.9
40	42.4	86	67	148	63	296	91	84	65	45.6	39.6	19.8
60	63.7	126	85	186	76	363	136	127	97	68.6	59.6	29.8
80	84.9	150	95	213	85	419	181	169	129	91.5	79.5	39.7
100	106.1	161	101	236	93	468	226	211	162	114	98.7	49.5
					$C = 0.974$ 1⅝-INCH SMOOTH NOZZLE						
20	21.8	45	38	85	42	257	58	53	39	34.5	29.9	14.9
40	43.7	89	69	152	66	363	116	107	77	68.6	59.7	29.8
60	65.5	131	87	192	79	445	174	160	116	104.0	89.7	44.8
80	87.3	154	97	220	88	514	232	214	154	137.9	119.7	59.8
100	109.1	165	103	243	96	574	285	267	193	161.6	149.0	74.5
					$C = 0.74$ 1½-INCH RING NOZZLE						
20	20.5	42	34	76	36	126	29	28	25	8.3	7.2	3.6
40	40.9	82	62	131	53	179	59	56	49	16.7	14.5	7.2
60	61.4	115	76	164	65	219	88	84	74	25.0	21.7	10.8
80	81.8	138	86	186	73	253	117	113	98	33.4	29.0	14.5
100	102.3	150	93	202	80	283	146	141	123	41.7	36.3	18.1
					$C = 0.74$ 1¼-INCH RING NOZZLE						
20	20.7	43	35	78	37	157	34	33	27	12.8	11.2	5.6
40	41.4	83	63	138	55	222	69	65	54	25.5	22.3	11.1
60	62.1	119	79	172	68	272	103	98	81	38.3	33.5	16.7
80	82.8	143	88	196	77	314	137	130	108	51.4	44.7	22.3
100	103.5	154	95	215	84	351	171	163	135	64.1	55.8	27.0
					$C = 0.74$ 1⅜-INCH RING NOZZLE						
20	21.1	43	36	79	38	192	41	39	30	19.2	16.7	8.3
40	42.1	85	64	144	59	271	82	77	61	38.3	33.3	16.6
60	63.2	123	81	180	72	332	124	116	91	57.3	50.0	25.0
80	84.2	147	90	206	81	383	165	155	122	76.5	66.6	33.3
100	105.3	158	97	227	88	429	206	193	152	96.0	83.4	41.7

(*c*) What is the discharge through a $1\frac{3}{8}$-inch ring nozzle under 70 pounds per square inch indicated pressure? The discharge is approximately proportional to the square root of p_c or p. From the table, for 100 pounds per square inch $G = 429$. Then, for 70 pounds,

$$G = (\tfrac{70}{100})^{1/2} \times 429 = 0.837 \times 429 = 359.$$

PROBLEMS

1. Develop the formula for discharge from a nozzle and find velocity of jet and coefficient of velocity C_v, for a $1\frac{3}{8}$-inch nozzle attached to a 2.5-inch play pipe and discharging 310.6 gallons per minute under an indicated pressure of 30 pounds per square inch.

2. Compute the discharge in gallons per minute through a smooth nozzle $1\frac{3}{8}$ inches in diameter under a pressure at the base of the nozzle of 60 pounds per square inch. Diameter of play pipe $2\frac{1}{2}$ inches.

3. A $1\frac{1}{4}$-inch smooth nozzle is attached to a $2\frac{1}{2}$-inch play pipe. Indicated pressure at base of play pipe is 80 pounds per square inch. Find discharge of nozzle in gallons per minute.

4. Find discharge in gallons per minute through $1\frac{1}{4}$-inch smooth nozzle on a 4-inch pipe with a base pressure of 70 pounds per square inch.

5. If the hydrant pressure is 100 pounds per square inch, find the pressure at the base (at same elevation as hydrant) of a $1\frac{1}{8}$-inch smooth nozzle attached to a $2\frac{1}{2}$-inch rubber-lined hose of best quality, 400 feet long. Also find discharge in gallons per minute.

6. What will be the discharge in gallons per minute from a smooth nozzle $1\frac{1}{4}$ inches in diameter attached to a $2\frac{1}{2}$-inch hose, the indicated pressure being 80 pounds per square inch at the base of the nozzle? To what vertical height would the stream rise?

7. What is the head at a fire hydrant when the discharge from a $1\frac{3}{8}$-inch ring nozzle at a distance of 400 feet is 257 gallons per minute through a $2\frac{1}{2}$-inch hose, when the nozzle is at same elevation as hydrant?

8. What head will be required in a hydrant to give a discharge of 250 gallons per minute 450 feet from the hydrant, through $2\frac{1}{2}$-inch rough rubber-lined pipe and $1\frac{1}{4}$-inch ring nozzle at same elevation as the hydrant?

9. Given a hydrant pressure of 80 pounds per square inch. What will be the discharge in gallons per minute through 700 feet of best quality rubber-lined fire hose, $2\frac{1}{2}$ inches in diameter, and a $1\frac{3}{8}$-inch smooth nozzle, supposing that the nozzle is held 10 feet above the elevation of the hydrant?

10. The head on a fire hydrant is 234 feet. To it is connected 500 feet of smooth rubber-lined hose, $2\frac{1}{2}$ inches in diameter, with a $1\frac{1}{4}$-inch smooth nozzle attached. Find the discharge in gallons per minute when the nozzle is at same elevation as the hydrant.

11. A nozzle is held 15 feet higher than the hydrant to which the hose is attached. Nozzle is $1\frac{1}{4}$-inch smooth, hose is $2\frac{1}{2}$-inch inferior rubber-lined, 300 feet long. Discharge is 250 gallons per minute. (*a*) Find pressure at hydrant. (*b*) Also determine the open butt discharge.

CHAPTER XIII

WEIRS

186. Weirs. A weir is an overfall opening or notch used in determining the volume of flow from measurements of the depths (*heads*) of water running over its crest or sill, of which the length and shape are known. Usually a weir is a dam, or a notch in the top of a dam, or in a bulkhead built across a stream. Weirs are usually classified according to their outlines and the profiles of their crests.

Rectangular weirs are the most common in outline; but trapezoidal, triangular, and even very irregular forms are often used.

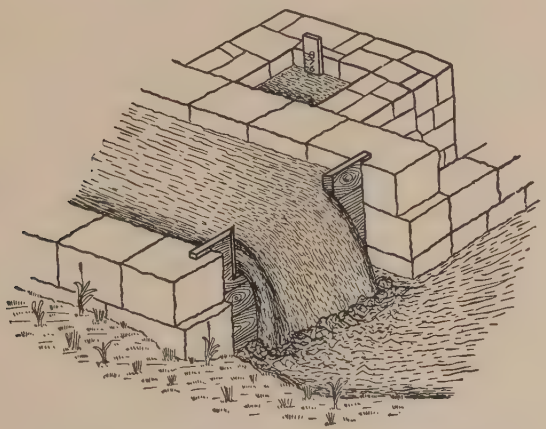


FIG. 53. — A RECTANGULAR CONTRACTED WEIR

Rectangular weirs are further distinguished as contracted or suppressed.

A contracted weir, or one with end contractions, has a length of crest less than the width of channel in which it is set. (See Fig. 53.)

A suppressed weir is one in which the end contractions are suppressed; that is, it has a crest equal in length to the width of the channel in which it is set. (See Fig. 54.)

A sharp-crested or thin-edged weir is one which has a crest on which the overfalling sheet of water, called the *nappe* (overhanging sheet), touches only a sharp, regular edge on its upstream face and springs entirely clear of the downstream parts of the weir. The actual thickness of the crest for high heads may be considerable, and yet allow the nappe to spring clear. For very low heads, however, the crest should be of metal and

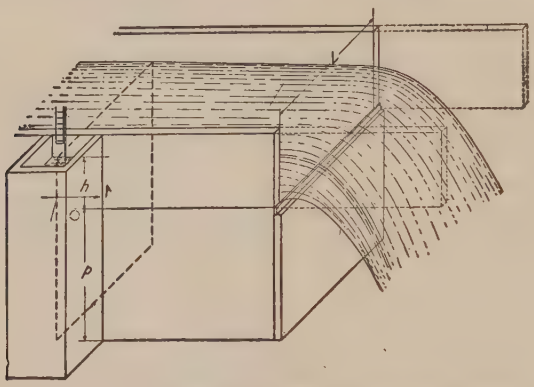


FIG. 54. — A RECTANGULAR SUPPRESSED WEIR

should be very thin. The upstream face should be vertical, and the downstream face so nearly vertical that the nappe cannot touch it.

Weirs of irregular section are those in which the profiles of the weirs and crests (measured in a vertical plane parallel to the direction of flow) are such as will cause the nappe, after passing the upstream edge, to adhere to or again to touch the crest, or the weir face; or will produce a form of nappe in any way different from that of a vertical, sharp-crested weir of equal height and under equal heads.

Free overfall implies tnat the nappe is formed with atmospheric pressure on all its surfaces, and is subjected to no modifying conditions after passing the crest.

Submergence implies that the nappe falls into water which is either above the crest level, or so near the crest level that the form of the nappe is thereby modified.

187. Notation. The symbols here given and explained, with the aid of Figs. 53 and 54, will be used throughout in discussing weirs.

Figure 53 represents a rectangular vertical weir with level crest and two end contractions, and having free overfall.

Figure 54 represents a suppressed weir with level crest. Prolonging the sides downstream along the crest level, as shown in light broken lines, very slightly decreases the discharge, but is common practice.

h = the observed head on the crest, being the difference in elevation in feet between the top of the crest and the surface of the water in the channel, at a point upstream, which should, if possible, be taken just beyond the beginning of the surface curve.

H = the observed head corrected to include the effect of the velocity of approach.

L = the length of the crest in feet.

p = the height of the crest, being the difference in elevation in feet between the top of the crest and bottom of the channel; strictly used only in rectangular channels, and measured in the same stream cross section as the head.

A = the cross-sectional area of the channel where h is measured, which is called the channel of approach.

V_A = the mean velocity of approach in feet per second.

h_v = the head due to the mean velocity of approach = $\frac{V_A^2}{2g}$.

N = the number of end contractions.

Q = the actual discharge in cubic feet per second.

G = the area of the channel of approach divided by the width of the nappe: for suppressed weirs, $G = A/L = (p + h)$; and for contracted weirs, $G = \left(\frac{A}{L - \frac{NH}{10}}\right)$.

188. Theory of the Weir Measurement. Since the weir is merely a special form of orifice, the theory of the orifice applies to it. The cross section of the nappe at the crest may be considered as a number of elemental orifices superposed one above the other. (Fig. 55.) The flow through each is $CL\ dy\ \sqrt{2\ gy}$. The total flow over the weir, then, is

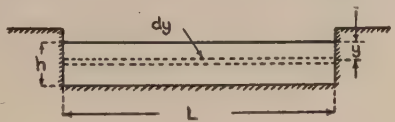


Fig. 55

$$Q = CL\ \sqrt{2\ g} \int y^{1/2}\, dy.$$

Integrating this between the limits $y = 0$ and $y = h$, we obtain

$$Q = 8.02\, CL\, \frac{2}{3}\, h^{3/2}.$$

This is the equation of the weir used by the early hydraulicians. The coefficient C was determined experimentally. This formula is not precise, since there are one or two complications which are not taken into account. Later experiments, however, have shown that it is approximately correct, and that the discharge varies very nearly with the three-halves power of the head. The fundamental form of all modern weir formulas is

$$Q = CLH^{3/2}. \tag{1}$$

189. Volume of Flow. To determine the volume of flow, it is therefore necessary to measure L and h, choose a suitable coefficient, and solve equation (1) or an equivalent formula. The length of the crest and the head may be measured with any desired degree of precision.

When a very high degree of precision is not required, or standard conditions exist, a suitable coefficient may be readily selected, and the computation of discharge is simple. When, however, a high degree of precision is required, or the standard conditions do not exist, a reasonable choice of a coefficient from among the large number available, some apparently contradictory, may present many difficulties.

The flow of water over weirs is a combination of the flow in an open channel and the flow through a form of orifice. Many elements tend to modify this flow, some of which in certain cases can be eliminated by duplicating experimental conditions; but in other cases must be considered more or less carefully as the desired degree of precision may demand. The important modifying elements are:

(1) The velocity of approach;
(2) The contractions of the nappe;
(3) The conditions surrounding the nappe below the crest, whether it has free overfall, is depressed, or is "wetted underneath."

190. The Velocity of Approach. The channel by which the water reaches the weir is called the *channel of approach.* Its cross-sectional area A should be measured in the same vertical

plane as the head, and at right angles to the direction of flow. The mean velocity of flow in the channel of approach is called *the velocity of approach* V_A. Hence

$$V_A = \frac{\text{discharge in cubic feet per second}}{\text{area of channel of approach}} = \frac{Q}{A} \text{ feet per second.}$$

The effect of velocity of approach is to increase the discharge above that due to the observed head h by an amount that can be determined only approximately for several reasons. The head due to the mean velocity of flow h_v, which equals $V_A^2/(2\,g)$, is not a precise measure of the kinetic energy of the moving water

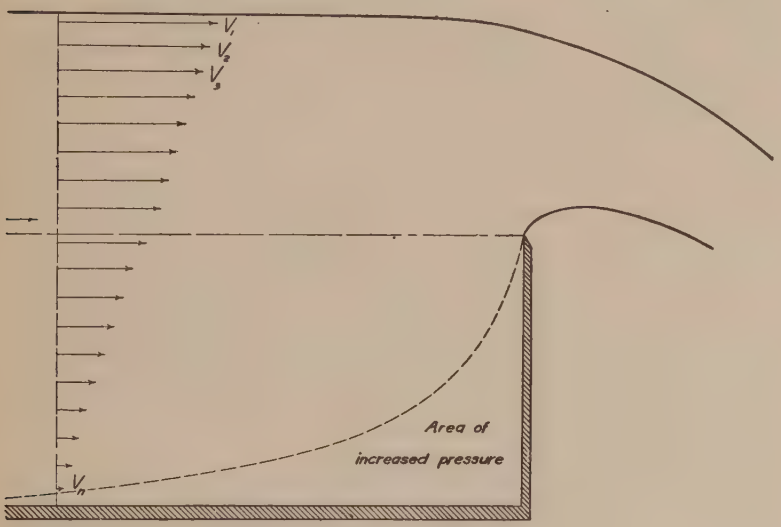


FIG. 56

in the channel. For in few, if in any, streams is the velocity uniform throughout any cross section. If the actual velocities represented by V_1, V_2, V_3, $\cdots$ V_n were determined in equal subdivisions of the cross sections by, say, a current meter, the summation of their velocity heads

$$\frac{V_1^2}{2\,g} + \frac{V_2^2}{2\,g} + \cdots \frac{V_n^2}{2\,g}$$

would not equal but would exceed $V_A^2/(2\,g)$. (See Fig. 56.) In every case this correction would depend very much upon the distribution of the velocities throughout the channel of approach.

Whatever value may be assigned to the velocity of approach, its full value is probably not available at the crest of the weir, some energy being lost in change of velocity and direction, in eddy currents, and in unknown ways. If the velocity of approach is great and the correction becomes an important part of the total head H, its effect cannot easily be distinguished from the effect of other factors, notably the effect of modified crest contraction. The correction for the effect of velocity of approach is applied by experimenters in their formulas either by increasing the observed head by some function of the velocity head, assigning a constant value to the coefficient for a given weir as did Francis and Fteley and Stearns; or by including this correction in a weir coefficient of varying value, as Bazin did. In using the weir formula or the coefficient of any experimenter, his own method of correcting for velocity of approach, upon which the value of his coefficients depends, should be faithfully duplicated.

191. The Contractions of the Nappe. The form of the nappe is determined by three factors, viz., the upper surface curve, the crest contraction establishing the shape of the lower surface curve, and the lateral or end contractions, if any.

The area of the section where the crest contractions and the end contractions (if any) reach a maximum in weirs is probably as important in establishing the volume of flow through a weir as the area of the vena contracta is in an orifice, being in both cases the effective area of the stream.

192. The Surface Curve. The free surface of the water in the channel as it approaches the crest curves downward at an increasing rate of curvature over and beyond the weir, thus forming the upper boundary surface of the nappe.

193. The Crest Contraction. The lower surface of the nappe detaches itself from the crest at some angle varying with the conditions and form of weir, rises to a summit, then descends. The diminution in sectional area of the nappe due to the influence of the lower surface curve is termed ***the crest contraction.*** Complete crest contraction is said to exist when a further increase of the ratio of the height of weir to the observed head p/h will not increase the contraction; a ratio of 3/1 is usually sufficient.

194. Effect of Contractions on Volume of Flow. The effects of the upper surface and lower surface curves in modifying the

volume of flow are important; but except for Bazin's experiments [1] on suppressed weirs, little accurate knowledge concerning this subject is available.[2] Bazin showed by a series of measurements of the profiles of nappes under different heads on vertical and inclined weirs, and by simultaneous measurements of the volume of flow on his standard weir, that the crest contraction furnishes fundamental data for computing the discharge, that ϵ/h (see Fig. 57) roughly represents a sort of coefficient of contraction, and that the section of the nappe just above the summit of the lower curve is the effective area of discharge.

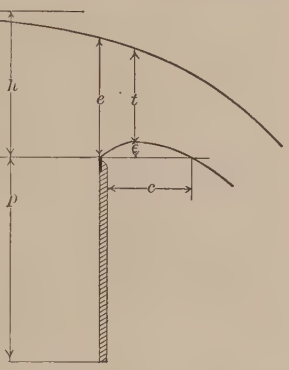


FIG. 57

He showed further that Boussinesq's[3] formula,

$$Q = 0.52(2\,g)^{1/2}(h - \epsilon)^{3/2},$$

was approximately correct, but that the determination of ϵ was too delicate an operation for practical use.

Bazin's experiments, while too voluminous for reproduction here, are most pertinent to an understanding of weir discharge, and some of his conclusions will be briefly stated.

195. Bazin's Conclusions concerning Contractions. If, by the use of coordinates, the profiles of the nappes are plotted for a given weir of constant height p, the forms of each curve will be almost identical for all heads, although the absolute dimensions change with the head. Replacing the abscissas x by x/h, and the ordinates y by y/h, for any value of x/h the corresponding value of y/h will be practically constant. For similar forms of weir an increase in the ratio h/p will increase the amplitude of the upper surface curve and decrease the crest contraction.

Inclining the weir face upstream (up to 1 on 1) has but little effect on the upper surface curve, but increases the crest contraction. Inclining the weir face downstream decreases the height of

[1] ANNALES DES PONTS ET CHAUSSÉES, Mém. et Doc., 1890, 1er semestre, pp. 1–82.

[2] See also Poncelet and Lesbros, *Expériences Hydrauliques*, plate 7; also Fteley and Stearns, TRANSACTIONS A. S. C. E., vol. 12, table XIII.

[3] COMPTES RENDUS DE L'ACADÉMIE DES SCIENCES, July 4, 1887.

the upper curve, elongating it both upstream and downstream. At slopes of 1 on 4 the crest contraction will be practically eliminated; and the opening ceases to be a sharp-crested weir.

The thickness relative to the head e/h of the stream at the weir crest, which is nearly constant for vertical weirs and for weirs inclined (up to 1 on 1) upstream, diminishes very rapidly as the weir is inclined downstream. The ratio e/h, for the same inclination, increases with the ratio h/p.

The relative crest contraction ϵ/h decreases from about 0.16 for weirs inclined 1 on 1 upstream to 0.003 for weirs inclined 1 on 4 downstream. Maximum crest contraction occurs at a distance of $0.25\,h$ downstream for weirs with vertical faces or with upstream inclination; at $0.17\,h$ for downstream inclination of 1 on 1, and at $0.05\,h$ for 1 on 4. The relative chord length c/h decreases with ϵ/h.

The relative thickness of the nappe t/h above the summit of the lower curve decreases as the weir face is inclined upstream up to 1 on 1, increases as the weir face is inclined downstream up to 1 on 1, and decreases with further downstream inclination. Increase or decrease in the rate of discharge follows closely upon increase or decrease of t/h.

The following ratios given by Bazin show how the changes in the form of surface curves caused by inclination of the face modify the flow as compared with a vertical weir. These ratios are used only as illustrating the principles of the effect of the upper surface curve and the crest contraction:

Inclination of Weir		
Toward Upstream	1 horizontal 1 vertical	0.93
	2 horizontal 3 vertical	0.94
	1 horizontal 3 vertical	0.96
	Vertical	1.00
Toward Downstream	1 horizontal 3 vertical	1.04
	2 horizontal 3 vertical	1.07
	1 horizontal 1 vertical	1.10
	2 horizontal 1 vertical	1.12
	4 horizontal 1 vertical	1.09

In developing existing weir formulas, however, no successful attempt has been made to separate these effects from that of velocity of approach, and they are ordinarily combined.

196. End Contractions. If the length of the weir crest is less than the width of the channel of approach, the nappe is contracted laterally after passing the crest and its width is reduced, and with it the discharge is diminished. Whether this contraction occurs in the same section as the summit of the lower surface curve so far as is known, has never been determined.

197. Complete End Contraction. If a weir has sharp-edged ends, an increase in the distance between an end and the adjacent side of the channel, up to a certain limit, will increase the degree of contraction. When this limit has been reached, the contraction is said to be complete. Francis said that the distance should be at least equal to the head. In the light of later experiments it appears that the distance should be at least 2 to 3 times the head.

198. Corrections for End Contractions. For incomplete end contractions, there is no satisfactory method of correction; hence in precise measurements the contraction should be completed or eliminated.

For complete end contraction, there is only one satisfactory method available, which was proposed by J. B. Francis,[1] as a result of his observations. The correction is based upon the assumption that the only effect of end contractions is to reduce the width of the nappe; and it consists in substituting for the measured length of crest L, in the formula for computing the discharge, the value

$$L - \frac{NH}{10}.$$

Later Fteley and Stearns,[2] by a series of experiments, corroborated this conclusion and recommended the continuation of the use of this factor.

199. Conditions surrounding the Nappe below the Crest. Prolonging the sides of the channel downstream above the level of the crest, as shown by dotted lines in Fig. 54, was found by Francis to diminish the flow about $\frac{1}{240}$. He recommended that the sides of the channel should not be prolonged beyond the crest.

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[1] *Lowell Hydraulic Experiments*, p. 74.
[2] TRANSACTIONS A. S. C. E., vol. 12, pp. 108–114.

Both Fteley and Stearns, and Bazin, on the other hand, used such prolongation in all their experiments on suppressed weirs. This is now the more common practice, excepting where there is difficulty in getting perfect aeration under the nappe.

200. Conditions beneath the Nappe. Special provision for free access of air underneath the sheet must be made, if the sides of the channel are prolonged downstream below the crest level. If the space underneath the nappe is cut off from atmospheric pressure, the imprisoned air becomes more rarefied as the flow continues, the water rises behind the nappe, and when the rarefaction is sufficient the nappe will be "depressed" toward the weir. If the head becomes high enough, all the air will be expelled and the space under the nappe is occupied by a mass of turbulent water. This condition is called by Bazin "wetted underneath." The effect of depressing and wetting is, in general, to expand the cross-sectional area of the nappe and increase the discharge. The results of Bazin's researches [1] on this subject are too voluminous to be reproduced and cannot be satisfactorily condensed.

At very low heads, perhaps half an inch or less, the nappe usually adheres to the crest and the lower face of the weir, making the measurement of discharge uncertain.

If free access of air at the ends is provided, the downstream water level may rise very nearly to or perhaps slightly above the crest without affecting the discharge. Figure 58, taken from Bazin's experiments, shows certain conditions that may occur.

201. Procedure in Weir Measurements. The procedure to be followed in weir measurements comprises the following steps:

(1) Constructing and setting up the weir and the gage for measuring the head, reproducing, if possible, the experimental conditions of the formula to be used.
(2) Measuring the length of the crest and determining its irregularities if any.
(3) Taking a profile of the crest if not sharp-edged.
(4) Determining by actual measurements the cross-sectional area of the channel of approach.
(5) Establishing by leveling the relative elevations of the crest of the weir, and the zero of the gage.

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[1] Annales des Ponts et Chaussées, Mém. et Doc., 1891, 2ième semestre, pp. 445–520; *idem*, 1894, 1er semestre, pp. 249–357; latter includes profiles of nappes.

A — Nappe depressed
B — Adhering nappe as formed under a low head
C — Adhering nappe as formed under a greater head
D — Nappe of form known as wetted underneath
E — Nappe adhering at crest and detached below
F — Adhering nappe detached at crest of weir
G — Adhering nappe with backing up of water in channel below weir
H — Wetted underneath form with backing up of water in channel below weir

FIG. 58. — FORMS OF NAPPE OCCURRING IF AIR IS CUT OFF FROM BENEATH THE SHEET. AS COMPARED WITH FREE OVERFALL, THE DISCHARGE MAY BE INCREASED WITH *A* AS MUCH AS 6%; WITH *D*, 15%; WITH *G*, 25%

(6) When the desired regulation of flow is established, determining the head by hook gage or other observations at intervals as frequent as the conditions require.

(7) If possible, measuring the actual velocity in the channel of approach by a current meter or some other direct method.

(8) Computing the discharge by the formula selected.

Three of these operations require especial considerations, viz.: *construction and setting, the measurement of the head,* and the *selection of the formula.*

202. Construction and Setting of Weirs. In order to eliminate as far as possible factors for which precise allowance cannot be made, the construction and setting should meet the following conditions:

(1) A sharp-crested weir with complete crest contraction should be used.

(2) The crest should be level, and its ends vertical.

(3) The end contractions should be complete, or, if suppressed, entirely suppressed.

(4) The upstream face should be vertical; the downstream so designed that the nappe has free overfall.

(5) Free access for air under the nappe should be made certain.

(6) The weir should be set at right angles to the direction of flow.

(7) The channel of approach should be straight for at least 25 feet above the weir, of practically uniform cross section, and of little slope (preferably none).

(8) Screens of coarse wire or baffles of wood should be set in the channel, if necessary, to equalize the velocities in different parts of the channel, but not nearer the crest than 25 feet.

(9) The channel of approach should have a large cross-sectional area in order to keep the velocity of approach low.

203. Measurement of the Head. Gages. Precise determinations of the head on the crest h necessitate accurate observations of a suitable gage, preferably a hook gage, so set as to give the true head.

The hook gage [1] (see Fig. 59) is a graduated sliding scale, with a sharp hook on the lower end, fitted with a vernier, and attached

[1] Called Boyden's Hook Gage, after its inventor. See *Lowell Hydraulic Experiments,* p. 18.

vertically to a frame. In order to make an observation the point of the hook is slowly raised by a screw from below the water surface until the point slightly elevates the surface of the water, but does not pierce it. Then, if with a slight lowering of the point, this distortion of the surface just disappears, the point is assumed to be at the water level. The relation of the zero of the scale to the elevation of the crest must be found by leveling or by a still water comparison. If possible, the zero of the scale should coincide with the average elevation of the crest. The head is usually taken to be the arithmetic mean of all observations for any one condition of steady flow.

Fig. 59. — Hook Gage

Other forms of gages, such as graduated tubes and stationary graduated wooden scales, are frequently used, but cannot be read so closely; dial floats, although extremely useful for continuous observations or registration, are usually subject to errors caused by inertia and by lost motion in the mechanism.

204. Errors of Observation. Ordinarily, by means of the vernier, differences of 0.001 of a foot may be read and smaller subdivisions estimated. The hook gage observation is liable to slight errors of setting and reading, which are ordinarily not important, but may be in the cases of low heads. For example, an error of 0.001 of a foot with a head of 0.1 foot will cause an error in discharge computations of 1.5 per cent; the same error in observation with a 1.0-foot head will cause an error of only 0.15 per cent.

Errors in head observations may be caused by improper location of the gage. The obvious position for a gage is directly in the channel of approach on one or both sides, and just beyond the beginning of the surface curve; unless a high degree of precision is required, it may be so placed. Even with a steady flow, however, the water surface may oscillate more rapidly than it can be followed, even with a hook gage. To remedy this difficulty as far as possible, the gage should be set in a recess or still box outside of, but in communication with, the channel. By throttling

the communication between the channel and the still box, the oscillations can be further reduced to facilitate observations. The still box is obviously a piezometer.

205. Location of Still Box. The location of a still box should meet the following essential conditions:

(1) The cross-sectional area of the communicating opening or pipe must be sufficient to allow free communication with the channel even when throttled.

(2) The channel end of this opening must be set into and exactly flush with the flat walls of the channel, or into a flat surface laid parallel to the direction of flow; and the pipe itself must be normal to the direction of flow. (See discussion of piezometers, Chapter VIII.)

(3) The channel end of this opening must be located far enough upstream to avoid the slope of the surface curve, and not far enough to increase the observed head by the natural slope of the stream.

The area of increased pressure (see Fig. 56), which forms above the bottom, beginning at the upstream face of the weir and extending upstream, perhaps about to the beginning of the surface curve,[1] once thought to be a location at which the observed head would include the velocity head, has been proved to be a poor location for the opening.

Avoid perforated pipes, no matter where the holes are bored, laid transversely or longitudinally in the stream at different depths; avoid so-called piezometers of any form which project in any direction into the stream. After the Lowell hydraulic experiments were made, Francis sometimes used pipes with holes bored in a vertical plane in order to secure an average pressure across the stream, in recognition of the fact that the surface is not transversely level. Since his time, this has been shown to be a vicious practice, which may introduce more errors than it was designed to obviate.

The essential conditions of location of a still box will in general be met if its opening is set well upstream from the beginning of the surface curve, and at or a few inches below the crest level.

If Francis's, Fteley and Stearns's, Bazin's, or any particular experimenter's formula is to be used, his location should be duplicated.

[1] See Fteley and Stearns, Transactions A. S. C. E., vol. 12, p. 42, Plate IV.

206. Weir Formulas. The practice of weir measurements in this country depends upon the original experiments of James B. Francis; of Fteley and Stearns; of H. Bazin (in France); of George W. Rafter, Gardiner S. Williams, R. E. Horton, and others at the Cornell Hydraulic Laboratory; and the investigations of H. Smith, Jr. (about the year 1880) of all the available data then existing; and the production therefrom of valuable and widely used coefficients.[1]

In order to furnish the student with a basis for a choice of formulas, the authors have endeavored, in so far as the nature of this book permits, to restate from the original records the essential conditions of the experiments and the methods by which the more important formulas were derived.

Sharp-crested rectangular weirs with free overfall and the derivation of their formulas will be first considered; and then the values of the coefficients by which the same formulas and occasional new ones have been made to suit weirs of irregular section and outline with submerged or other modified conditions.

207. Summary of Formulas. The general weir formula may be expressed by the equation $Q = CLH^{3/2}$. To this form all the equations in use may be reduced; but it is better practice, in view of the several methods of correcting for the velocity of approach followed by the various experimenters, to use their forms of the equation.

The Francis formula.

(2) $$Q = 3.33(L - 0.1\,NH)H^{3/2},$$
$$H^{3/2} = [(h + h_v)^{3/2} - h_v^{3/2}].$$

See § 214.

The Fteley and Stearns formula.

(3) $Q = 3.31\,LH^{3/2} + 0.007\,L,$

$H = (h + 1.50\,h_v)$ for suppressed weirs,

$H = (h + 2.05\,h_v)$ for contracted weirs.

For contracted weir, make $L = (L - 0.1\,NH)$.

See §§ 221–226.

[1] For the most complete compilation and examination of existing weir data, as well as for hitherto unpublished results, the invaluable U. S. Water Supply and Irrigation Paper No. 200, by R. E. Horton, *Weir Experiments, Coefficients*, etc., should be consulted.

The H. Smith, Jr., formula.

(4) $Q = (C_s \text{ or } C_c)^{2/3} L(2\,g)^{1/2} H^{3/2}$,

$H = (h + 1\frac{1}{3}\,h_v)$ for suppressed weirs,

$H = (h + 1.4\,h_v)$ for contracted weirs.

See § 227.

The Bazin formula.

(5) $Q = mLh(2\,gh)^{1/2}$ (for suppressed weirs only),

m = coefficient including effects of crest contraction and of velocity of approach.

See §§ 228–239.

208. The Francis Experiments. James B. Francis made at Lowell, Massachusetts, during the period of 1848 to 1852, elaborate investigations in the flow of water over sharp-crested vertical weirs with free overfall, which, although limited in range, have rarely, if ever, been excelled in precision. They were published among his other notable hydraulic researches in the *Lowell Hydraulic Experiments*.

209. Development of the Francis Formula.[1] In order to establish a formula for computing the discharge over sharp-crested weirs with end contractions, Francis caused a constant but unknown volume[2] of flow to pass over various weirs of different lengths from 3.5 to 17 feet, and having from 2 to 8 lateral (end) contractions. From simultaneous measurements of head and observations of the effects of lateral contractions, he adopted the following formula[3]:

(6) $$Q = C(L - 0.1\,NH)H^{3/2}.$$

To establish a value for the coefficient C, Francis made a series of 88 experiments measuring the volume of flow in a large basin.[4]

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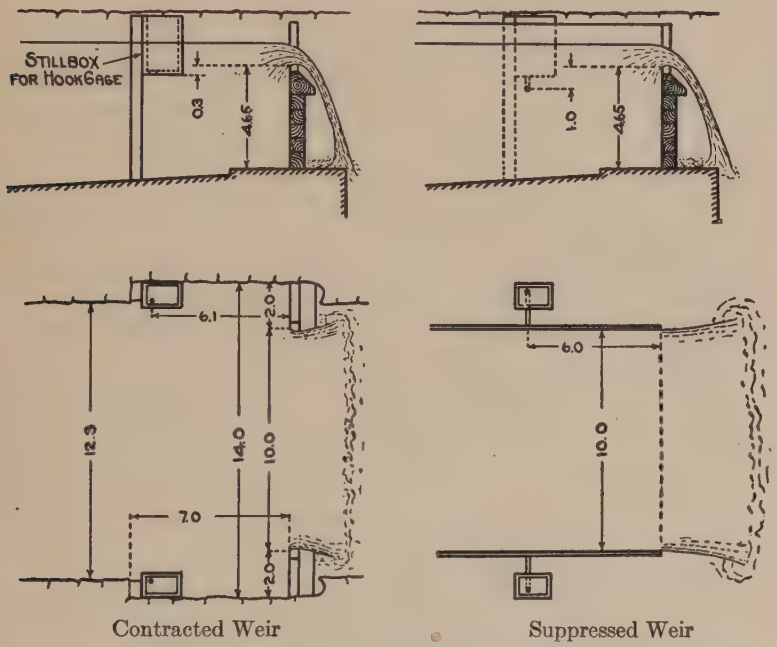
[1] *Lowell Hydraulic Experiments*, pp. 71–102.

[2] Maintained constant by a constant head on set of wheel orifices.

[3] *Lowell Hydraulic Experiments*, p. 118. He stated that the 1.47th power, while inconvenient to use, was more exact than the 1.5 power used.

[4] These experiments were made at Lowell, Massachusetts, at the "lower locks," so-called, of the water power corporation known as the Proprietors of the Locks and Canals on the Merrimack River, which company bore the expense of the investigation.

210. Weir Settings. The same weir was used in all experiments, and was set at the end of a lock chamber about 22.5 feet long having a plank bottom and masonry side walls. The weir with end contractions is shown in Fig. 60. The weir without end contractions (suppressed weir) was obtained by building two parallel timber walls inside the original channel, each 20 feet in length and separated by a space equal to the length of the weir.



Contracted Weir Suppressed Weir

FIG. 60. — FRANCIS WEIRS

Figure 60 also shows the exact arrangements for the suppressed weir experiments. For experiments 36–43, 62–66, 79–84, and 85–88, a level false bottom 20 feet long and 2 feet lower than the crest was added; and for experiments 34–35, and 85–88, a two-foot partition with sharp edges was set in the middle of the crest to cause two additional end corrections.

211. Measurements of Head. The head was observed by two hook gages, one on each side of the channel, set in still boxes, which were 18 inches long by 11 inches wide, as shown in Fig. 60. Communication with the channel was made for the contracted

weir measurements by a one-inch diameter hole in the bottom of each box, located 6 feet upstream from the weir and 4 inches lower than the level of the crest. For the suppressed weir, communication was established by pipes, opening into the sides of the channel one foot lower than the level of the crest. All openings used were therefore 6 feet upstream from the weir. To prevent rapid oscillations, the openings were throttled by a perforated plug set on the inside of the still boxes.

212. Measurements of Volume. The volume of flow was measured by the rise of the water surfaces in a rectangular wooden basin built into the lower locks, into which, when the desired rate of flow was established by means of head gates, the stream over the weir was deflected by a swinging hood. In the periods between experiments, the hood caused the water to be wasted through a by-pass channel. The volume of the measuring basin, which was determined by careful measurements, was over 12,000 cubic feet, and was nearly filled during each experiment. In every case a correction was made for the slight amount of leakage.

213. Measurements of Time. The duration of each experiment was measured on a marine chronometer. The beginning and end of each experiment was announced by a bell operated by an electric circuit which was opened and closed when the swinging arm carrying the level reached the position at which half of the stream was flowing into the measuring basin.

The duration of individual experiments was from 182 to 822 seconds. The extreme error in the observations used in the determining the value of the Francis coefficient C probably did not exceed one half of one per cent.

214. The Francis Formulas. The conditions, setting, and the results of the eighty-eight experiments are summarized in Table XII. Francis fixed on 3.33 as the mean value of C, so that the general formula becomes

$$Q = 3.33(L - 0.1\,NH)H^{3/2}.$$

The corrected H is to be computed by the expression

$$H^{3/2} = (h + h_v)^{3/2} - h_v^{3/2}.$$

The Francis formulas are then as follows.

For contracted weirs, neglecting velocity of approach:

$$Q = 3.33(L - 0.1\,Nh)h^{3/2}. \tag{7}$$

For contracted weirs, head corrected for velocity of approach:

$$Q = 3.33(L - 0.1\,NH)[(h + h_v)^{3/2} - h_v^{3/2}]. \tag{8}$$

NOTE. The use of h instead of H in the factor $(L - 0.1\,NH)$ used in correcting for end contractions is as precise as ordinary practice warrants.

For suppressed weirs, neglecting velocity of approach:

$$Q = 3.33\,Lh^{3/2}. \tag{9}$$

For suppressed weirs, head corrected for velocity of approach:

$$Q = 3.33\,L[(h + h_v)^{3/2} - h_v^{3/2}]. \tag{10}$$

215. Corrections for Velocity of Approach. The correction for the velocity of approach may be made by successive approximations by solving for Q by formula (7) or (9), then dividing Q thus found by the area of the channel of approach, which gives a trial value of V_A from which $h_v(=V_A^2/(2\,g))$ is determined. A second determination of Q by formula (8) or (10) will usually be as precise as is necessary.

Messrs. Hunking and Hart [1] suggested a simpler equivalent for H, viz., making $H^{3/2}$ equal to

$$Kh^{3/2} = (h + h_v)^{3/2} - h_v^{3/2}. \tag{11}$$

The value of K will be very nearly $1 + 0.2489\,(h/G)^2$, or nearly enough for most cases, $1 + [h/(2\,G)]^2$. For G, see § 187.

216. Conditions for Applicability of the Francis Formulas. The Francis formulas are strictly applicable only to vertical sharp-crested rectangular weirs with complete contractions and with free overfall and

When the head H is not greater than one-third the length L;

When the head is not less than 0.5 foot nor more than 2 feet;

When the velocity of approach is 1 foot per second or less;

When the height of the weir is at least 3 times the head.

In all probability the formulas are usable with higher heads than 2 feet but not much lower than 0.5 foot, as shown by experiments of Fteley and Stearns.

[1] JOURNAL FRANKLIN INSTITUTE, August, 1884, pp. 121–126.

TABLE XII

Francis Weir Experiments

Dimensions in feet. Velocities in feet per second

Experiment Nos.	Number of End Contractions	Length of Weir in Feet	Width of Channel at Hook Gage	Depth from Level of Crest to Bottom of Channel		Location of Piezometer for Meas. Hd.		Range of Observed Head		Range of Velocity of Approach		Mean Value of C
				At Hook Gage	At Crest	Up-stream	Below Crest Level	From	To	From	To	
1– 4	2	9.997	13.96	5.048	4.60	6	0.33	1.524	1.569	0.768	0.789	3.3181
5–10	2	9.997	13.96	5.048	4.60	6	0.33	1.237	1.255	0.590	0.600	3.3338
11–33	2	9.997	13.96	5.048	4.60	6	0.33	0.916	1.069	0.395	0.486	3.3223
56–61	2	9.997	13.96	5.048	4.60	6	0.33	0.777	0.819	0.317	0.341	3.3246
72–78	2	9.997	13.96	5.048	4.60	6	0.33	0.592	0.655	0.218	0.251	3.3275
36–43	2	9.997	13.96	2.014	2.014†	6	0.33	1.028	1.079	0.950	1.005	3.3527
62–66	2	9.997	13.96	2.014	2.014†	6	0.33	0.771	0.889	0.669	0.796	3.3403
79–84	2	9.997	13.96	2.014	2.014†	6	0.33	0.631	0.660	0.519	0.550	3.3262
34–35	4	7.997*	13.96	5.048	4.60	6	0.33	1.010	1.026	0.353	0.360	3.3601
85–88	4	7.997*	13.96	2.014	2.014†	6	0.33	0.669	0.688	0.438	0.453	3.3368
44–50	None	9.995–7	9.992‡	5.048	4.60	6	1 to 4.8	0.975	0.987	0.538	0.546	3.3409
51–55	None§	9.995–7	9.992‡	5.048	4.60	6		0.992	1.006	0.548	0.559	3.3270
67–71	None	9.995–7	9.992‡	5.048	4.60	6		0.736	0.815	0.366	0.421	3.3393

Arithmetical mean of values of $C = 3.3318$.

* Virtually 2 weirs of equal length with a partition 2 feet wide between them.
† Bottom level for 23 feet upstream from weir crest.
‡ Sides parallel for 20 feet upstream from weir crest.
§ Sheet prevented from expanding downstream by prolonging the sides downstream above crest level.

EXAMPLE. Given a weir with two contractions, length = 10.24 feet; height of crest = 3.8 feet; width of channel of approach = 15 feet; observed head = 1.2 feet. Compute the discharge in cubic feet per second, making correction for velocity of approach. By formula (9), $Q = 4.38$ (approx.) cubic feet per second per foot of crest. Hence

$$Q = 4.38(10.24 - 0.2 \times 1.2) = 43.8 \text{ (approx.)},$$

whence

$$V_A = \frac{43.8}{(3.8 + 1.2)15} = 0.59 \text{ feet per second},$$

and

$$h_v = \frac{0.59^2}{2g} = 0.0054$$

Then

$$(h + h_v)^{3/2} = (1.2054)^{3/2} = + 1.3234,$$
$$- h_v^{3/2} = - 0.0005,$$

and

$$(h + h_v)^{3/2} - h_v^{3/2} = 1.3229$$

Hence $Q = 3.33 \times 10 \times 1.3229 = 44.053$ cubic feet per second. In this example the correction due to velocity of approach is but 0.25 cubic feet or about 0.6 of one per cent.

By Hunking and Hart's method of correcting for velocity of approach,

$$G = \frac{15 \times (3.8 + 1.2)}{(10.24 - 0.2 \times 1.2)} = 7.5,$$

$$K = 1 + \left(\frac{1.2}{2 \times 7.5}\right)^2 = 1.0064,$$

and

$$Q = 3.33(10.24 - 0.24) \times 1.2^{3/2} \times 1.0064 = 44.05 \text{ cu. ft. / sec.}$$

217. Experiments of Fteley and Stearns. In 1877–79 Alphonse Fteley and Frederic P. Stearns [1] made investigations in the flow of water over weirs which not only verified Francis's results, but also contributed new and accurate data concerning velocity of approach and the flow of water over weirs, especially with low heads.

[1] TRANSACTIONS A. S. C. E., vol. 12 (1883), pp. 1–114. Experiments were made in and at the entrance to the Sudbury Aqueduct of the Boston (now Metropolitan) Waterworks.

The conditions of these experiments may, in so far as they apply to sharp-crested vertical weirs, be summarized as shown in Table XIII.

218. Measurement of Head. In all the five sets of experiments the head was measured by hook gages set in still boxes which were connected with the channel by pipes. Although the actual form of piezometer openings varied, the essential condition that the opening be at or below the crest in and normal to a flat surface parallel to the direction of flow, was in all cases maintained. The location of each opening is stated in the table.

219. Measurement of Volume. In the experiments (indicated in Table XIII as A) upon the suppressed weir 5 feet in length, the volumes were measured by observing the rise of the water surfaces in two basins. One had a capacity of about 360 cubic feet, and the other a capacity of about 6400 cubic feet. These basins were formed by partitioning off convenient portions of the Sudbury aqueduct.

In the experiments indicated as B and C, a constant but not determined volume of flow was maintained by establishing a steady head upon certain orifices above the experimental weir. In the experiments indicated as D, the discharge was measured on the suppressed weir (5 feet long) previously calibrated. In the experiments (indicated as E) upon the suppressed weir 19 feet long, the volume was measured by the rise of water surfaces in another portion of the Sudbury aqueduct, 11,600 feet long, which had a measuring capacity of 300,000 cubic feet.

220. Measurements of Time. The time for all experiments was observed by a stop watch reading to fifths of seconds, checked for long intervals by a chronometer. The duration of experiments on the 5-foot weir varied from 200 to 3481 seconds; on the 19-foot weir from 2193 to 14,852 seconds. The period of flow was regulated by the use of stop planks and gates.

221. The Fteley and Stearns Formula for Suppressed Weir 5 Feet Long. The results of the experiments of Fteley and Stearns briefly stated are as follows. From the thirty-one experiments on the 5-foot weir, the formula found was:

$$Q = 3.33\,LH^{3/2} + 0.0065\,L. \tag{12}$$

In this case, H was computed to be equal to $(h + 1.8\,h_v)$.

TABLE XIII

Fteley and Stearns' Weir Experiments

	Year	Number of Experiments	Number of End Contractions (N)	Length of Weir in Feet (L)	Width of Channel of Approach in Feet	Length of Channel of Approach of Rectangular Cross Section and Level Bottom, Feet	Height of Weir Crest Above Bottom of Channel, Feet	Range of Observed Heads (h)		Range of Velocity of Approach (V_A)		Piezometer Located (Feet)		Volume of Flow, How Determined
								From	To	From	To	Upstream	Below Crest Level	
A	1877	31	None	4.994 to 5	5	12	3.17	0.074	0.820	0.023	0.639	6	2.4	In basin
B	1878	94	None	5	5	18 to 25	0.5; 1.0; 1.7; 2.6 and 3.56	0.19	0.94	0.077	2.35	6	0.4	Kept constant not measured
C	1878	17	1 1 2	4 3.3 3	5 5 5	18 to 25	0.5; 1.0; 1.7; 2.6 and 3.56	0.56	0.93	0.23	1.41	6	0.4	Kept constant not measured
D	1878	54	None 1 2	5 3.3 and 4 2.3 and 3	5 5 5	25	3.56	0.15	0.95	0.054	0.556	6	2.4	By suppressed weir
E	1879	10	None	18.966	19	7.5	6.55	0.47	1.6	0.15	0.84	6	0.0	In basin

On all suppressed weirs, the sides of the channel were prolonged downstream above the level of the crest.

222. Corrections for Velocity of Approach. From ninety-four experiments on suppressed weirs and seventeen experiments on contracted weirs, in which a false bottom was used to vary the heights of the crest, the following values of α to be used in the expression $(h + \alpha h_v)$, by which the corrected head H is to be computed, were determined for suppressed weirs. For weirs with two end contractions add 0.55 to the value of α given in Table XIV.

TABLE XIV

VALUES OF α FOR SUPPRESSED WEIRS TO BE USED IN THE FORMULA $H = (h + \alpha h_v)$ FOR VARIOUS VALUES OF p

OBSERVED HEAD IN FEET	$p = 0.5$	$p = 1.0$	$p = 1.7$	$p = 2.6$ AND HIGHER
0.2	1.70	1.87	1.66	1.51
0.3	1.53	1.83	1.65	1.50
0.4	1.53	1.79	1.63	1.49
0.6	1.52	1.71	1.60	1.47
0.8	1.50	1.65	1.57	1.45
0.9	1.49	1.63	1.56	1.44
1.0	1.48	1.61	1.54	1.43
1.2		1.57	1.51	1.41
1.4		1.54	1.48	1.39
1.6		1.51	1.44	1.37
1.8			1.41	1.35
2.0			1.38	1.33

Values below the line are beyond the limits of actual experiment.

The above values are to be used where considerable refinement of calculation is desired. For average values of a correction for velocity of approach to be used in a general formula, Fteley and Stearns deduced the following expressions:

For suppressed weirs, $H = h + 1.5\, h_v$.

For weirs with two end contractions, $H = h + 2.05\, h_v$.

223. End Contractions. Fifty-four experiments were made on the 5-foot weir: first with no end contractions, and afterwards with the same weir shortened to provide one contraction and later two. The discharge was made steady and measured over the 5-foot suppressed weir. Then, without changing the volume of flow the contractions were set in one at a time and the effect of each change noted by the change in head. From these experiments Fteley and Stearns reach the following conclusions.

The contraction did not increase regularly with the increase in head.

With different forms of weirs the effect of contraction had a variable effect, not sufficiently determined to be defined as a law.

In the Francis factor $L - NH/10$, the value of $H/10$ was found to be not a constant value. It varied with perfect contraction from 0.061 H to 0.124 H.[1]

Since the end contractions cannot be properly compensated for, they should be avoided for accurate measurements; or, if unavoidable, one instead of two should be used.

Being unable to offer a satisfactory substitute, they recommended Francis's method of making this correction as being reasonably correct.

224. The Fteley and Stearns Formula for the Suppressed Weir 19 Feet Long. From the ten experiments on the suppressed weir, 18.966 feet in length, the following formula was derived:

(13) $$Q = 3.291\, LH^{3/2} + 0.004\, L,$$
$$H = (h + 2.4\, h_v).$$

225. Recomputation of Francis's Experimental Results. From a recomputation of J. B. Francis's experiments, the following formula was found to fit his observations:

(14) $$Q = 3.313\, LH^{3/2} + 0.006\, L.$$

226. The Fteley and Stearns General Formula. From an examination of their own and Francis's results they proposed the general formula:

(15) $$Q = 3.31\, LH^{3/2} + 0.007\, L;$$
H for suppressed weirs $= (h + 1.5\, h_v)$,
H for contracted weirs (2 ends) $= (h + 2.05\, h_v)$,
For the net length of contracted weirs use $(L - 0.1\, NH)$.

The present practice largely follows the Fteley and Stearns formula for heads from 0.07 to 0.5 foot and beyond this the Francis formula. Table XXXI (Appendix) is computed in this manner. Fteley and Stearns also devoted considerable attention to the question of measuring the head somewhere within the area of

[1] H being equal to $(h + \alpha h)$; α being equal to 1.5 and 2.05 according to form of weir.

increased pressure and of no velocity (see Fig. 56) upstream from and just behind the weir, which Francis had mentioned; but they they did not recommend measuring the head in this area. They suggested that the piezometer orifice may be set at any depth if located upstream from the weir at least $2\frac{1}{2}$ times p; but if much nearer than this, the opening should not be much below mid depth.

Examples. (*a*) Compute the discharge through a suppressed weir 6 feet long, the crest of which is 1.50 feet above the bottom of the channel, when the observed head is 0.50 ft.

Neglecting velocity of approach, we find

$$Q = 3.31 \times 6 \times 0.5^{3/2} + 0.007 \times 6 = 7.06.$$

Correcting for velocity of approach, we have

$$V_A = \frac{7.06}{6 \times (1.50 + 0.50)} = 0.59; \; h_v = 0.0054;$$

$$H = 0.5000 + 1.5(0.0054) = 0.508;$$

Then $Q = 3.31 \times 6 \times 0.508^{3/2} + 0.007 \times 6 = 7.24$ cu. ft. sec.

(*b*) Compute the discharge through a weir with two end contractions 6 feet long, with its crest 1.50 feet above the bottom of the channel; channel 9 feet wide. Observed head 0.50 foot.

$$Q = 3.31[6 - (0.2 \times 0.5)] \times 0.5^{3/2} + 0.007[6 - (0.2 \times 0.5)]$$
$$= 6.94.$$

$$V_A = \frac{6.94}{9 \times (1.5 + 0.5)} = 39; \; h_v = 0.0024;$$

$$H = 0.500 + 2.05 \times 0.0024 = 0.5049;$$

$$Q = 3.31 \times 5.9 \times 0.5049^{3/2} + 0.007 \times 5.9 = 7.05 \text{ cu. ft. sec.}$$

227. Investigations of Hamilton Smith, Jr. H. Smith, Jr.[1] reviewed and recomputed from the results of Lesbros, Poncelet and Lesbros, Francis, and Fteley and Stearns, together with twelve experiments of his own on a weir 2.6 feet long with two end contractions, values of c in the fundamental formula

$$Q = \tfrac{2}{3} c (2g)^{1/2} L H^{3/2}. \tag{16}$$

For suppressed weirs, $H = (h + 1\frac{1}{3} h_v)$.

For weirs with two end contractions,

$$H = (h + 1.4 h_v),$$

[1] *Hydraulics.*

whence

$$c = \frac{Q(\text{known by experiments})}{\frac{2}{3}(2\,g)^{1/2}LH^{3/2}}.$$

The values of c thus determined were plotted and through them curves were drawn for weirs of different form and length. Finally these values of c were tabulated as given in Table XXX (Appendix); different coefficients c_s and c_c were established for suppressed and contracted weirs.

228. The Bazin Experiments on Sharp-Crested Weirs with Free Overfall. Henri Bazin's experiments, beginning in 1886 [1] and extending over many subsequent years, are the most notable contribution to the science of weir measurements. His early investigations furnished many new data in regard to sharp-crested weirs, especially concerning the effects of variations in height of crest and in velocity of approach. His subsequent work on weirs of irregular sections and modified nappes opened a field of knowledge almost unexplored.

Bazin's experiments on sharp-crested vertical weirs having no end contractions comprised:

(1) The calibration of a vertical, sharp-crested weir, the nappe falling freely in the air; the volume of flow being measured in a large basin. This became his standard weir.

(2) Calibration of sharp-crested weirs having different heights p of crest; the volume of flow being determined by placing them in tandem with the standard weir.

229. Calibration of the Standard Weir. The weir was set in a rectangular channel of smooth-faced concrete 6.56 feet (2 m.) wide,

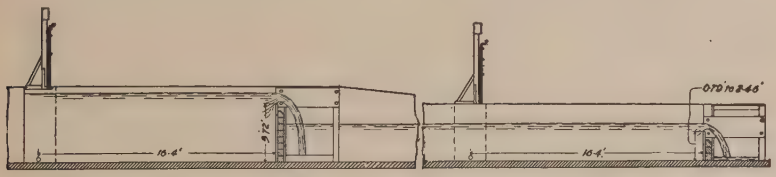


Fig. 61. — Bazin's Weirs set in Tandem

5.25 feet (1.6 m.) deep, and straight for 49.2 feet (15 m.) upstream from the weir. See Fig. 61.

[1] Annales des Ponts et Chaussés, Mém. et Doc., 1888, 2ième semestre, pp. 393–448.

At the downstream face of the weir two recesses were built in the sides of the channel to provide free access of air underneath the nappe, but to prevent lateral expansion of the nappe the sides of the channel were prolonged downstream from above the level of the crest.

230. Measurements of Volume. Below the weir was a channel used as a measuring basin 6.56 feet (2 m.) wide, 3.94 feet (1.2 m.) deep, and 656 feet (200 m.) long, having a bottom slope of about 1 in 1000, made of concrete with a smooth cement lining. The volume of the basin at different elevations and the proper allowance for leakage being determined, the rise in the elevation of the water surface gave the discharge. The flow was regulated by means of sluices set upstream. A wooden stop-gate was also placed directly on the top of the weir. The duration of experiments was from 8 to 40 minutes.

231. Measurement of Head. At a distance 16.4 feet (5 m.) upstream from the weir crest there was constructed a lateral chamber 1.64 feet (0.5 m.) square, communicating with the channel of approach by an opening 0.33 foot (0.1 m.) in diameter, at the bottom of the channel and normal to and flush with the side wall of the channel.

A hook gage and a dial float for observing the head were set in this chamber. By the dial float the fluctuations of the water surface were observed at very frequent intervals and from these the mean position of the hand of the dial was determined; and by the hook gage the elevation of the water in the chamber corresponding to that mean position. The hook gage and weir crest were referenced to permanent iron bench marks set in the walls of the channel.

232. Limits of Experiments on Standard Weir. The rating of the standard weir comprised three series of experiments.

Series No.	Number of Experiments	L Length of Weir	p Height of Weir	Range of Observed Head in Feet	Range of Velocity of Approach in Feet per Second
1	67	6.56 ft.	3.72 ft.	0.194 to 1.012 ft.	0.08 to 0.74
2	38	3.28 ft.	3.72 ft.	0.188 to 1.339 ft.	0.07 to 1.02
3	48	1.64 ft.	3.30 ft.	0.191 to 1.780 ft.	0.08 to 1.62

233. Formula Adopted. Bazin adopted the common formula for discharge

$$Q = mLh(2gh)^{1/2}, \tag{17}$$

where m is the coefficient of discharge and includes the effect of velocity of approach and of all variable factors excepting the head.

From the values of Q determined volumetrically, values of m corresponding to the different values of observed heads were computed by the equation

$$m = \frac{Q}{Lh(2gh)^{1/2}}.$$

The mean values of m for series 1 and 2, with values for series 3 (which were less precise) for heads above 1.3 feet, were plotted against the corresponding observed heads. Through the midst of these points a smooth curve was drawn averaging errors of observation; and from this curve values of m were tabulated. See column 2 in Table XV. Bazin found that these values from the curve, on the average, agreed within $\frac{1}{300}$ with the original observations; in a few extreme cases within only $\frac{1}{100}$.

234. Experiments on Weirs of Different Heights p of Crest. Bazin, having established coefficients for his standard weir, set up, one at a time, four other vertical sharp-crested weirs in tandem with his standard weir. These weirs were erected below the original weir in the channel, which was used in the first experiments as a measuring basin. The head was read on each of these lower weirs, through a hole 16.4 feet upstream in the manner described for the standard weir.[1] See Fig. 61. Thus, as the same volume of flow Q passed over both weirs in succession, the value of m (which may be called m') for each of the four lower weirs was computed by comparison as follows:

For the weir p' feet high, $Q = L'm'(2g)^{1/2}h'^{3/2}$.

For the standard weir, $Q = Lm(2g)^{1/2}h^{3/2}$.

Therefore

$$m' = m\frac{L}{L'}\left(\frac{h}{h'}\right)^{3/2}. \tag{18}$$

[1] During these experiments the head was also measured on the standard weir 16.4 feet upstream, except in series 4, 6, and 8, when it was moved to 32.8 feet, upstream.

The lengths L', L, etc., were constructed as nearly as possible equal to 6.56 feet (2 m.); but as discrepancies occurred, actual lengths had to be used in computations.

The experiments comprised the following series:

Series No.	Number of Experiments	p Heights of Weirs in Feet	Range of Observed Heads (h) in Feet
4 and 5	58	2.46	0.48 to 1.44
6 and 7	58	1.64	0.34 to 1.41
8 and 9	68	1.15	0.29 to 1.34
10	44	0.79	0.30 to 1.34

The values of m', having been computed by comparison with the standard weir, by (18) were plotted to scale against the observed heads. Curves drawn as for the standard weir, from which values of the coefficient m' were tabulated for each of these four weirs. See columns 3, 4, 5, and 6 in Table XV.

235. Bazin's General Formula. Bazin then sought a general formula by which the value of m which should include a correction for the velocity of approach could be calculated. Replacing h by $h + \alpha V_A^2/(2g)$ in formula (17), and calling μ the value of m when there is no velocity of approach, we may write (17) in the form

(19)
$$Q = \mu L\left(h+\alpha\frac{V_A^2}{2g}\right)\left[2g\left(h+\alpha\frac{V_A^2}{2g}\right)\right]^{1/2} = \mu L h(2gh)^{1/2}\left(1+\alpha\frac{V_A^2}{2gh}\right)^{3/2},$$

whence [1]

$$(20)\quad m = \mu\left(1+\alpha\frac{V_A^2}{2gh}\right)^{3/2} = \text{(very nearly) } \mu\left(1+\frac{3}{2}\alpha\frac{V_A^2}{2gh}\right).$$

Or since

$$(21)\qquad V_A = \frac{Q}{L(p+h)}; \qquad m = \mu\left[1+K\left(\frac{h}{p+h}\right)^2\right].$$

From an elaborate study of his own observations and of Fteley and Stearns's experiments, Bazin established a mean value of α

[1] For a mathematical reduction of this formula, not given by Bazin, see translation, by Marichal and Trautwine, Proceedings of the Philadelphia Engineers' Club, pp. 280–281.

equal to $\frac{5}{3}$, and a mean value of K equal to 0.55. Hence he adopted as his general formulas:

$$m = \mu\left[1 + 0.55\left(\frac{h}{p+h}\right)^2\right], \tag{22}$$

$$Q = \mu\left[1 + 0.55\left(\frac{h}{p+h}\right)^2\right]Lh(2\,gh)^{1/2}. \tag{23}$$

TABLE XV

EXPERIMENTAL VALUES OF m IN BAZIN'S FORMULA[1]

$Q = mLh\ (2\,gh)^{1/2}$ for different values of height p and head h. Obtained by a replotting of Bazin's curves. Horizontal lines indicate limits of experiment.

OBSERVED HEADS IN FEET	2 $p = 3.72'$	3 $p = 2.46'$	4 $p = 1.64'$	5 $p = 1.15'$	6 $p = 0.79'$
0.164	.4485	.4487	.4490	.4495	.4505
0.2	.4428	.4435	.4441	.4451	.4466
0.3	.4349	.4359	.4369	.4389	.4442
0.4	.4308	.4325	.4349	.4397	.4476
0.5	.4283	.4316	.4360	.4428	.4527
0.6	.4268	.4321	.4379	.4467	.4584
0.7	.4260	.4332	.4403	.4511	.4646
0.8	.4258	.4347	.4430	.4555	.4707
0.9	.4262	.4361	.4459	.4600	.4769
1.0	.4267	.4377	.4489	.4646	.4832
1.1	.4272	.4393	.4519	.4692	.4896
1.2	.4278	.4410	.4550	.4740	.4961
1.3	.4285	.4428	.4581	.4788	.5026
1.4	.4293	.4444	.4612		
1.5	.4301				
1.6	.4309				
1.7	.4318				
1.8	.4326				
1.9	.4335				
2.0	.4344				
	Series 1–2–3	4 and 5	6 and 7	8 and 9	10

236. The Value of μ. Theoretically, when there is no velocity of approach, that is, when the value of p becomes infinite, m will attain its limiting value, which Bazin called μ. He computed

[1] Bazin's values were plotted for h and p in meters, inconvenient for use and interpolation by American readers. We have therefore replotted these curves from their tabulated coordinates (tables on pp. 413 and 433, ANNALES DES PONTS ET CHAUSSÉES, 2ième semestre, 1888) to an open scale and from these curves taken values of m corresponding to (h) and (p) in feet, further checking them by interpolation from the same original tables.

TABLE XVI[1]

VALUES OF m AND $8.02\,m$; μ AND $8.02\,\mu$ CORRESPONDING TO HEADS h AND HEIGHTS OF WEIR p IN FEET FOR USE IN BAZIN'S FORMULA $Q = mL(2g)^{1/2}h^{3/2}$

h	$p =$ 0.656 m $8.02\,m$	$p =$ 1.0 m $8.02\,m$	$p =$ 1.5 m $8.02\,m$	$p =$ 2 m $8.02\,m$	$p =$ 2.5 m $8.02\,m$	$p =$ 3 m $8.02\,m$	$p =$ 4 m $8.02\,m$	$p =$ 5 m $8.02\,m$	$p =$ 6 m $8.02\,m$	$p =$ 6.56 m $8.02\,m$	μ $8.02\,\mu$
0.2	.456	.449	.446	.444	.444	.443	.443	.443	.443	.443	.4423
	3.657	3.601	3.577	3.561	3.561	3.553	3.553	3.553	3.553	3.553	3.547
0.3	.457	.446	.440	.438	.436	.436	.435	.435	.434	.434	.4337
	3.665	3.577	3.529	3.513	3.497	3.497	3.489	3.489	3.481	3.481	3.478
0.4	.463	.448	.439	.435	.433	.432	.431	.430	.430	.430	.4288
	3.713	3.593	3.521	3.489	3.473	3.465	3.457	3.449	3.449	3.449	3.439
0.5	.469	.451	.440	.435	.432	.430	.428	.427	.427	.427	.4254
	3.761	3.617	3.529	3.489	3.465	3.449	3.433	3.425	3.425	3.425	3.412
0.6	.476	.455	.442	.435	.431	.429	.427	.425	.425	.424	.4227
	3.818	3.649	3.545	3.489	3.457	3.441	3.425	3.409	3.409	3.400	3.390
0.7	.482	.460	.444	.436	.432	.429	.426	.424	.423	.423	.4207
	3.866	3.689	3.561	3.497	3.465	3.441	3.417	3.400	3.392	3.392	3.374
0.8	.489	.465	.447	.438	.433	.430	.426	.424	.423	.422	.4193
	3.922	3.729	3.585	3.513	3.473	3.449	3.417	3.400	3.392	3.384	3.363
0.9	.495	.470	.451	.440	.434	.430	.426	.424	.422	.422	.4182
	3.970	3.769	3.617	3.529	3.481	3.449	3.417	3.400	3.384	3.384	3.354
1.0	.501	.475	.454	.443	.436	.432	.426	.424	.422	.421	.4172
	4.018	3.810	3.641	3.553	3.497	3.465	3.417	3.400	3.384	3.376	3.346
1.1		.479	.457	.445	.438	.433	.427	.424	.422	.421	.4164
		3.842	3.665	3.569	3.513	3.473	3.425	3.400	3.384	3.376	3.340
1.2		.483	.461	.448	.439	.434	.428	.424	.422	.421	.4154
		3.874	3.697	3.593	3.521	3.481	3.433	3.400	3.384	3.376	3.332
1.3		.487	.464	.450	.441	.435	.428	.424	.422	.421	.4146
		3.906	3.721	3.609	3.537	3.489	3.433	3.400	3.384	3.376	3.325
1.4		.491	.467	.452	.443	.437	.429	.425	.422	.421	.4137
		3.938	3.745	3.625	3.553	3.505	3.441	3.409	3.384	3.376	3.318
1.5		.495	.470	.455	.445	.438	.430	.425	.422	.421	.4129
		3.970	3.769	3.649	3.569	3.513	3.449	3.409	3.384	3.376	3.311
1.6			.473	.457	.447	.440	.431	.425	.422	.421	.4121
			3.793	3.665	3.585	3.529	3.457	3.409	3.384	3.376	3.305
1.7			.475	.459	.448	.441	.431	.426	.422	.421	.4112
			3.810	3.681	3.593	3.537	3.457	3.417	3.384	3.376	3.298
1.8			.478	.461	.450	.442	.432	.426	.422	.421	.4104
			3.834	3.697	3.609	3.545	3.465	3.417	3.384	3.376	3.291
1.9			.480	.463	.452	.444	.433	.427	.423	.421	.4096
			3.850	3.713	3.625	3.561	3.473	3.425	3.392	3.376	3.285
2.0			.483	.465	.453	.445	.434	.427	.423	.421	.4090
			3.874	3.729	3.633	3.569	3.481	3.425	3.392	3.376	3.280

[1] This table was constructed by the authors as follows. Values of Bazin's μ were plotted to an open scale against the value of head in metres expressed as feet; through the plotted points a curve was drawn, and from this curve values of μ corresponding to $\frac{1}{10}$ ft. were taken, the values being further checked by proportional differences from Bazin's table. Based on these values of μ this table was computed by Bazin's formula exactly as his own original table, with which by interpolation the values were checked.

this value from values of m for the standard weir ($p = 3.72$); and he found very nearly

$$\mu = \frac{m}{1 + 0.55\left(\frac{h}{3.72 + h}\right)}. \tag{24}$$

237. Tabulated Computations of m and 8.02 m; μ and 8.02 μ. Using the values of μ so computed, Bazin calculated values of m for various heads h and heights of weir p in meters by his formula (19), and tabulated the results.[1] Bazin found the values of m thus computed to agree with his experimental coefficients within $\frac{1}{100}$. In Table XVI, Bazin's coefficients in terms of feet are given.

238. An Approximation. Bazin computed an approximate formula for discharge, agreeing with the table within $\frac{1}{100}$. He found

$$\mu = 0.405 + \frac{0.00984}{h},$$

$$Q = \left(0.405 + \frac{0.00984}{h}\right)\left[1 + 0.55\left(\frac{h}{p + h}\right)^2\right] L(2\,g)^{1/2} h^{3/2}. \tag{25}$$

239. Another Approximation. A rougher approximate formula, usable for heads of 0.33 to 1 foot when errors of 3 per cent in the formula are admissible, is

$$m = 0.425\left[1 + \tfrac{1}{2}\left(\frac{h}{p + h}\right)^2\right] = 0.425 + 0.21\left(\frac{h}{h + p}\right)^2,$$

$$Q = \left[0.425 + 0.21\left(\frac{h}{p + h}\right)^2\right] L(2\,g)^{1/2} h^{3/2}. \tag{26}$$

NOTE. All Bazin's formulas given in this book are in terms of feet.

EXAMPLE. Given a suppressed weir 10 feet long, with its crest 2 feet high. Compute the discharge in cubic feet per second when the observed head is 0.9 foot. By formula (23) and Table XVI, we have

$$Q = 0.4182\left[1 + 0.55\left(\frac{0.9}{2.0 + 0.9}\right)^2\right] 10 \times 0.9 \times 8.02 \times 0.9^{1/2},$$

or

$$Q = 3.529 \times 10 \times 0.9^{3/2} = 30.13.$$

[1] ANNALES DES PONTS ET CHAUSSÉES, vol. 16 (1888), p. 446.

By formula (25), we find

$$Q = \left(0.405 + \frac{0.00984}{0.9}\right)\left[1 + 0.55\left(\frac{0.9}{2.0+0.9}\right)^2\right] 10 \times 8.02 \times 0.9^{3/2}$$
$$= 29.99$$

By formula (26), we find

$$Q = \left[0.425 + 0.21\left(\frac{0.9}{2.0 + 0.9}\right)^2\right] 10 \times 8.02 \times 0.9^{3/2} = 30.48$$

NOTE. Due to the experimental inaccuracies in determining the experimental constants in these formulas, the computed results cannot be relied upon to more than two significant figures.

240. United States Deep Waterways Experiments [1] on Sharp-Crested Weirs with Free Overfall. A series of experiments was carried out at the Cornell University Hydraulic Laboratory in 1899 to determine the value of C in the Francis formula for heads above 2 feet. The discharge was measured on a weir 16 feet long

TABLE XVII

U. S. D. W. EXPERIMENTS ON SHARP-CRESTED WEIRS

STANDARD WEIR, 16′ LONG 13.13′ HIGH	LOWER THIN-EDGED WEIR: $p = 5.2$, $L = 6.56$					
Q, Bazin Formula, in Cubic Feet Per Second	h in Feet	$\frac{h}{p+h}$	K, Hunking and Hart Formula	$H^{3/2}$	Q, Cubic Feet Per Second, Per Foot of Length (corrected)	$C = \frac{Q}{H^{3/2}}$
14.12	0.7462	0.1255	1.0041	0.6469	2.1066	3.256
19.42	0.9139	0.1495	1.0056	0.8787	2.9143	3.317
25.35	1.0885	0.1731	1.0075	1.1434	3.8183	3.331
32.24	1.2933	0.1992	1.0099	1.4849	4.8685	3.279
37.86	1.4436	0.2173	1.0122	1.7564	5.7252	3.260
45.13	1.6306	0.2387	1.0141	2.1116	6.8333	3.236
52.62	1.8115	0.2583	1.0166	2.4787	7.9750	3.218
52.77	1.8088	0.2581	1.0166	2.4730	7.9977	3.234
73.46	2.2389	0.3010	1.0225	3.4254	11.1516	3.226
92.79	2.6434	0.3370	1.0283	4.4193	14.0960	3.190
143.90	3.4660	0.4000	1.0398	6.6095	21.8902	3.312
202.37	4.2747	0.4512	1.0504	9.2867	30.8008	3.317
233.81	4.6773	0.4735	1.0557	10.6789	35.5933	3.333

[1] G. W. Rafter and others, at the Cornell Hydraulic Laboratory. TRANSACTIONS A. S. C. E., vol. 44, p. 397. Recomputed by R. E. Horton, U. S. GEOLOGICAL SURVEY W. S. AND I. PAPER, No. 200.

and 13.13 feet high, and computed by Bazin's formula; and the stream was allowed to flow unchanged in volume over a weir placed in tandem with it 6.56 feet long and 5.2 feet high. The results of these experiments, as recomputed by R. E. Horton, are given in Table XVII.

241. King's Formula. Professor H. W. King recently deduced a weir formula from a study of the experiments of Francis, Fteley and Stearns, and Bazin.[1] On the whole it shows a very satisfactory agreement with these, and is of a simpler form than Bazin's formula. Like Bazin's, this formula does not require a separate correction for the velocity of approach.

$$Q = 3.34\, Lh^{1.47}\left[1 + 0.56\left(\frac{Lh}{A}\right)^2\right].$$

where A denotes the area of section of the channel of approach. The correction for end contractions is made as in the Francis formula. This formula is largely based on Bazin's experiments, and, generally speaking, it shows a closer agreement with these than does Bazin's own formula.

242. Choice of Formulas. When Francis weir settings can be duplicated or when the velocity of approach is very low, one foot per second or less, there is general willingness on the part of both engineers and laymen to accept his formula for heads for from 0.5 to 2 feet, and the same is true of the Fteley and Stearns formula for heads of 0.07 to 0.5 foot. For higher heads the Cornell experiments, which are the only guides, indicate that the Francis formula may be used with reasonable accuracy up to heads of 5 feet.

Bazin's formula, or King's modification of it, is the best where Bazin's conditions can be reproduced. If the velocity of approach is high and the height of weir low, his formula is the only one sufficiently flexible. For this reason it is the most useful.

Smith's coefficients are the result of the most thorough study, but they are based upon experimental data of somewhat unequal accuracy. They do, however, furnish means for satisfactory interpolation to suit cases not covered precisely by the data which he used.

If possible, contracted weirs should be avoided, but they are often necessary to insure atmospheric pressure underneath the

[1] King, H. W., *Handbook of Hydraulics*, McGraw-Hill, 1918.

nappe. If end contractions are unavoidable, the Francis formula should be used.

For rough measurements, there has never appeared to be any good reason for departing from the Francis formula, which has the advantage of long usage and consequent familiarity, especially in legal cases; although it has often been used far beyond the limits laid by Francis himself. It should be borne in mind, however, that his formula applies only to sharp-crested weirs.[1]

243. Precision obtainable in Measurements with Suppressed Weirs. Where conditions match the original experiments and are otherwise favorable, measurements by weir and those made by other methods have repeatedly shown an agreement within 1 per cent. Higher precision can rarely be attained, even with the utmost care; if conditions are nearly duplicated, but the surroundings are unfavorable, errors of from 1 to 5 per cent may occur.

For contracted weirs duplicating Francis conditions, errors of about 2 per cent may occur. In other cases the accuracy will vary.

244. Triangular Weirs. For any orifice the discharge through any elementary lamina such as ldy equals the area times the velocity due to the head on its center. See Fig. 62.

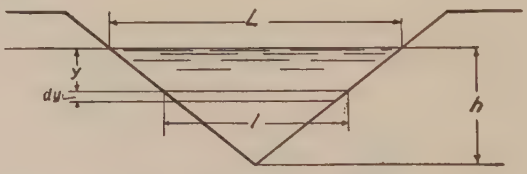


FIG. 62. — TRIANGULAR WEIR

The theoretical discharge is $dQ_t = ldy\ (2\,gy)^{1/2}$. But

$$\frac{l}{L} = \frac{h - y}{h};$$

Hence

$$Q_t = \int_0^h (2\,g)^{1/2} \frac{L(h - y)}{h}\, y^{1/2} dy,$$

whence

$$Q_t = \tfrac{4}{15} L(2\,g)^{1/2} h^{3/2}. \tag{27}$$

[1] The student is advised to read the discussion on weirs by the late F. P. Stearns (of Fteley and Stearns) in TRANSACTIONS A. S. C. E., Vol. 83, p. 169.

Professor Thompson[1] (of Belfast) deduced experimentally a value of $C = 0.593$ for heads of 0.15 to 0.60 foot. Hence, for right-angled notches ($L = 2\,h$)

$$Q = 0.593 \times \frac{4\,L}{15}\,(2\,g)^{1/2}h^{3/2} = 1.27\,Lh^{3/2}, \tag{28}$$

and

$$Q = 2.54\,h^{5/2}. \tag{29}$$

Later experiments[2] on right-angled notches have shown this formula to be very nearly correct.

245. Trapezoidal Weirs. As will be seen by Fig. 63, the discharge is equivalent to the sum of the discharges of a suppressed

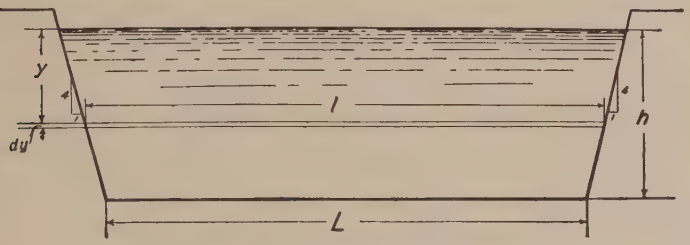


FIG. 63. — TRAPEZOIDAL WEIR

weir and of one triangular weir of height (h) and each side having a batter z; hence, neglecting contractions,

$$Q_t = \tfrac{2}{3}(2\,g)^{1/2}Lh^{3/2} + \tfrac{4}{15}\,2\,z(2\,g)^{1/2}h^{5/2}. \tag{30}$$

When the slope $z = \frac{1}{4}$, which, as has been shown by Horton,[3] is just about sufficient to offset the effect of end contractions, the weir is called a Cippoletti weir, after an Italian engineer, who first described it and derived from the Francis experiments a coefficient which (or its equivalent 3.367) is now used. Hence

$$Q = 3.367\,Lh^{3/2}. \tag{31}$$

Correction for velocity of approach with the Cippoletti weir is made as for the Francis formula. The advantage of this weir lies in that if a suppressed weir cannot be used, with this form

[1] *Experiments on Triangular Weirs*, BRITISH ASSOCIATION REPORTS, 1861.
[2] *Flow of Water over Triangular Notches*, James Barr, ENGINEERING, April 8 and 15, 1910.
[3] *W. S. & I. Paper* 200.

of weir a constant value of L may be taken, thus obviating the tedious computations required to compute the effect of end contractions. For continuous use where many varying heads must be registered and computed, it is convenient; and it is much used in irrigation practice in the western part of this country.

By adding 1 per cent to the values of Q in the weir table (XXXI) for Francis's formula, the discharge for a Cippoletti weir may be computed.

246. The Herschel Weir. Clemens Herschel devised a special form of weir. The crest consists of a 4-inch brass pipe set at the apex of two 2:1 slopes, the upstream one of which extends down to the bottom of the channel, forming the bulkhead portion of the weir. The downstream slope extends some 2 feet beyond the crest. The pipe forming the crest is perforated by a number of pressure orifices, drilled normal to the plane of the upstream slope. The difference in head between the pressure inside the pipe and the water surface upstream from the weir is used to determine the discharge. The discharge curve for this weir is a straight line, varying directly with the head. The experimental weir tested by Mr. Herschel gave $Q = 5.50\,d$, where d = difference between the two observed heads. The only difficulty about this type of weir seems to be that the degree of smoothness of the two slopes has a marked effect on the discharge. Once such a weir had been calibrated, it should give very excellent results. Mr. Herschel automatically corrected for the velocity of approach, by inserting a Pitot tube in the channel of approach, and connecting it to the upstream water column.

247. Weirs with Crest not Level. When the weir crest is not about level, accurate measurement of discharge cannot be made; but unless the crest is too uneven, the average depth on it may be used. With the form of weir shown in Fig. 64 the method suggested by R. E. Horton is probably as good as any. The flow through the elementary width dl is

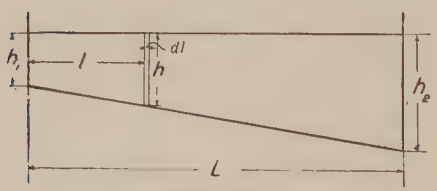


Fig. 64

$$dQ = Ch^{3/2}\,dl,$$

and

$$h = h_1 + \frac{h_2 - h_1}{L} l.$$

Hence the total discharge is

$$Q = \int_0^L Ch^{3/2} dl = C \int_0^L \left(h_1 + \frac{h_2 - h_1}{L} l \right)^{3/2} dl.$$

Integrating, we find

$$Q = \frac{2\,CL}{5(h_2 - h_1)} (h_2^{5/2} - h_1^{5/2}). \tag{32}$$

In this formula, if for a sharp-crested weir, either the mean coefficient deduced by Thompson for a triangular weir, or that of Francis, may be used. If there are end contractions, reduce the length by

$$0.2 \left(\frac{h_1 + h_2}{2} \right).$$

If the crest is not sharp, use an experimental value of C which most nearly fits the conditions. Satisfactory results cannot be expected in any case where the crest is not nearly level.

248. Submerged Weirs. If the water on the downstream face of a weir is higher than the crest, the weir is said to be submerged. See Fig. 65. With our present knowledge of coefficients, such an apparatus is not to be selected for water measurements if other types of weirs can be installed. Not infrequently, however, the existence of a submerged weir in a stream provides a very valuable element in getting a permanent record of flow. Moreover, the effect of a submerged dam in raising the water may be determined approximately by computing the head needed to discharge the required volume of water, and offers a means of determining the effect of back water.

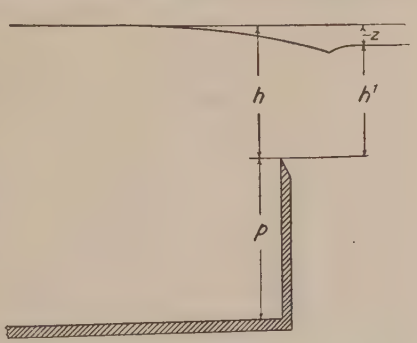


FIG. 65. — SUBMERGED WEIR

The available experimental information is chiefly derived from a few experiments by Francis, some by Fteley and Stearns, and

experiments of considerable range by Bazin. We shall use the following notation:

h = depth of water measured to the crest level on the upstream side.

h' = depth of water measured to the crest level on the downstream side.

$z = h - h'$.

p = height of weir.

249. Experiments of Francis and of Fteley and Stearns. The Francis experiments [1] (1848), six in number, included values of h from 0.85 to 0.97 foot, and of h' from 0.02 to 0.49 foot. The Fteley and Stearns experiments [2] (1877), twenty-two in number, included values of h from 0.33 to 0.81 foot, and of h' from 0.01 to 0.79 foot. In Fteley and Stearns's experiments end contractions were suppressed; and the heads h and h' were measured by piezometers each 6 feet from the crest. The method of conducting the experiments was to submerge the weir after a steady known flow had been established and note the effect of submergence upon the head. The Francis formula was used by Fteley and Stearns in computing the flow passing the measuring weir. From their own experiments and those of Francis, Fteley and Stearns derived the following formula, called the Fteley and Stearns

TABLE XVIII

COEFFICIENTS C FOR THE FTELEY AND STEARNS SUBMERGED-WEIR FORMULA

$\frac{h'}{h}$	0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	—	3.33	3.33	3.34	3.34	3.36	3.37	3.37	3.37	3.37
0.1	3.37	3.36	3.35	3.34	3.34	3.33	3.32	3.31	3.30	3.29
0.2	3.29	3.28	3.27	3.26	3.26	3.25	3.24	3.23	3.23	3.22
0.3	3.21	3.21	3.20	3.19	3.19	3.18	3.18	3.17	3.17	3.16
0.4	3.16	3.15	3.15	3.14	3.14	3.13	3.13	3.12	3.12	3.12
0.5	?.11	3.11	3.11	3.10	3.10	3.10	3.10	3.10	3.10	3.09
0.6	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09	3.09
0.7	3.09	3.09	3.10	3.10	3.10	3.10	3.11	3.11	3.11	3.12
0.8	3.12	3.13	3.13	3.14	3.14	3.15	3.16	3.16	3.17	3.18
0.9	3.19	3.20	3.21	3.22	3.23	3.25	3.26	3.28	3.30	3.33
1.0	3.36									

[1] *Lowell Hydraulic Experiments*, p. 102.

[2] TRANSACTIONS A. S. C. E., vol. 12, pp. 101–108.

formula for sharp-crested submerged weirs having no end contractions:

$$Q = CL\left(h + \frac{h'}{2}\right)(z)^{1/2}, \tag{33}$$

where C is a coefficient whose values are given in Table XVIII.

The correction for the effect of velocity of approach may be made by the Francis method, or simply by adding to h the amount $V_A^2/(2\,g)$. The Fteley and Stearns formula is not applicable for values of h'/h less than 0.08 unless air has free access under the nappe; but is probably applicable beyond the limits of the experiments.

250. Herschel formula for sharp-crested submerged weirs, derived from a recomputation of the Francis and the Fteley and Stearns experiments, may be expressed as follows: [1]

$$Q = 3.33\,L(nh)^{3/2}, \tag{34}$$

where n denotes a factor depending on the ratio, h'/h. Table XIX contains values of n.

TABLE XIX

COEFFICIENT n, HERSCHEL'S SUBMERGED-WEIR FORMULA [2]

$\frac{h'}{h}$	0	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	1.000	1.004	1.006	1.006	1.007	1.007	1.007	1.006	1.006	1.005
0.1	1.005	1.003	1.002	1.000	0.998	0.996	0.994	0.992	0.989	0.987
0.2	0.985	0.982	0.980	0.977	0.975	0.972	0.970	0.967	0.964	0.961
0.3	0.959	0.956	0.953	0.950	0.947	0.944	0.941	0.938	0.935	0.932
0.4	0.929	0.926	0.922	0.919	0.915	0.912	0.908	0.904	0.900	0.896
0.5	0.892	0.888	0.884	0.880	0.875	0.871	0.866	0.861	0.856	0.851
0.6	0.846	0.841	0.836	0.830	0.824	0.818	0.813	0.806	0.800	0.794
0.7	0.787	0.780	0.773	0.766	0.758	0.750	0.742	0.732	0.723	0.714
0.8	0.703	0.692	0.681	0.669	0.656	0.644	0.631	0.618	0.604	0.590
0.9	0.574	0.557	0.539	0.520	0.498	0.471	0.441	0.402	0.352	0.275

251. Bazin Formula [3] for Sharp-Crested Submerged Weirs. Bazin proposed as a general formula for submerged weirs the following expression:

$$Q = m_s Lh(2\,gh)^{1/2}, \tag{35}$$

[1] R. E. Horton, *W. S. & I. Paper* 200, p. 140.
[2] VAN NOSTRAND'S MAGAZINE, vol. 34, p. 176.
[3] ANNALES DES PONTS ET CHAUSSÉES, Mém. et Doc., 2ième trimestre, 1898, p. 235.

where

$$m_s = m \times 1.05\left(1.0+0.2\frac{h'}{p}\right)\left(\frac{z}{h}\right)^{1/3} = m\left(1.05+0.21\frac{h'}{p}\right)\left(\frac{z}{h}\right)^{1/3},$$

and where m denotes the coefficient for a similar sharp-crested weir with free overfall and full crest contraction, having the same value of h and p. Bazin gave also two formulas more limited and more precise; but it is probable that the formula given is as precise as the ordinary use of submerged weirs will warrant.

EXAMPLE. Given a submerged sharp-crested suppressed weir 10 feet long; the water upstream stands 0.49 foot above the crest, and downstream 0.16 foot; the height (p) = 1.5 feet. Compute the discharge by (a) the Fteley and Stearns, (b) the Herschel, (c) the Bazin formulas.

(a) Since

$$\frac{h'}{h} = \frac{0.16}{0.49} = 0.326,$$

we have, by the table,

$$C = 3.194.$$

$$Q = 3.194 \times 10\left(0.49 + \frac{0.16}{2}\right)(0.49 - 0.16)^{1/2} = 10.45 \quad \text{cu. ft./sec.}$$

To allow for velocity of approach,

$$V_A = \frac{10.45}{10(1.5 + 0.49)} = 0.52 \quad \text{ft./sec.}$$

The corrected head H is

$$H = 0.49 + \frac{0.52^2}{64.32} = 0.4942;$$

hence

$$Q = 3.196 \times 10\left(0.4942 + \frac{0.16}{2}\right)(0.4942 - 0.16)^{1/2} = 10.6 \text{ cu. ft./sec.}$$

(b) By the table, we have $n = 0.9512$; hence

$$Q = 3.33 \times 10(0.9512 \times 0.49)^{3/2} = 10.60 \quad \text{cu. ft./sec.}$$

To allow for velocity of approach,

$$V = \frac{10.60}{10(1.5 + 0.49)} = 0.533$$

The corrected head H is

$$H = 0.49 + \frac{0.533^2}{64.32} = 0.4944$$

$$Q = 3.33 \times 10(0.952 \times 0.4944)^{3/2} = 10.7 \quad \text{cu. ft./sec.}$$

(c) For $h = 0.49$, and for $p = 1.50$, we have $m = 0.440$; hence

$$Q = 0.440\left(1.05 + 0.21\,\frac{.16}{1.50}\right)\left(\frac{0.33}{0.49}\right)^{1/3} \times 10 \times 8.02 \times 0.49^{3/2}$$
$$= 11.4 \text{ cu. ft./sec.}$$

252. Discharge of a Weir under a Varying Head. Reservoir of Prismatic Form. Given a reservoir of prismatic form of area F filled to a height h_0 above the crest of a weir; required the time necessary to lower the water surface to a height h_t above the crest of the weir by discharging over it. See Fig. 66.

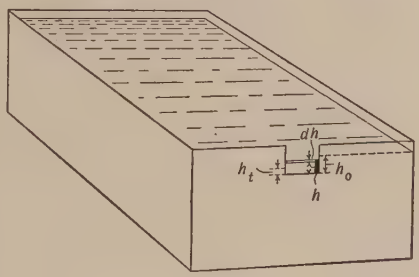


FIG. 66

Let h be the head at the time t. Let Fdh be the volume discharged from the reservoir in time dt. Then the rate of flow at the time t will be $dQ = CLh^{3/2}\,dt$, and $dQ = -Fdh$, the minus sign being used because Q decreases with h. It follows that

$$CLh^{3/2}dt = -Fdh,$$

whence

$$dt = -\frac{Fdh}{CLh^{3/2}},$$

and

$$\int_{t=0}^{t=T} dt = -\frac{F}{CL}\int_{h_0}^{h_t} h^{-3/2}dh. \tag{36}$$

Integrating (36) between the limits h_0 and h_t corresponding to $t = 0$, and $t = T$, we find that the total time required to lower the surface from h_0 to h_t is found to be

$$T = \frac{2F}{CL}\left(\frac{1}{h_t^{1/2}} - \frac{1}{h_0^{1/2}}\right) \text{ seconds;} \tag{37}$$

hence h_t must have a finite value, since the problem is otherwise indeterminate. The value of C must be selected both to suit the particular form of the weir and its setting, and it must have an average value which will represent varying conditions of head during the drawing off.

EXAMPLE. Given a prismatic reservoir 100,000 square feet in area. Required the time necessary to lower the water by flow

over a sharp-crested suppressed weir 20 feet long, from a head of 3.24 feet to a head of 0.36 foot.

Let C for the weir be 3.33; then the time required is

$$T = \frac{2 \times 100000}{3.33 \times 20}\left(\frac{1}{(0.36)^{1/2}} - \frac{1}{(3.24)^{1/2}}\right) = 3333 \quad \text{seconds.}$$

253. No Inflow, Varying Outflow, from a Reservoir of Irregular Volume. Given a reservoir in which the water surface varies in area as it varies in elevation; the time required to lower the water surface is to be computed.

Determine the volume contained between successive differences in elevation (as small as may seem necessary).

Divide the volume between two successive elevations by the discharge through a weir for the average head due to the average elevation of the water surface between two successive differences, and the approximate time required for this depletion will result.

The total time is the summation of the times required to make each step.

EXAMPLE. A reservoir is filled to elevation 103; what time will be required to lower its surface to the crest of a weir at elevation 100? The overflow weir is a sharp-crested suppressed weir 20 feet long. The required computations in tabular form are as follows. The results deduced are only approximately correct; but quite as accurate as practice would usually require.

Elevation of Water Surface	F Area of Water Surface, Square Feet	S Volume of Each 0.5-Ft. Lowering	h Average Head at Each 0.5-Ft. Lowering	Q Average Each 0.5-Ft. Lowering by 20-Ft. Weir	$t = \frac{S}{Q}$ For Each 0.5-Ft. Lowering, Seconds
103.0	400,000				
		190,000	2.75	303.7	626
102.5	360,000				
		170,000	2.25	224.8	756
102.0	320,000				
		150,000	1.75	154.2	973
101.5	280,000				
		130,000	1.25	93.1	1,396
101.0	240,000				
		110,000	0.75	43.3	2,540
100.5	200,000				
		90,000	0.25	8.3	10,843
100.0	160,000			Total time,	17,134 seconds
				Call it,	17,130 seconds

254. Uniform Inflow, Varying Outflow. Given a reservoir with a uniform inflow from a stream, and variable outflow through a weir;[1] compute the difference in the rate of inflow and outflow. Divide the volume between successive elevations of the surface by this difference; the result will be the time for each interval of rise. The total time is the summation of the time required for each step.

EXAMPLE. A reservoir is filled to the elevation of its weir crest (100). Water is flowing in at the rate of 300 cubic feet per second; and begins to overflow a sharp-crested suppressed weir 20 feet long. To what elevation will the water rise and in what time? The required computations in tabular form are as follows. The results, while only approximate in any case, are sufficiently accurate for such problems; and a greater accuracy can be obtained by increasing the number of steps.

Elevation of Water Surface	F Area of Water Surface, Sq. Ft.	S Volume of Each 0.5-Ft Rise	h Average Head at Each 0.5-Ft. Rise	Q_0 Outflow Average for Each 0.5-Ft. Rise	Q_1 Inflow Rate	$Q_1 - Q_0$	$t = \frac{S}{Q_1 - Q_0}$ Seconds
100.0	160,000						
		90,000	0.25	8.3	300	292	308
100.5	200,000						
		110,000	0.75	43.3	300	257	428
101.0	240,000						
		130,000	1.25	93.1	300	207	628
101.5	280,000						
		150,000	1.75	154.2	300	146	1,027
102.0	320,000						
		170,000	2.25	224.8	300	75.0	2,267
102.5	360,000						
		(92,500)*	(2.63)*	(283)*	300	(17)*	5,441
		190,000	2.75	303.7			
103.0	400,000						
							10,099 seconds
							Call it, 10,100 seconds

* For rise 102.50 to 102.75. The water will rise to about 102.75 because at that elevation the rate of inflow and outflow about balance.

In the same general way and on a diagram made especially for this purpose, the rise or fall of a reservoir by the discharge over a weir through a given interval of time can be calculated where both the inflow and outflow are irregular.

[1] The same kind of computations will apply to orifices or sluices combined with weirs or alone.

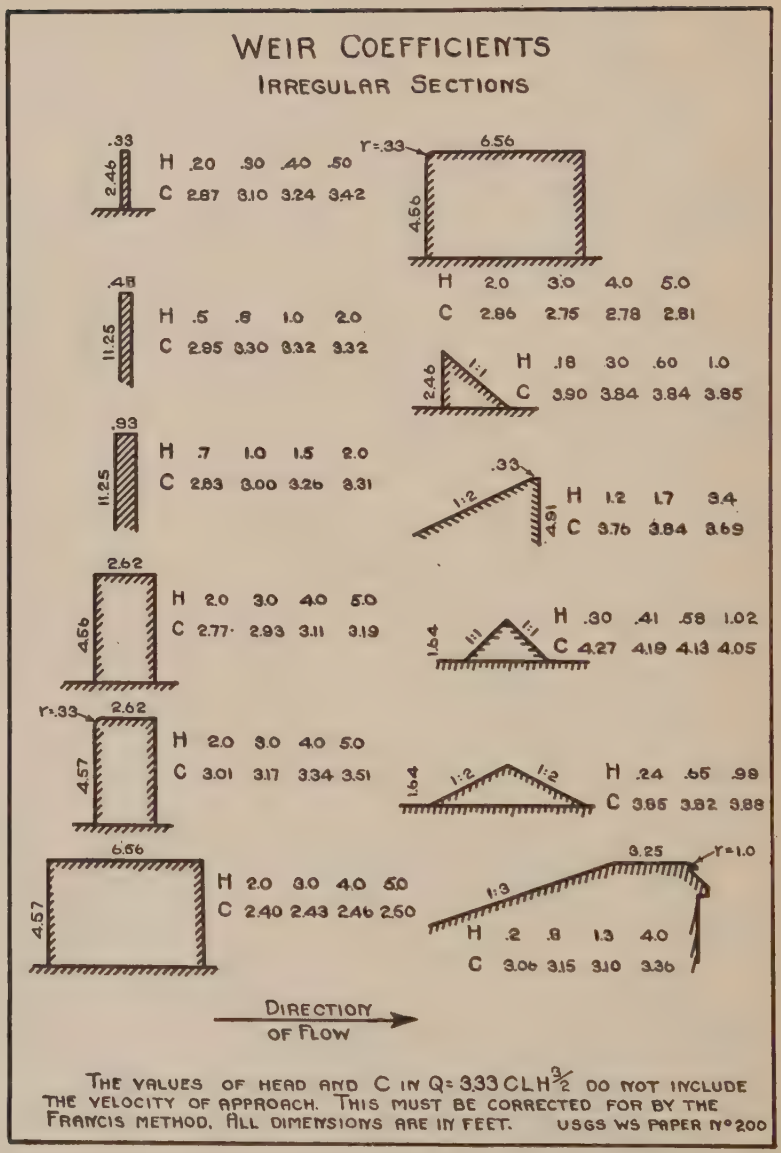

WEIR COEFFICIENTS
IRREGULAR SECTIONS
.33
2.46
H .20 .30 .40 .50
C 2.87 3.10 3.24 3.42
r=.33
6.56
4.56
H 2.0 3.0 4.0 5.0
C 2.86 2.75 2.78 2.81
.48
11.25
H .5 .8 1.0 2.0
C 2.95 3.30 3.32 3.32
2.46
1:1
H .18 .30 .60 1.0
C 3.90 3.84 3.84 3.85
.93
11.25
H .7 1.0 1.5 2.0
C 2.83 3.00 3.26 3.31
.33
1:2
4.91
H 1.2 1.7 3.4
C 3.76 3.84 3.69
2.62
4.56
H 2.0 3.0 4.0 5.0
C 2.77 2.93 3.11 3.19
1.64
1:1
1:1
H .30 .41 .58 1.02
C 4.27 4.19 4.13 4.05
r=.33
2.62
4.57
H 2.0 3.0 4.0 5.0
C 3.01 3.17 3.34 3.51
1.64
1:2
1:2
H .24 .65 .99
C 3.85 3.82 3.88
6.56
4.57
H 2.0 3.0 4.0 5.0
C 2.40 2.43 2.46 2.50
3.25
r=1.0
1:3
H .2 .8 1.3 4.0
C 3.06 3.15 3.10 3.36
DIRECTION OF FLOW
THE VALUES OF HEAD AND C IN Q = 3.33 C L H^3/2 DO NOT INCLUDE THE VELOCITY OF APPROACH. THIS MUST BE CORRECTED FOR BY THE FRANCIS METHOD. ALL DIMENSIONS ARE IN FEET. USGS WS PAPER N°200

FIG. 67

255. Weirs of Irregular Section. Previous to Bazin's experiments, the knowledge of the discharge over weirs of irregular section was not extensive; and in Bazin's experiments the heads were relatively low. Several series of experiments made at the Cornell University laboratory extended Bazin's work, especially in the matter of coefficients for higher heads.

Since it is necessary to reproduce absolutely the conditions obtaining during any set of experiments in order to use the coefficients at all satisfactorily, and since it is almost never possible to reproduce in practice the exact form of each weir crest, it is not thought advisable to go into the reasons for obtaining the results which were found in the experiments referred to, but simply to give the results in figures showing the weirs and the important conditions governing them, and in diagrams showing the relation between observed head and discharge.

256. Horton's Analysis. R. E. Horton made a careful analysis of all the good available experiments, including Bazin's; the latter he reduced to English measures, and certain others he recomputed. Horton also reduced all data to terms of the Francis formula ($CLH^{3/2}$), and computed values of C for use in this formula. These values of C, which are based upon the Francis method of making correction for the velocity of approach, together with all the necessary data, are given in the *Water Supply and Irrigation Paper No.* 200.

PROBLEMS

1. A weir 6 feet long is set in a channel 6 feet wide, and the crest is 6 feet high. Compute the discharge by the Francis formula when the observed head is 1.916 feet.

2. A weir 6 feet long is set in a channel 16 feet wide, and the crest is 6 feet high. Compute the discharge by the Francis formula when the observed head is 1.916 feet.

3. A weir 1.5 feet long is set in a channel 8 feet wide, and the crest is 4 feet high. Compute the discharge when the observed head is 0.69 foot. Use (*a*) Francis, and (*b*) Fteley and Stearns formulas.

4. It is desired to build a sharp-crested weir so that head on its crest shall not exceed 2 feet and which shall deliver 420 cubic feet per second. The stream is 60 feet wide, and when the weir is in place, is not to be more than 7 feet deep, upstream. Determine length of weir by the Francis formula.

5. A weir 19 feet long is set in a rectangular canal 19 feet wide. The crest of the weir is 9 feet above the bottom of canal. The hook gage reading showed a head of 0.83 foot. Compute (*a*) the discharge, (*b*) the velocity of

approach, and (*c*) the head due to same. Use Hamilton-Smith coefficients and compare with results computed by Fteley and Stearns formula made especially for a 19-foot weir.

6. Length of weir, 8 feet, width of channel, 10 feet, observed head, 0.36 foot. Height of crest, 1.0 foot. Compute the discharge (*a*) by the Francis, and (*b*) by the Fteley and Stearns formulas.

7. A weir 15 feet long is set in a channel 15 feet wide, the crest is 3.28 feet high, and the observed head is 1.181 feet. Compute the discharge by Bazin's formulas.

(*a*) The more precise formula.

(*b*) The approximate formula.

(*c*) The rougher approximate formula.

(*d*) By King's formula.

8. Given a suppressed weir 7 feet long, with the crest 4.5 feet above the bottom of the channel. The observed head is 1.36 feet. Compute the discharge by the Francis, Fteley and Stearns, and Bazin formulas: (*a*) not correcting for the velocity of approach; (*b*) correcting for the velocity of approach.

9. Compute the discharge through a Cippoletti weir under a head of 0.9 foot; length of the crest, 6 feet.

10. Compute the discharge through a triangular notch, the sides of which meet at 90 degrees, when the head is 0.64 foot.

11. Given a submerged weir with crest 9 feet long, h is 2.4 feet, and h' is 0.64 foot. Compute the discharge (*a*) by Fteley and Stearns formula; (*b*) by the Herschel formula; (*c*) by the Bazin formula.

12. How high should a submerged sharp-crested weir be built to deepen by 25 per cent on the upstream side, the water of a stream 4 feet deep by 20 feet wide in original section, if the velocity in original section equals 3 feet per second?

CHAPTER XIV

FLOAT AND CURRENT METER MEASUREMENTS

257. The Velocity-Area Method. The method of water measurements most used in the field is the velocity-area method. The cross-sectional area of a stream is determined and the velocity of the water through that area is measured. The product of these two is the quantity of water passing the section. Floats, current meters, and sometimes Pitot tubes, are used to determine the velocity, but the principle is the same for all three instruments. The only difference is that, while the current meter and Pitot tube are held stationary at one point and the velocity past that point is measured, the float moves with the water through a certain distance, while its velocity of transit is measured. This means that for a float a straight reach of river or canal of sufficient length and uniform section is necessary. Such a condition is not often met with, and the use of the float is therefore usually restricted to special cases. The actual procedure in the velocity area method of measurement consists first of determining the cross section at the point where the measurement is to be made. The section should be at right angles to the flow of the water, although a slight angularity is often allowable, though undesirable. The profile of the bottom and of the sides of the channel is determined by leveling on it when possible. Often this cannot be done, and soundings must be made instead. It is very essential to obtain an exact profile of the bottom. This is simple when the channel can be measured while dry and its irregularities are visible. When the profile must be determined by soundings, it becomes difficult and liable to inaccuracies. Particular care should be taken to explore the whole bottom and to locate all rocks, sunken logs, or other such obstructions. When the depth at the measuring station is small, the profile must be taken very carefully, since it is often the controlling factor which determines the accuracy of the measurement under such conditions.

The profile is plotted on cross-section paper, and the resulting area is subdivided into a number of arbitrary vertical subdivisions, their width depending on the total width of the channel and the number of separate measurements that can be made.

Generally speaking, the smaller these divisions are, the greater the accuracy of the measurement; but a compromise must be made on account of the time element involved. Suppose the channel shown in Fig. 68 is 63 feet wide. Divide it into 15 sections 4 feet wide and one section 3 feet wide. If the measurement must be more hurried and somewhat less accurate, it might be divided in ten six-foot sections and one three-foot one. The size of these sections is entirely arbitrary and can be determined only by experience. Generally speaking, for small channels up to ten or twenty feet, sections should be one or two feet wide; for large rivers, of say five hundred feet width, they may be twenty-five feet, or even fifty if the measurement must be completed in a minimum of time.

Fig. 68. — Cross Section of Channel

It is usually desirable to have all the sections of uniform width, except the last one, where the slack is taken up. Having determined the sections, one must next measure the mean velocity in each section. This velocity multiplied by the width of the section and its average depth will give the quantity flowing through that section. The summation of the flows through the component sections is the flow through the total channel. This is the method upon which all velocity-area measurements are based. The various minor complications and refinements will be discussed later.

Water flowing in a channel moves at different velocities at different depths. It moves slowest at the bottom, while its greatest velocity is usually a little below the surface.

258. Surface Floats. A float immersed in water will move at practically the velocity of the water which surrounds it. A *surface float* will record only the velocity of the water at the surface. This is always greater than the mean velocity of the water at that point measured on a vertical plane unless it is affected by wind. A subsurface float consists of a body slightly heavier than water suspended at a certain depth by a wire attached to a surface float. Neither surface nor subsurface floats are accurate and results obtained by using them may be considered only as a rough approximation. Surface floats are sometimes used for rough reconnaissance work. A coefficient must be used to determine the mean velocity from that at the surface. This coefficient will

run from perhaps 0.7 for rough channels to 0.90 for very smooth and uniform ones.

259. Rod Floats. *Rod floats* are long rods, usually one to three inches in diameter, weighted so that they float upright in water. A rod float should extend very nearly to the bottom. Since the rod float extends throughout practically the entire depth, it moves at very nearly the mean velocity of the water. Rod floats give very accurate results, but they have several disadvantages. Since they must extend very nearly to the bottom, a great many lengths are necessary to meet varying conditions. They are bulky and difficult to transport. They require a long straight

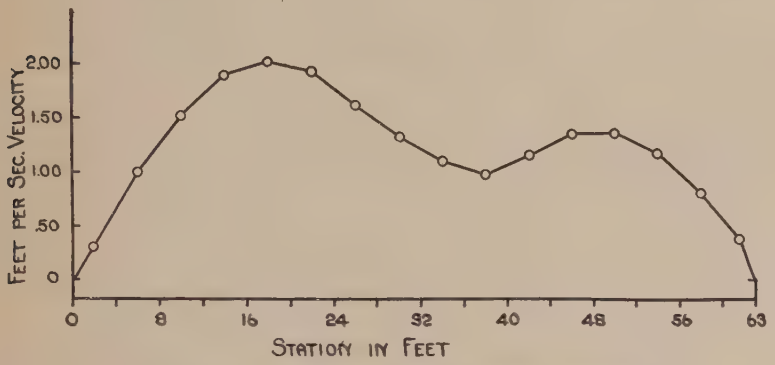


Fig. 69. — Velocity Curve across Channel

reach of nearly uniform section. For certain conditions, however, such as systematic use in power or irrigation canals, these objections are not serious. Under such conditions rod floats will give excellent results.

A light bridge or beam with foot divisions clearly marked on it should extend across the channel at either end of the measuring section, which should be at least 50 feet long. A float is put into the stream a short distance above the upper bridge. The time when the float passes the first bridge is noted as well as the "station," or distance out from one bank (by convention this is the *LH* bank looking downstream). This is repeated at the lower bridge. The time of transit is thus obtained, and the two "stations" averaged to give the average "station." This process is repeated until the whole width of the stream has been explored.

The rod float differs slightly from the current meter in that the course of the float cannot be predetermined exactly. The result is that the mean velocity of a subdivision cannot be determined directly. The velocities observed are plotted on cross-section paper as ordinates against the stations as abscissas, and are joined by lines (Fig. 69). From this curve the average velocity over each sub-area is determined. This can be done by inspection after a little practice. The velocity observations can be connected either by straight lines or by curved ones (Fig. 70). There is equal justification for either method and the final result will usually be very nearly the same.

Fig. 70

The average velocities are multiplied by their respective areas to give the total flow. In cases where the section is rectangular or nearly so the mean velocity for the whole section may be determined and multiplied by the total cross-sectional area. Where the profile of the bottom is irregular or the sides slope much, this short cut is not allowable. It is often very convenient to plot all the different observations on one sheet as in Fig. 79.

260. Rod-Float Correction. Since the rod float does not extend all the way to the bottom, it will record a mean velocity slightly higher than the truth. Francis made some experiments with rod floats, passing water through a measuring flume and then over a weir. He developed the following formula for correcting the observed velocity of the floats, where C is a coefficient to be applied to the observed velocity.

$$C = 1 - 0.116\,(\sqrt{D} - 0.1)$$

where

$$D = \frac{\text{depth of water in flume} - \text{depth of submergence of float}}{\text{Depth of water in flume}}.$$

It is desirable to have D as small as possible, but great care must be taken that the float does not strike the bottom in the course of its transit. When a uniform rectangular canal is used as a flume, D may be kept very low, say 0.02, giving a C of 0.995, while a more irregular channel may require a D of 0.10, giving a C of 0.975. Francis's correction is not strictly applicable to

non-rectangular, earth-lined channels, but it may be used for such conditions with results more nearly correct than if the correction were omitted. It is customary to use an average D for the whole section, rather than a separate one for each vertical.

There is no real need that the sub-areas be determined before the measurement is made. If the profile is carefully measured and the velocity determined at regular intervals across the whole width, the sub-areas may be determined when the profile and the velocity curve have been plotted.

The velocity always drops at the sides of a channel. It is desirable to make a velocity observation as close to each side as is possible, and to extend the curve arbitrarily until the sides are met.

This is the general principle upon which all velocity-area measurements are carried out. When a current meter is used, the exact stationing of the velocity observations is within the observer's control, and the velocity at the center of each arbitrary section can be determined. This can be multiplied directly by the width and mean depth of the sub-area and the flow through that area is found. It is usually desirable, however, to plot the velocity curve, so that the velocity distribution can be visualized and studied. This is particularly true for the two end sections where the curve must be extended.

261. Summary. To recapitulate, the velocity-area method of water measurement requires the following steps, not, however, necessarily in the order named.

1. Selection of a suitable station.
2. Exact measurement of the cross-sectional profile of the bottom and the sides of the channel. In the case of the rod float method a number of sections must often be taken and averaged.
3. The subdivision of the cross-sectional area into a number of subdivisions, usually of equal width, and of varying depth.
4. The measurement of the mean velocity in each one of these subdivisions, by float, current meter, or Pitot tube.
5. The plotting of the velocity curve and the cross section.
6. The computation of the discharge.

The level of the water surface will always fluctuate somewhat in the course of a measurement. A gage of some sort, preferably a staff gage, should be installed at the measuring station and

readings taken on it at intervals during the measurement. The profile of the section must be referred to the gage. The area of the section below the water surface can then easily be determined for any gage height. The gage heights observed during the measurement are averaged and the result is used as the mean gage height. If the water surface varies considerably during the measurement, this method is not exact, but under such conditions of varying stage, an exact measurement cannot be made.

262. Current Meters. Current meters are small water wheels which revolve freely when held in moving water. Each meter

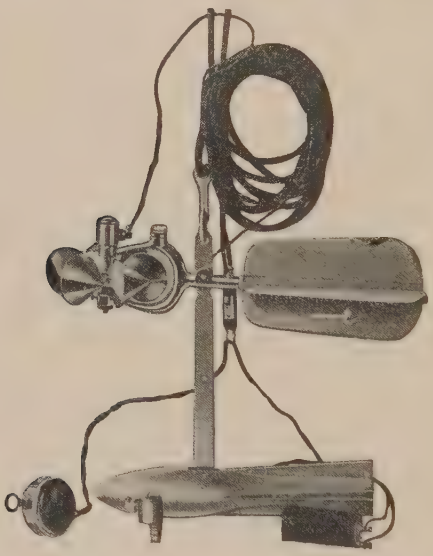


FIG. 71. — PRICE CURRENT METER
(Courtesy of W. & L. E. Gurley)

has a definite relation between its revolutions per second and the velocity of the water causing it to revolve. If the meter is held in moving water and its revolutions recorded against time, the velocity of the water may be determined.

Current meters are of two general types, those with a vertical axis, like a cup anemometer, and those with a horizontal axis, like a ship's propeller. The Price form is practically the only one of the first class used today, the second class contains the forms originated by Fteley, Haskell, and Ott.

263. The Price Meter. The Price meter is the official meter of the U. S. Geological Survey (Fig. 71). In its latest form it is quite rugged, very simple, and easy to operate. It is usually suspended from a cord, but may also be used on a rod in shallow channels. Its revolutions are recorded electrically. It has two disadvantages, the first being that, as the cup wheel is not guarded, it cannot be used close to the side walls of the channel for fear of damaging the cups, the second, that it records sidewise eddy currents and negative velocities as if they were true velocities. In turbulent water it will over-register considerably. The Price meter is particularly adapted to water containing considerable silt, as its bearings are extremely well protected. Its construction is such that it is not easily clogged by leaves and refuse.

264. The Fteley Meter. The Fteley meter, as improved by Safford, is shown in Fig. 72. The runner is entirely enclosed by a thin band which is integral with it, and is surrounded by heavy guard bars. The result is that the meter is very rugged and will withstand hard service. The revolutions are recorded on a set of dials which are thrown in and out of mesh by pulling a cord. Some of the meters also have an electric recording device.

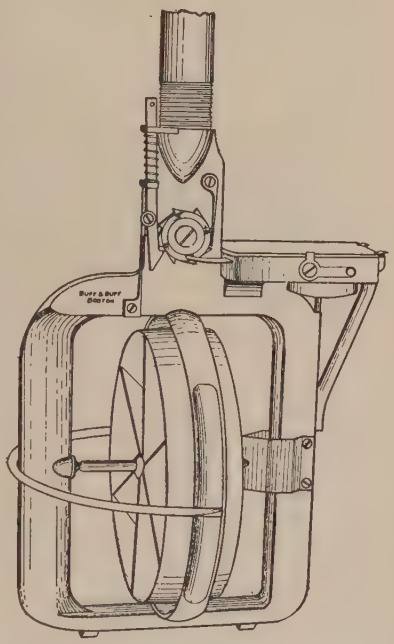


FIG. 72. — IMPROVED FTELEY METER

The Fteley meter is always used on a rod and is therefore limited in the depths to which it can reach. A wooden pointer clamped to the top of the rod shows the direction in which the meter points. Since the meter has no vane, it must be kept pointed upstream by the operator. The Fteley meter will record velocities very accurately from 0.5 ft. per sec. to 8 ft. per sec. and it has been used for velocities as high as 16 ft. per sec. without

damage to the meter. At this velocity, however, the results become much less reliable.

265. The Haskell and Ott Meters. The Haskell and the Ott meters are also used in this country. The Haskell meter (Fig. 73) is simple and rugged but it is somewhat susceptible to eddies. The design of its runner, however, is such that it is practically impossible to clog it with leaves or refuse. The Ott meter (Fig. 74) is German and is not used to a very great extent in this country. It is very beautifully made and appears to be capable

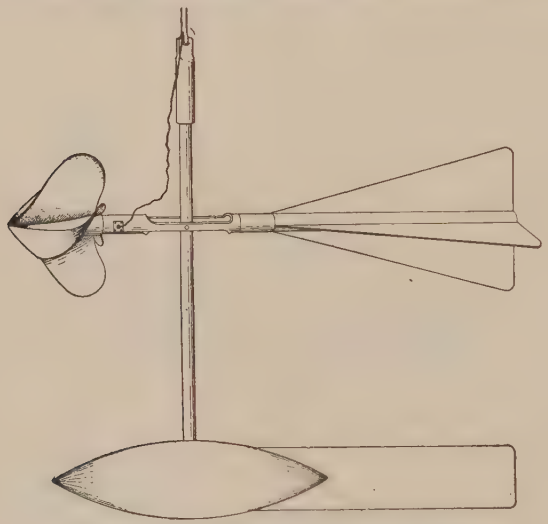


FIG. 73. — HASKELL CURRENT METER

of good work. Various other meters have been used in the past, but practically the only ones in use today are those that have been described.

266. Comparison of Meters. In turbulent water a Price meter will over-register, while a meter of the screw type will record something less than the true velocity. Of the screw meters the Fteley will give the most nearly correct results under such conditions, due to the band surrounding the runner, but there also is the disadvantage that this band increases the tendency of the meter to clog. No meter can be expected to give good results in very turbulent water, but, when such conditions are met with, the Fteley will give the most nearly correct answer. Under favorable conditions any of the meters mentioned will give good results.

The velocity which it is desired to measure is the forward velocity parallel to the axis of the stream. A meter held rigidly facing upstream will record only the forward component of the velocity, whereas a freely pivoted meter with a tail vane will register the maximum velocity regardless of its direction. For accurate work the meter should be rigidly held and kept always pointing directly upstream. The Fteley meter is the only one which is usually operated in this manner, but there is no reason why any of the other meters cannot be so used. A meter must

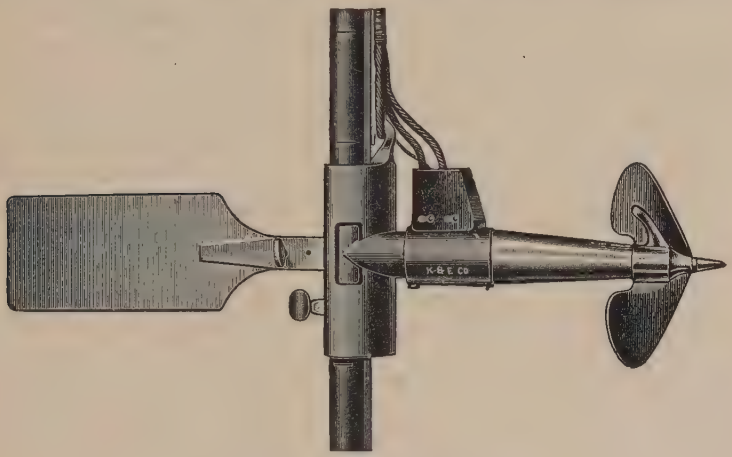


FIG. 74. — OTT CURRENT METER
(Courtesy of Keuffel & Esser Co.)

necessarily be suspended from a cable when working at considerable depths, but the results will not be so accurate as if the meter were rigidly supported.

267. Current-Meter Rating. A current meter must be calibrated before it can be used. This is known as rating. The meter is drawn through still water at various known speeds, and the relation between this speed and the revolutions per second of the meter determined. This information is then plotted as a curve, or a rating table is constructed. A meter may be suspended from a boat which is rowed across a still pond, but it is better to rate the meter at a regular rating station. This consists of a small track along the edge of a body of still water, such as a reach of canal, or a large pond. A small car runs on the

track and the meter is attached to it and suspended some two feet below the surface of the water. The car is moved over a given distance, usually 100 ft. or more, at varying speeds either by hand or by an electric motor. When the canal is small it is best to make a run in one direction, and then to return, making another run at the same velocity, and to use the average of the two runs. This is to avoid the effect of any currents induced by the passage of the meter or caused by wind. Excellent results have been obtained with a circular rating station at the Worcester (Mass.) Polytechnic Institute. A long arm, pivoted in the middle of a pond, is rotated by a water wheel. The meter is suspended from the beam at any radius desired and the speed of the arm can be varied.

The rating curve of a current meter may vary somewhat from time to time, depending upon the type of meter and the use to which it is subjected. An improved Fteley meter which has been rated a great many times showed a change of rate as shown in the accompanying table.

VELOCITY *vs.* R. P. S.

	1	2	3	5	R. P. S.
March, 1914 . . .	1.00	1.98	2.98	4.97	
April, 1924 . . .	1.03	2.01	3.00	4.96	Ft. per sec.
Highest	1.06	2.02	3.00	4.97	
Lowest	1.00	1.98	2.96	4.92	
Per cent variation .	6	2	1.3	1	

During this period the meter was used a great deal, slight changes have been made in the mechanism, and it has been overhauled several times. It is always desirable, however, to rate a meter every so often to make sure that no change has taken place. In measurements which are of great importance it is desirable to rate the meter both before and after making the measurement. Some meters are less sturdy and foolproof than the improved Fteley, and their ratings must be watched more carefully.

It will sometimes be found that a meter will have some change in rating when it is first put to use. This is usually due to the bearings getting broken in. It is desirable to let a new meter

remain in running water for a period before it is rated. If a meter is treated with the care given to any other engineering instrument and is rated occasionally, it will, in the hands of a skillful operator, give very excellent and consistent results.

268. Choice of Gaging Section. While some mechanical skill is required in operating a current meter, the most important requisite for a good measurement is a good gaging section. The choice of this will be determined largely by experience. The section should preferably be near the end of a straight reach of channel. The flow should appear smooth and free from undue surging and turbulence. The profile of the bottom should be as uniform as possible, without undue irregularities, and the depth should be fairly large compared with the width.

A channel 20 feet wide averaging four or five feet deep is better than one 50 feet wide and two feet deep.

When the section has been chosen and its profile taken, velocity observations are made at as many points as time and the desired accuracy permit. The multiple-point measurement should be used whenever time allows, but a complete-point measurement requires considerable time. The 0.2 and 0.8 method, and the integrating method, both require much less time and allow the velocity to be observed on a greater number of verticals. (See §§ 269–273.) The vertical velocity-distribution of moving water has been given considerable study and it has been found that the velocities at various percentage depths maintain nearly constant ratios to the mean stream velocity, but the velocity distribution across the channel will be different in every case. For this reason it is usually desirable somewhat to abbreviate the study of the vertical velocity-distribution for the sake of devoting more time to the study of the velocity across the width of the channel.

Velocity observations should be made as near each bank as possible, and the distribution of velocity around any obstructions, such as bridge piers, should be studied.

Current-meter measurements can be made from bridges, from boats, or, in shallow water, by wading. The first method is always preferable when possible. Sometimes a bridge will be so high above the water that it is best to work from a boat which is managed from the bridge. This is often the case when large rivers are to be measured. When a measurement is made from

a boat, the boat should be held in position by a rope, rather than by rowing. The meter should be operated from a support extending some distance away from the boat to avoid any effect on the meter of the obstruction to the flow caused by the boat. This is particularly important where the water is fairly shallow. Wading can be resorted to in depths up to about four feet where the velocity is not too high. The operator should face sidewise to the stream so that he will form as little obstruction to the flow as possible, and the meter should be held out at arm's length. In small cross sections a deduction should be made for the area obstructed by the operator's body.

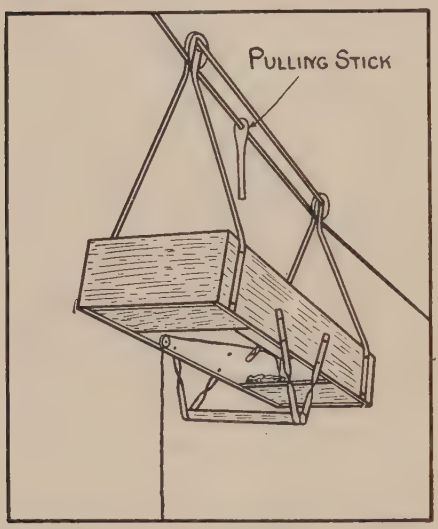


Fig. 75. — Cable Car for Stream Gaging

Another method used considerably in permanent gaging stations consists of stretching a cable across the channel. A small car in which the operator sits is suspended from the cable (Fig. 75), and the meter is operated from it. This method, of course, is only adapted to meters suspended on a cable, such as the Price.

269. Multiple-Point Method. In this method the current meter is lowered to a certain depth below the surface and the velocity measured. The meter is then lowered to another depth and another observation made. This is repeated until the whole depth of the channel has been explored (Fig. 76). The distance apart of the various observations depends upon the depth at the point and the time available. Generally speaking, at least five should be made, one just below the surface, one just above the bottom and the other three evenly distributed between. If the depth is great, or if plenty of time is available, more observations are desirable; for shallow sections two or three will suffice. As in

the case of the spacing of the observations across the stream, the number and vertical separation of the observations at different depths vary with the conditions and with the desired accuracy.

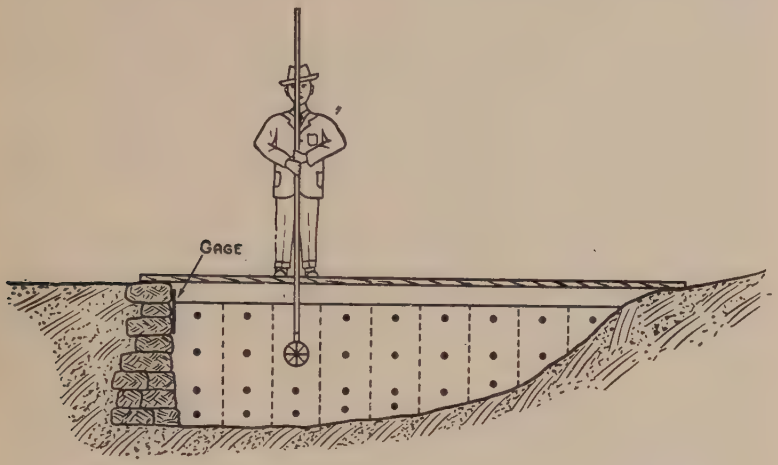


FIG. 76. — MULTIPLE-POINT CURRENT METER MEASUREMENT

The object is to determine the velocity at as many points as possible over the whole cross section (Fig. 76).

The observations are then plotted on cross-section paper with the velocities as abscissas against the depths as ordinates (Fig. 77).

The mean velocity is then determined from the curve. Usually, if the observations are evenly spaced, this step may be eliminated and the velocity observations averaged directly. It is often desirable, however, to construct the vertical velocity curve in order to study its distribution. Sometimes the observations cannot be spaced uniformly. In such cases the vertical velocity curve must be constructed and its mean velocity determined.

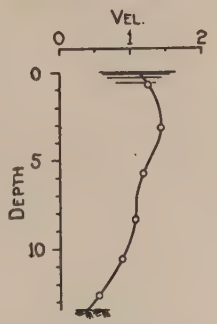


FIG. 77. — VERTICAL VELOCITY CURVE

270. The 0.2 and 0.8 Method. The study of a great number of vertical velocity curves has shown that the general form of the curve is such that if the velocity at 0.2 and 0.8 of the depth is measured, the average of these two will very nearly equal the mean velocity. This method

is very quick and convenient. It is the official method used by the U. S. Geological Survey in its stream gaging. It is not so exact as the multiple-point method, but for most cases it is sufficiently accurate.

271. The 0.6 Method. Experience has shown that the mean velocity in a vertical section usually occurs at a point very near 0.6 of the depth. The 0.6 method consists of making a single observation at this point and of using it for the mean velocity of that vertical. This method cannot be depended upon and must be considered as an approximation.

272. The Integrating Method. In this method the meter is held just below the surface. The observation is then started and the meter is slowly lowered until the bottom is reached. It is then slowly raised to the starting position and the observation stopped. The velocity recorded is very nearly the mean. The vertical motion must be slow, not over 10% of the velocity of the current and preferably 5%. Only a meter of the screw type can be used for this type of measurement. The Fteley form is the best meter for this method, particularly since it is used on a rod.

273. Choice of Method. A multiple-point measurement should always be made when results of the greatest accuracy are desired and plenty of time is available.

The 0.2 and 0.8 method is best suited to gaging the flow in rivers and natural channels, and under such conditions will give very good results. It should not be used in cases where the flow is not normal, such as just upstream from a dam or weir, or just downstream from some submerged obstruction. Such locations should not be chosen for gaging sections under any conditions, if they can be avoided.

The integrating method is best adapted to gaging the flow in small channels and power canals of depths of not over ten or fifteen feet. Under such conditions the meter is used on a rod and is entirely under the control of the operator. Integrating measurements have been made to greater depths with a meter suspended from a cable. The integrating method has been found to have a tendency to overestimate the flow, due to the fact that the slowest velocities at the very bottom are not given full weight (Fig. 78).

Due to its size a current meter cannot measure the velocity close to the sides and near the bottom of the channel, the places

where the velocity is usually lowest. In other words, there is a narrow strip along the whole wetted perimeter which is not measured. The velocities in this strip are lower than in the rest of the section, and the result is that the measured mean velocity through the section is greater than actually exists. So long as the area of this strip is a comparatively small part of the whole, the

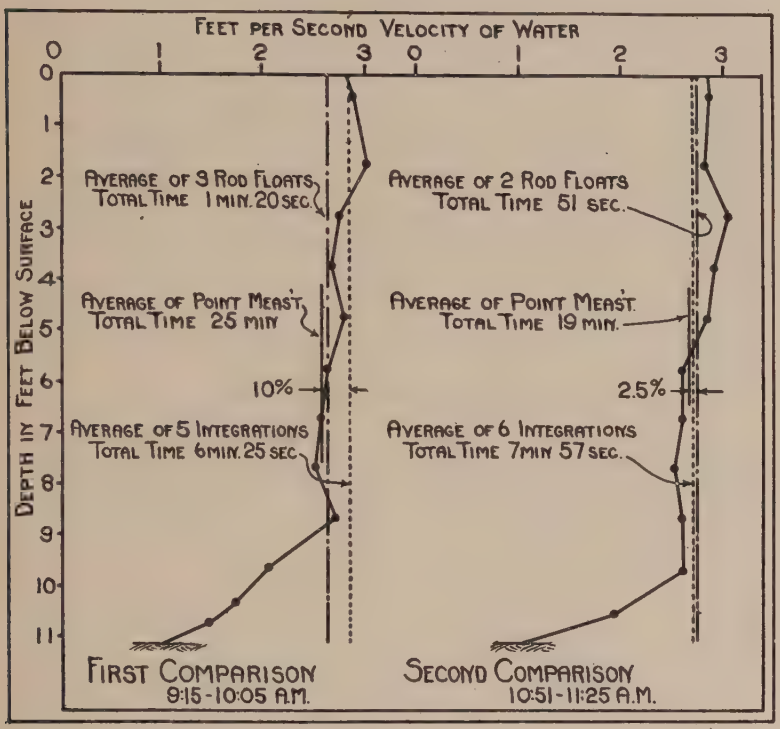

FEET PER SECOND VELOCITY OF WATER
0 1 2 3 0 1 2 3
DEPTH IN FEET BELOW SURFACE
0 1 2 3 4 5 6 7 8 9 10 11
AVERAGE OF 3 ROD FLOATS TOTAL TIME 1 MIN. 20 SEC.
AVERAGE OF POINT MEAS'T. TOTAL TIME 25 MIN
10%
AVERAGE OF 5 INTEGRATIONS TOTAL TIME 6 MIN. 25 SEC.
AVERAGE OF 2 ROD FLOATS TOTAL TIME 51 SEC.
AVERAGE OF POINT MEAS'T. TOTAL TIME 19 MIN.
2.5%
AVERAGE OF 6 INTEGRATIONS TOTAL TIME 7 MIN 57 SEC.
FIRST COMPARISON 9:15 - 10:05 A.M.
SECOND COMPARISON 10:51 - 11:25 A.M.

FIG. 78. — DETERMINATION OF VELOCITY IN A VERTICAL BY ROD FLOAT, AND BY POINT AND INTEGRATING CURRENT METER MEASUREMENTS

effect on the accuracy is small, if not practically negligible. When the section becomes shallow, say three feet deep or less, there is a decided effect upon the accuracy. The above does not apply to the 0.2 and 0.8 method nor to the 0.6 method, so much as it does to the multiple-point and integrating methods.

The velocity-area method of water measurement is only an approximation, due to the fact that the flow is always more or less turbulent, occurring in a succession of surges (Fig. 22). An

observation of short duration may catch the high or the low point, and not record the true average velocity. To overcome this the observation should extend over as long a period as possible, and no observation should be less than forty-five seconds in duration. The longer the duration of the measurements, the greater the accuracy, provided no change in flow occurs. The average of two or three good measurements usually comes very near the truth. A single measurement may vary as much as five per cent, or even more, from the truth, but the average of three good measurements will usually be within three, or even two, per cent. To secure such accuracy, however, a good gaging section, a skilled operator, and a carefully rated meter in good mechanical condition are all essential. Eliminate any one of these and the error may become twenty-five per cent or greater. Figure 79 and the accompanying tables show the complete field notes and computation of flow of an integrating current-meter measurement.

COPY OF FIELD NOTES

CURRENT-METER MEASUREMENT

Mill River at Smith's Mills, November 17, 1924

Measurement made from footbridge about two hundred yards downstream from lower mill. The elevation of the water surface was read on the vertical staff gage on the L. H. bridge abutment.

CROSS SECTION AT BRIDGE

TIME	GAGE HEIGHT	STATION	DEPTH
9:56	257.93	0	0
		4	4.9
		8	7.1
		12	8.7
		16	10.0
	.94	20	10.2
		24	10.0
		28	9.1
		32	8.0
		36	6.3
	.91	40	7.5
		44	8.6
		48	8.3
		52	6.4
		56	4.0
		60	2.0
10:16	257.92	63	0

VELOCITY OBSERVATIONS

Integrating method. Smith, meter; Jones, notes. Buff & Buff Fteley Meter No. 8163, Watch No. 5.

TIME	STATION	GAGE	PERIOD (Sec.)	REVOLUTIONS OF METER	R. P. S.	VELOCITY
10:23	2	257.92	58.3	7	0.12	0.35
	6	.92	81.6	76	0.93	0.99
	10	.93	90.6	136	1.50	1.52
	14	.95	99.0	185	1.87	1.89
	18	.94	110.0	220	2.00	2.02
	22	.92	108.2	208	1.92	1.94
	26	.91	107.0	171	1.60	1.62
	30	.90	105.3	136	1.29	1.33
	34	.90	102.8	110	1.07	1.11
	38	.91	90.2	83	0.92	0.97
	42	.92	95.5	107	1.12	1.16
	46	.92	97.0	129	1.33	1.36
	50	.91	91.8	123	1.34	1.37
	54	.90	88.5	101	1.14	1.18
	58	.93	79.7	71	0.89	0.82
11:19	61.5	257.94	50.0	7	0.14	0.37

Average = 257.92

NOTE. The last two columns in this table are usually worked up later, and are not, strictly speaking, part of the field notes, but rather part of the discharge computation.

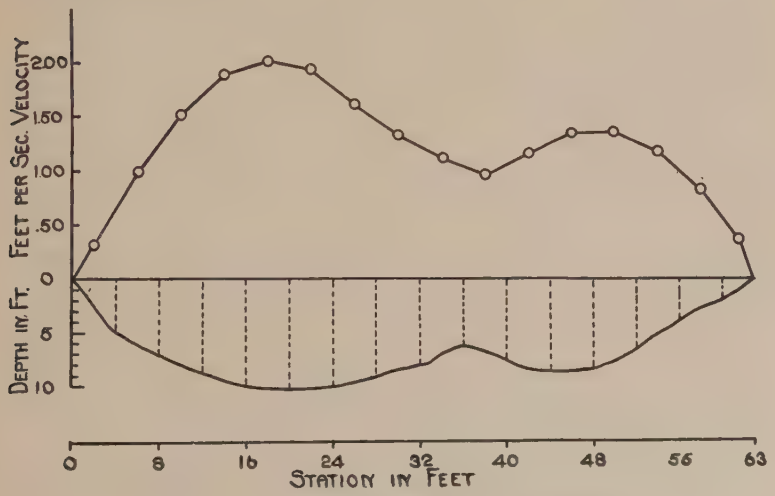


FIG. 79. — CURRENT-METER MEASUREMENT

COMPUTATION OF DISCHARGE (See Fig. 79)

SECTION	WIDTH	DEPTH *	VELOCITY	QUANTITY
0– 4	4	2.45	0.35	3.4
4– 8	4	6.0	0.99	23.8
8–12	4	7.9	1.52	48.0
12–16	4	9.35	1.89	70.7
16–20	4	10.1	2.02	81.6
20–24	4	10.1	1.94	78.4
24–28	4	9.55	1.62	61.8
28–32	4	8.55	1.33	45.5
32–36	4	7.15	1.11	31.7
36–40	4	6.9	0.97	26.8
40–44	4	8.05	1.16	37.4
44–48	4	8.45	1.36	45.9
48–52	4	7.35	1.37	40.2
52–56	4	5.2	1.18	24.5
56–60	4	3.0	0.82	9.8
60–63	3	1.0	0.37	1.1
			Total discharge = 630.6 C. F. P. S.	

* This is the average depth of each 4 ft. section obtained from the plot.

NOTE. In this example there was not enough change in gage height during the time required for taking the cross section and the velocity observations to require any correction. Actually the gage height often changes considerably, and must be corrected for.

PROBLEMS

1. After the above measurement had been made and completely worked up, it was found that the stopwatch had a constant error of + 5 seconds. Subtract five seconds from each of the observed periods, and using these corrected values, compute the discharge. Assume, for this problem, that the rating curve of the current meter is a straight line, $V = 0.985$ R. P. S.

2. The following observations were made in a given vertical during a current-meter measurement in a large power canal, 17 feet deep.

DEPTH	0	1	2	3	4	5	6	7	8
VELOCITY	2.44	2.70	2.66	2.75	2.83	2.66	2.75	2.79	2.86
DEPTH	9	10	11	12	13	14	15	16	
VELOCITY	2.75	2.68	2.73	2.57	2.40	2.33	1.84	1.59	

A 0.2 and 0.8 measurement at the same point gave velocity 2.84 at depth 3.4, and velocity 2.24 at depth 13.6. Plot the observations and determine the velocity in the section by both methods.

CHAPTER XV

FLOW OF WATER IN PIPES AND CONDUITS

274. Flow in Pipes. The determination of the volume of flow in existing channels may be made by any suitable method of measurement; but the predetermination of the probable volume of flow in channels which are to be constructed must be based upon such knowledge of the laws of flow as has been derived from previous experiments upon channels of similar material, form, and dimensions.

Problems concerning the flow in channels are complicated by many influencing factors of which the precise effects are frequently not predictable. Since formulas used in calculating discharge depend upon coefficients based upon published experiments, of which there are many, accuracy from the use of such formulas depends upon the selection of suitable coefficients by comparing the known or assumed conditions of any problem with the supposedly similar conditions under which the flow was measured in determining the coefficients. Such comparisons may be in error because of the difficulties in duplicating the interior surfaces of new channels, in predicting the effects of *wear* and *age* in old channels, and because of uncertainties either in original investigations on which coefficients are based, or of errors in interpreting data. The effects of bends, of changes in cross-sectional areas, and of obstructions are known only from a limited number of experiments. Moreover, in a majority of cases, in designing channels, not only is the actual volume of flow which the channel must deliver subject to temporary fluctuations, but the conditions of use may be entirely changed.

275. The Volume of Flow — the Discharge. The volume of flow, commonly called the discharge, is the quantity of water passing through any stream cross section in a unit of time, usually a second. The discharge Q equals the cross-section area A of the stream multiplied by the mean velocity of flow V. That is,

$$Q = AV.$$

Unless otherwise stated, the area A will be assumed to be in

square feet, and the mean velocity V in feet per second; hence Q will be in cubic feet per second.

In addition to the area and velocity, two other factors which depend upon the dimensions are commonly used in formulas for stream flow, *i.e.*, the wetted perimeter and mean hydraulic radius.

276. Wetted Perimeter. The wetted perimeter (*w.p.*) is the linear dimension of that part of the boundary line of the cross section of a channel which is in contact with the water.

277. Mean Hydraulic Radius. The mean hydraulic radius R may be found by dividing the cross-sectional area A of a stream by its wetted perimeter (*w.p.*).

EXAMPLES. (*a*) Compute the mean hydraulic radius of a stream in a rectangular canal 50 feet wide if the water is 10 feet deep.

$$A = 50 \times 10 = 500 \text{ square feet.}$$
$$w.p. = 10 + 50 + 10 = 70 \text{ feet.}$$
$$R = \frac{500}{70} = 7.14 \text{ feet.}$$

(*b*) Compute the mean hydraulic radius of

(1) A stream flowing in a circular pipe entirely full:

$$A = \frac{\pi D^2}{4}; \quad w.p. = \pi D; \quad R = \frac{A}{w.p.} = \frac{\frac{\pi D^2}{4}}{\pi D} = \frac{D}{4}.$$

(2) A stream flowing in a circular pipe half full:

$$A = \frac{\pi D^2}{8}; \quad w.p. = \frac{\pi D}{2}; \quad R = \frac{A}{w.p.} = \frac{\frac{\pi D^2}{8}}{\frac{\pi D}{2}} = \frac{D}{4}.$$

278. Open and Closed Channels. Channels for conveying water may be classified as (1) closed channels; or (2) open channels.

Closed channels, or channels under pressure, are those in which the flowing water has no continuous free surface, and in which the intensity of pressure at every point in the stream is in excess of that due to the depth of water; that is, the flowing water entirely fills the channel and exerts an upward pressure on the entire top cover. Such channels may be of any cross-sectional form, but are usually circular.

Supply and distribution pipes in water supply systems, pumping

mains, hot-water heating pipes, penstocks, draft tubes of turbines, and fire hose are common examples of closed channels.

Open channels are those in which the upper surface of the flowing water is exposed to atmospheric pressure, and in which the intensity of pressure at every point in the stream is due alone to the depth of the point below the water surface.

Rivers, canals, and, ordinarily, aqueducts and sewers are open channels; but a river or canal may become a closed channel if covered with ice, and an aqueduct or sewer may become a closed channel if the water flows under pressure. See Chapter XVI for flow in open channels.

The fundamental laws applying to flow in closed and open channels are probably identical. The same general formulas are, with proper modifications, equally applicable to open or closed channels. Nevertheless, whether the water in a channel is to flow under pressure or open, must be determined in advance of construction for other reasons. Any wrong assumption may result in failure to deliver the required volume of flow, or in delivering the water at undesired pressures, or in actual failure of the structure itself, due to excessive internal intensity of pressure.

279. Relation of Area to Velocity. A change in the cross-sectional area of either closed or open channels is accompanied by a change in the mean velocity; but there is a difference between closed and open channels in the manner in which adjustments of velocity to changes in cross-sectional area take place. In open channels, although a stream must conform to the shape of its bottom and sides, yet the elevation of its free surface, the cross-sectional area, and the velocity will vary in a manner that can only be accurately determined by observations. Knowledge of the behavior of a stream in an open channel with variable flow can rarely be accurately used for another channel with variable flow. In closed channels, on the other hand, excepting short stretches near points where abrupt changes in sections occur, the areas of the channels and the areas of the streams are identical. Changes in the velocity conform to known changes in area; and comparison with other channels of similar size, form, and nature is easier and more trustworthy.

280. Available Energy. The total energy available at a given point in a stream is the sum of the kinetic energy due to velocity

of flow, the energy due to the intensity of pressure in the water, and the energy due to elevation of the given point with reference to some other point within or without the stream. Hence we shall have

$$(1) \quad E = wQ\left(\frac{V^2}{2g} + \frac{p}{w} + \frac{p_a}{w} + h_e\right).$$

= the total available energy in foot pounds per second, where

p = the intensity of pressure measured from atmospheric pressure; if more than atmospheric, p is $+$, if less, p is $-$,

p_a = atmospheric pressure,

h_e = difference in elevation between the given point and the point of reference.

The total head at any given point, therefore, is

$$(2) \quad H = \frac{V^2}{2g} + \frac{p}{w} + \frac{p_a}{w} + h_e.$$

281. Lost Head. The movement of a body of water sets up various resistances, in overcoming which energy is transformed into heat, just as energy is transformed on a locomotive in overcoming track and other resistances. Energy so transformed cannot be recovered mechanically. It cannot reappear in any useful form, or do further work. The work done on a moving body in overcoming a resistance may be found by multiplying the resistance, if known, by the distance through which the body moves against the resistance. It follows, then, whether the resistance is constant or variable, that if the stream supplies the energy to overcome resistance, every successive downstream section will have less energy and therefore less head than those which precede it. The difference between the total head at one section in a stream, and the total head in another section is called ***lost head.*** By Bernoulli's theorem, we have

$$\frac{V_0^2}{2g} + \frac{p_0}{w_i} + \frac{p_a}{w} + h_0 = \frac{V_2^2}{2g} + \frac{p_2}{w} + \frac{p_a}{w} + h_2 + h_\lambda.$$

Hence

$$(3) \quad h_\lambda = \frac{V_0^2 - V_2^2}{2g} + \frac{p_0 - p_2}{w} + h_0 - h_2.$$

Mere changes in velocity, intensity of pressure, or elevation must not be confused with lost head. While at one point in a stream nearly all the energy may be due to velocity, and in another section of the same stream the energy may be nearly all due to pressure, or to elevation, yet the total energy and equivalent head may be nearly the same; and the only loss of head would

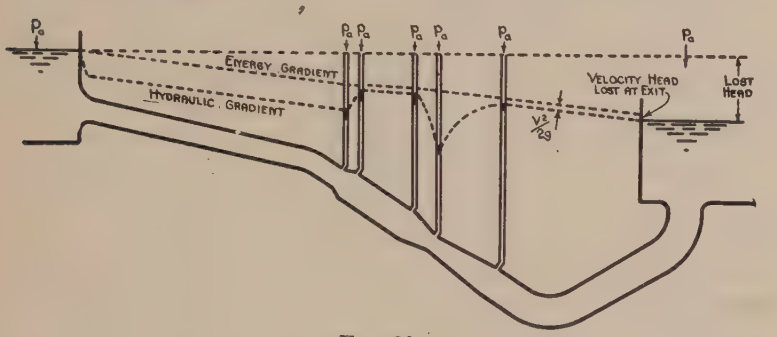


FIG. 80

be that due to the energy necessarily dissipated in heat in overcoming resistances and that due to the transformation of energy for useful purposes.

282. Hydraulic Grade Line. The hydraulic grade line is a profile on the axis of a stream, so drawn that a vertical distance from any point on the profile to any point in the stream is a measure of the intensity of pressure at the latter point.

The hydraulic grade line of a channel under pressure is a profile of the water surfaces in a series of actual or imaginary piezometers set in the channel.

The hydraulic grade line of an open channel is a profile of the water surface.

An imaginary line drawn from the water surface in a reservoir or piezometer at the source of flow to the water surface at its destination, may be called the hydraulic grade line between these points. If a channel is to flow under pressure, it must be so built that no part of the stream lies above this line, otherwise the channel will not flow entirely full, air will collect in the summit, and the flow will be modified. Where such an arrangement is necessary, an air pump may be installed to pump out the air; but this is neither a permanent nor satisfactory remedy.

If a pipe is laid entirely below the hydraulic grade line, its depth is immaterial so far as the discharge is affected; but increased depth means increased internal intensity of pressure, and this requires a stronger channel.

283. Graphical Representation of the Head Due to Elevation. If on the profile of the hydraulic grade line a horizontal line is drawn representing the elevation of the point to which the total head is referred, the length of the vertical from any point in the stream to this horizontal line (or datum plane) represents the head due to the elevation of that point.

The length of the vertical from the datum plane to the hydraulic grade line is obviously a measure of the sum of the heads due to intensity of pressure, and to elevation at any point in the stream which is located on this vertical.

The neglect of atmospheric pressure in drawing the hydraulic grade line will cause no practical error, if the intensity of pressure is everywhere in a stream equal to or in excess of atmospheric pressure. If, however, the pressure anywhere drops below atmospheric pressure, the grade line must, in order to show the actual intensities of pressure, include atmospheric pressure.

284. Slope of the Hydraulic Grade Line. The slope of the hydraulic grade line, or its inclination, is commonly measured by the sine of the angle of inclination; viz.:

$$S = \frac{h}{L}. \tag{4}$$

S = the sine of the angle of inclination measured from the horizontal, and commonly called *the slope*.

h = the difference in elevation between two successive points on the hydraulic grade line.

L = the distance between the two successive points measured along the axis of the channel.

285. The Determination of the Most Economical Size of Channel to use involves many practical considerations. A channel may be so designed that practically all the total head, or only a small part of it, is lost. If the cross-sectional area is large, the head lost in delivering a given volume of flow will be relatively small; but the cost of construction will be relatively large. If the cross-sectional area is small, the head lost will be relatively

great; and the loss of head may often be more costly than to increase the size of channels. The lost head in a water power plant reduces the output of energy from the wheels; in a water supply system it reduces the pressure, which may be needed to deliver water at high points. The problem is usually one of deciding whether an increased fixed charge and maintenance charge due to larger channels is more or less than the value of the loss of head or the cost of reproducing the required head by pumping. A thorough knowledge of the factors which affect resistances to flow is essential in the design of channels.

286. Conditions affecting Resistances to Flow. Resistance is independent of the intensity of pressure in the flowing water.

Resistance increases with an increase in the area of the rubbing surface; with increased roughness of the lining of the channel; with the square of the velocity (within 15%), for velocities with which the engineer has to deal (for critical velocity see Chapter VII); with abrupt changes in the cross-sectional area of the channel; at bends or at junctions with other channels; with decrease in depth in certain open channels, without an increase in the rubbing surface; with an increase in suspended matter in the water. Resistance decreases with rise of temperature.

The various resistances will assume different proportions in every different problem, and may have to be considered, or may be neglected, according to their relative magnitude. Variations due to suspended matter and to temperature changes, except in the case of very small pipes, are commonly not considered.

287. Formulas for Flow in Channels. It is extremely doubtful whether any single formula which is satisfactory and generally applicable can be evolved even after a considerable increase in existing data. The utmost that can be expected is a set of formulas each strictly applicable to its own set of conditions, and generally applicable, but with less precision, to a wider range. Two formulas are widely used in American and English practice, virtually two statements of the same formula, and based upon the assumption that the resistances vary as some function of the square of the mean velocity of flow. They have wide use in spite of several well-known inconsistencies, because they are convenient and have given as high precision as many practical problems appear to warrant.

288. Summary of Formulas.

Chezy.

$$V = C\sqrt{RS}$$

V = velocity in feet per second.
R = hydraulic radius in feet.
S = slope of the hydraulic grade line.
C = a coefficient determined by experiment or one of the following formulas.

Kutter.

$$C = \frac{41.66 + \dfrac{1.811}{n} + \dfrac{0.00281}{S}}{1 + \left(41.66 + \dfrac{0.00281}{S}\right)\dfrac{n}{R^{1/2}}},$$

where n is a coefficient of roughness determined by experiment.

Manning.

$$C = \frac{1.486}{n} R^{1/6},$$

where n is the same as in Kutter's formula.

Bazin.

$$C = \frac{157.6}{1 + \dfrac{m}{R^{1/2}}},$$

where m is Bazin's coefficient of roughness.

Hazen-Williams

$$V = C_w R^{0.63} S^{0.54}\, 0.001^{-0.04},$$

where C_w is a coefficient of roughness.

The Chezy and the Hazen-Williams formulas are generally used for flow in pipes, the other three for flow in open channels.

289. The Chezy Formula. This formula is probably more used than any other. It may be used either for open or for closed channels. For steady uniform flow it is usually stated as follows:

$$V = C(RS)^{1/2}. \tag{5}$$

C = coefficient increasing with the mean hydraulic radius or with increasing diameter, and for new clean channels usually increasing with the mean velocity of flow, and decreasing with the roughness of the channel.
R = mean hydraulic radius.
S = sine of the inclination of the hydraulic grade line.

290. Another Form of the Chezy Formula. The common textbook formula which, as nearly as may be determined, was proposed by Weisbach,[1] and which is used only for pipes under pressure, is as follows. For steady uniform flow under pressure in circular pipes,

$$h_f = f \frac{LV^2}{D\,2\,g}, \tag{6}$$

where

h_f = the head lost in friction (in feet) in a pipe of length L and of diameter D.

f = the friction factor or coefficient of friction, decreasing with an increase in the diameter of the pipe and commonly with an increase of velocity of flow; and increasing with the age and roughness of the surface in contact with the water.

L = the length (in feet) of the pipe measured on its axis.

D = the internal diameter (in feet) of the pipe.

V = the mean velocity of flow in feet per second.

g = the acceleration due to gravity, taken here as 32.16.

Equations (5) and (6) are merely different expressions for the same formula. Either may be used to suit convenience.

291. Exponential Formulas. Several formulas are in use which recognize the fact that resistances to flow do not vary exactly as the square of the mean velocity, but according to some other exponential value. Of these formulas, which are commonly called exponential formulas, the best in use in this country is Williams and Hazen's, which was worked out from the best published data; it is as follows:

$$V = C_w R^{0.63} S^{0.54} 0.001^{-0.04}. \tag{7}$$

This coefficient C_w is based chiefly upon the roughness of the channel lining, and should not be confused with C in the Chezy formula. The inconvenience in computing by this formula may be obviated by the use of Williams and Hazen's *Hydraulic Tables* or their slide rule.

292. Pipes. A pipe is a hollow cylindrical tube of sufficient length to show during flow an appreciable loss of head in friction, in addition to the head lost in entrance and in acquiring velocity.

[1] *Mechanics*, Coxe's translation, p. 866. The formula is not uncommonly designated as Weisbach's, Darcy's, Weston's, or Chezy's.

The materials used in constructing pipes are cast iron, wrought iron, steel, lead, brass, rubber, wood, glass, concrete, vitrified clay, and various other substances.

293. Uniform Flow under Pressure. Uniform flow under pressure requires the following conditions: a steady volume of flow which entirely fills a length of straight pipe, which has a constant cross-sectional area and is of uniform roughness. This condition can never wholly be satisfied, due to the fact that the flow is almost always more or less turbulent and therefore varying somewhat with respect to time. For all practical purposes, however, the flow may be assumed to be uniform if the other conditions are satisfied.

Under the conditions of steady, uniform flow in a pipe under pressure, the cross-sectional area, the mean velocity, and the discharge are all constant in every section of the pipe.

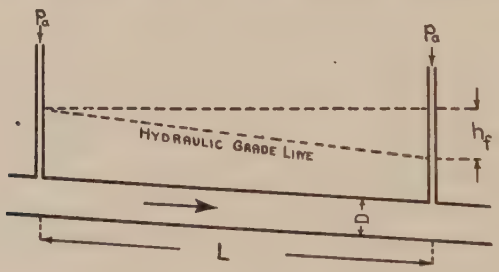


FIG. 81

Let Fig. 81 represent a straight length of pipe with steady uniform flow under pressure. Let D denote the internal diameter of the pipe in feet. Then

(8) $A = \frac{\pi D^2}{4}$ = cross-sectional area of stream in square feet;

(9) $V = \frac{Q}{A} = \frac{4\,Q}{\pi D^2}$ = mean velocity of flow in feet per second;

(10) $Q = AV = \frac{\pi D^2}{4} V$ = discharge in cubic feet per second.

294. The Friction Head. Between any two consecutive sections, as 0 and 1, the water will part with some of its energy; and the head at 1 will be less than at 0. As the pipe is straight

and the flow both uniform and steady, such loss of head as may occur will be due to the resistance to motion, which is in part rubbing friction between the pipe surface and the water, and in part is internal resistances in the water itself due to disturbances which are incident to flow. This resistance is called *friction*. The loss of head due to resistances in a straight pipe with uniform flow is called ***friction head***, and may be designated h_f.

The friction head may be actually determined in an existing pipe by measuring the pressure heads and elevations at two consecutive sections of a pipe and making the following computations. Let

$\frac{p_0}{w}$ = the pressure head at the center of the pipe at 0.

h_0 = the elevation of the center of the pipe above the datum plane at 0.

$\frac{p_1}{w}$ = the pressure head at the center of the pipe at 1.

h_1 = the elevation of the center of the pipe at 1.

Then, by (2), the total head at 0 will be

$$H_0 = \frac{V^2}{2g} + \frac{p_0}{w} + \frac{p_a}{w} + h_0,$$

and the total head at 1 will be

$$H_1 = \frac{V^2}{2g} + \frac{p_1}{w} + \frac{p_a}{w} + h_1;$$

hence the head lost in friction is

$$(11) \qquad h_f = H_0 - H_1 = \frac{p_0}{w} + h_0 - \frac{p_1}{w} - h_1 = \text{here } h_\lambda.$$

295. Formulas for Uniform Flow in Pipes under Pressure. If at the same time the volume of flow and the area of the pipe are also determined, the relation between the friction head and the velocity of flow may be computed. From a series of observations covering a sufficient range of conditions, the law of flow for a particular pipe may be deduced. From experimental data obtained in this way, the probable flow in other pipes may be predicted. For many particular cases fairly complete information is available and reliable predictions may be made, but for a large number of cases dependence has to be placed upon empirical formulas.

Problems concerning the flow in pipes arise in determining either the head required, the mean velocity, the discharge, or the diameter, if the length and the other essential factors are given. Two of the common forms of the Chezy formula are given by equations (5) and (6):

$$V = C(RS)^{1/2}; \qquad \text{or} \qquad h_f = f\frac{LV^2}{D\,2\,g}.$$

For uniform steady flow in circular pipes, the mean hydraulic radius is

$$R = \frac{D}{4}, \tag{12}$$

the slope of the hydraulic grade line is

$$S = \frac{h_f}{L}, \tag{13}$$

and the area of the stream is

$$A = \frac{\pi D^2}{4}.$$

Hence the friction head is

$$h_f = \frac{4\,V^2L}{C^2D}; \qquad \text{or} \qquad h_f = f\frac{LV^2}{D\,2\,g}. \tag{14}$$

Comparing of coefficients C and f, we find

$$C = \left(\frac{8\,g}{f}\right)^{1/2} = \frac{16.04}{(f)^{1/2}}; \qquad \text{and} \qquad f = \frac{8\,g}{C^2} = \frac{257.28}{C^2}. \tag{15}$$

Both of the above sets of equations are used as may be convenient; coefficients have in some instances been derived in terms of f, in others in terms of C, but they are strictly convertible.

296. Formulas for Flow in Pipes. A great many formulas have been developed, particularly in recent years, for determining the flow in pipes under pressure. Each is based on experiments of greater or lesser extent and has its own special coefficients.

Cast-iron pipes deteriorate in service to an extent depending upon their size and upon the quality of the water flowing through them. When a new pipe is installed, a substantial allowance, say 30–50%, must be added to the necessary discharge to allow for this future decrease in carrying capacity. When the flow through an old pipe is to be estimated, there is always the question as to the actual state of the pipe. This decrease in carrying

capacity becomes of less importance as the size of the pipe increases.

It can easily be seen that it is difficult to estimate the flow through a pipe. Such an estimate can be made quite accurately for a *new* pipe, but it is seldom that this is finally desired. The pipe must be made large enough to handle the necessary quantity *after it has deteriorated*, and the amount of this deterioration is difficult to predict. It is usually a waste of time to split hairs with various special exponential formulas, when an arbitrary allowance must also be added.

The Chezy formula is not exact, but it is a good approximation and is convenient to use. Also a great many coefficients have been determined for various sizes and ages of pipe. Of the two forms of the formula, the first, $V = C\sqrt{RS}$, is to be preferred, since it is easier to remember, and C is given more commonly than f. The coefficient must be picked from some of the many recorded experiments and due allowance must be made for deterioration. For new cast-iron pipe well laid a good average value of C is 130, but it may run as high as 140 when exceptionally well laid. For computations, a value of about 100 is apt to be used, but if the pipe is to be used for a long period without cleaning, 70 or 80 is a better value, especially if the pipe is small.

Approximate Values of C

Kind of Pipe	C for New Pipe	C Allowing for Deterioration
Cast iron	130	90
Riveted steel	110	95
Wood stave	120	110
Fire hose, unlined linen	80	—
Fire hose, rubber lined	120	—

Cleaning will generally restore the pipe to a condition practically equivalent to that of new pipe. With new pipe the value of C usually increases somewhat with the velocity, while with old pipe the reverse is true.

Riveted steel pipe has a lower coefficient than cast-iron pipe. For large steel pipes, or penstocks (2 feet or larger) a value of 110 is about right. Such pipe is already rough on the inside, so that deposits do not increase the friction much, and the coefficient seldom falls below 95.

Wood-stave pipe has a smooth inner surface, and is little affected by age. The value of C will usually run between 120 and 130, with a deterioration in the course of years of perhaps 10%.

It must be realized that the above values are only approximate and will vary considerably with individual cases. A small pipe will deteriorate more in carrying capacity than will a larger pipe.

Cast-iron pipe is usually coated on the inside with an asphaltum compound of some sort, which not only gives it a smoother surface but also decreases the rate of deterioration.

For most pipe problems, very satisfactory results may be obtained by the use of Williams and Hazen's Pipe Tables, and the use of these is to be recommended for the majority of cases when waterworks pipes are to be designed.

297. Wrought-Iron and Steel Riveted Pipes. The discharge for wrought-iron and steel riveted pipes is less than for cast-iron pipes of the same age and diameter, because the overlapping plates and projecting rivet heads at the joints usually offer more resistance than the joints in cast-iron pipes. By countersinking the rivets and using taper joints with beveled edges, a greater capacity can be obtained. For coefficients see Table XXVI.

298. Old Cast-Iron and Steel Pipes. As iron or steel pipes age with service, not only will their interior surfaces usually become roughened by the destruction of the preservative coating, and the formation of vegetable and other growths increase the friction head; but also these growths may increase with time to such an extent as to diminish the actual discharge area of the pipe. See Fig. 82.

There are at the present time sufficient experimental data to forecast the depreciation of iron and steel pipes sufficiently accurately for ordinary waterworks and water-power purposes, but hardly enough to formulate satisfactorily these results. The increasing number of large and small pipes and aqueducts which are being measured periodically will probably make this possible in the near future. Empirical formulas for predicting the deterioration of pipes should be cautiously used.

The friction head for a given discharge is often assumed to increase in service according to the following multipliers:

Years in service	5	10	15	20	25	30	35	40
Multipliers	1.16	1.31	1.47	1.63	1.78	1.94	2.11	2.25

FIG. 82. — A CAST-IRON (WATER) PIPE CUT APART TO SHOW THE ACCUMULATION OF TUBERCLES IN 54 YEARS OF SERVICE

299. Losses of Head in Variable Flow. If a pipe is not straight, nor of uniform cross-sectional area, in addition to the friction head there will be other losses due to changes in velocity or direction of flow. These losses vary in magnitude, and should be considered in any problem if their magnitude is sufficient to affect the degree of accuracy required in the result. These losses may be expressed conveniently as a function of the velocity head, or directly as lost head in feet.

300. Head Lost at Entrance. For a short distance at the end where a pipe takes out of a reservoir, experiments have shown

that the stream first contracts and then enlarges, and finally fills the pipe as in a short tube of similar form. The losses in a short tube depend on the relation between velocity in the contracted section and the velocity in the pipe where it is flowing full; and this relation is largely determined by the form of the opening. It is fair to assume that the losses in the short distance at the entrance to a pipe may be computed as for a short tube. Therefore the head lost at entrance is

$$h_0 = \left(\frac{1}{C^2} - 1\right)\frac{V^2}{2g} = \xi_0 \frac{V^2}{2g}, \tag{16}$$

where the following notation is used:

V = the mean velocity of flow in the pipe.

C = the coefficient of discharge of a short tube having a form similar to the entrance end of the pipe.

ξ_0 = factor for entrance loss $= \left(\frac{1}{C^2} - 1\right)$.

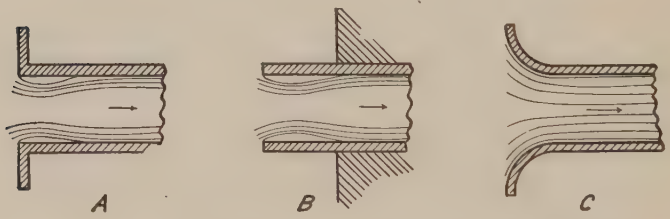


Fig. 83. — Three Common Forms of Pipe Inlets

The common forms of entrance are as follows:

A. The end of the pipe set flush in a flat surface. (See Fig. 83 *A*.) This form corresponds to a standard short tube. Hence $C = 0.815$ and $\xi_0 = 0.5$.

B. The end of the pipe projecting inward. (See Fig. 83 *B*.) This form corresponds to a reentrant short tube. Hence $C = 0.72$, and $\xi_0 = 0.93$.

C. The end of the pipe shaped like a bell mouthpiece. (See Fig. 83 *C*.) This corresponds to a rounded orifice. Hence $C = 0.95$ to 0.99, and $\xi_0 = 0.11$ to 0.02.

301. Loss because of Enlargement of Section. Abrupt Enlargement. Figure 84 represents a section of a pipe through which a steady discharge is flowing under pressure. At sections *OB*

and EF the pipe is entirely filled; and the following relations exist:

$$Q = A_0V_0 = A_1V_1,$$

$$\frac{V_0}{V_1} = \frac{A_1}{A_0}, \quad V_0 = \frac{A_1V_1}{A_0},$$

$$V_1 = \frac{A_0V_0}{A_1}. \tag{17}$$

The change in velocity from V_0 to V_1 is accompanied by a dissipation of energy which causes the sum total of energy in the stream

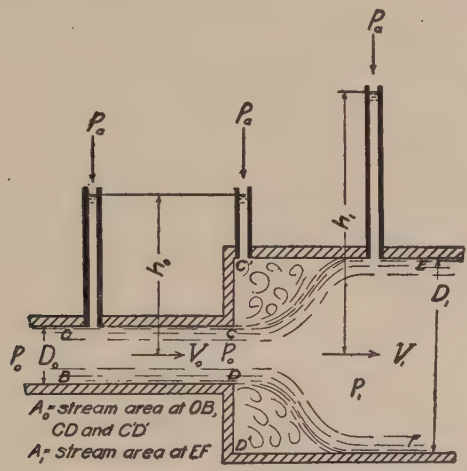


FIG. 84. — ABRUPT ENLARGEMENT IN A PIPE

at EF to be less than at OB. Let us use the following notation:

W = weight of volume of flow in pounds per second $= wQ$,

h_e = the head lost because of enlargement,

$$h_e = \xi_e \frac{V^2}{2g},$$

ξ_e = factor for enlargement loss.

By Bernoulli's theorem, we have

$$W\left(\frac{p_a}{w} + \frac{V_0^2}{2g} + h_0\right) = W\left(\frac{p_a}{w} + \frac{V_1^2}{2g} + h_1 + h_e\right); \tag{18}$$

therefore the head lost is

$$h_e = \frac{V_0^2 - V_1^2}{2g} + h_0 - h_1. \tag{19}$$

The force required to change W pounds of water per second from V_0 to V_1 (see § 82) is

$$F = \frac{W}{g}(V_1 - V_0) = \frac{wA_1V_1}{g}(V_1 - V_0). \tag{20}$$

Theoretically the intensity of pressure at CD and at $C'D'$ (neglecting the space filled by eddies) should be equal to the intensity of pressure at OB because the sectional areas of *the stream* are equal. There should be therefore at the entrance to the large pipe a total pressure in the direction of flow equal to $+ h_0wA_1$ (the plus sign indicating direction of motion). At EF there is a total pressure opposed to flow equal to $- h_1wA_1$ (the minus sign indicating direction opposed to motion). The algebraic sum of the total pressures at these two sections should theoretically be the force causing change in velocity; therefore

$$F = + h_0wA_1 - h_1wA_1 = wA_1(h_0 - h_1). \tag{21}$$

From (20) and (21), we have

$$wA_1(h_0 - h_1) = \frac{wA_1V_1}{g}(V_1 - V_0), \tag{22}$$

whence

$$h_0 - h_1 = \frac{V_1^2 - V_0V_1}{g} = \frac{2V_1^2 - 2V_0V_1}{2g}. \tag{23}$$

Substituting from (23) into (19), we find

$$h_e = \frac{V_0^2 - V_1^2 + 2V_1^2 - 2V_0V_1}{2g} = \frac{(V_0 - V_1)^2}{2g}. \tag{24}$$

This loss is in addition to the friction head. Since we have, by (17),

$$V_1 = \frac{A_0V_0}{A_1},$$

we may write

$$h_e = \left(1 - \frac{A_1}{A_0}\right)^2 \frac{V_1^2}{2g} = \left(1 - \frac{D_0^2}{D_1^2}\right)^2 \frac{V_1^2}{2g} = \xi_e \frac{V_0^2}{2g}. \tag{25}$$

But few experimental data confirming this formula are available. Weisbach states that he made experiments which confirm this theory of loss of head, and that Borda first found it. Gibson at a

later date also made experiments the results of which tend to confirm the theory. Archer [1] made some experiments on enlargement losses in pipes. He used commercial brass pipes ranging

TABLE XX

TABLE OF VALUES OF $\xi_e = \left[1 - \frac{D_0^2}{D_1^2}\right]^2$

$\frac{D_0}{D_1}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
ξ_e	0.98	0.92	0.83	0.70	0.56	0.41	0.26	0.13	0.04	0.0

from 1 to 3 inches in diameter and the ratio $\frac{D_0}{D_1}$ varied from .33 to .83. He derived the following formula:

$$h_e = 1.098 \frac{(V_0 - V_1)^{1.919}}{2g}.$$

302. Gradual Enlargement. If the section is enlarged uniformly through a sufficient distance and the edges rounded (see Fig. 85), the loss due to enlargement will be practically eliminated.

FIG. 85. — GRADUAL ENLARGEMENT IN A PIPE

For the purposes of the present book the loss due to gradual enlargement may be considered to be negligible, unless the angle of divergence is over perhaps 15°. For a divergence of from 15–30° one-half the loss for a sudden expansion of equivalent dimensions should be used; and above 30°, the full sudden expansion loss applies. Gibson [2] gives the results of several experiments on loss in diverging tubes, and Parker [3] gives a résumé of Andres' experiments.

303. Loss because of Contraction of Section. Abrupt Contraction. Figure 86 represents a section of a pipe through which a steady discharge is flowing under pressure. At sections *OB* and

[1] TRANSACTIONS A. S. C. E., vol. 76 (1913).
[2] A. H. Gibson, *Hydraulics and its Applications*, Van Nostrand, 1915.
[3] P. à M. Parker, *Control of Water*, Van Nostrand, 1915.

GH the pipe is assumed to be entirely filled; at section *EF* the stream cross section is smaller than the area at the entrance edge of the smaller pipe, or at *GH*. The following relations exist between velocity and area at these three sections:

$$Q = A_0V_0 = A_cV_c = A_1V_1, \tag{26}$$

whence

$$V_c = \frac{A_1V_1}{A_c}. \tag{27}$$

Since the edge at entrance to the smaller pipe corresponds to a sharp-edged orifice, the area of the contracted section will

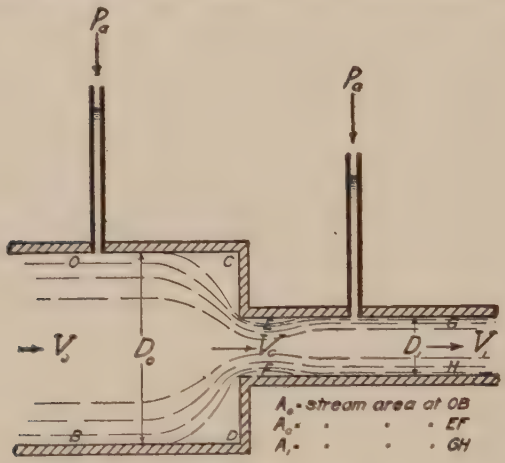


Fig. 86. — Abrupt Contraction in a Pipe

depend, as for an orifice, on the ratio, A_0/A_1; from which C, the coefficient of contraction, may be determined. If C_c is known, we have

$$A_c = C_cA_1, \quad \text{and} \quad \frac{A_1}{A_c} = \frac{1}{C_c}; \quad \text{hence} \quad V_c = \frac{V_1}{C_c}. \tag{28}$$

The change in velocity from V_0 to V_1 is accompanied by a dissipation of energy and causes a loss of head. Let h_κ denote the head lost because of contraction $= \xi_\kappa V^2/2\,g$, and let ξ_κ denote the factor for contraction loss. Then the head lost because of contraction may be divided into two parts: one loss between *OB* and *EF*, and the other between *EF* and *GH*. The first is similar to the loss in an orifice, is relatively small, and is frequently neglected; the

second is the loss due to sudden enlargement. The sum of these losses is

$$h_\kappa = \frac{V_c^2}{2g}\left(\frac{1}{C_v^2} - 1\right) + \frac{(V_c - V_1)^2}{2g}. \tag{29}$$

Substituting the value of V_c from (28), we find

$$h_\kappa = \frac{V_1^2}{2g}\left[\frac{1}{C_c^2}\left(\frac{1}{C_v^2} - 1\right) + \left(\frac{1}{C_c} - 1\right)^2\right], \tag{30}$$

and

$$\xi_\kappa = \frac{1}{C_c^2}\left(\frac{1}{C_v^2} - 1\right) + \left(\frac{1}{C_c} - 1\right)^2. \tag{31}$$

304. Values of ξ_κ. Very few determinations have been made of the value of ξ_κ. The following approximate values are based upon Freeman's experiments with ring nozzles and three sets of experiments by Gibson [1] upon models of sudden contraction in pipes.

$\frac{D_1}{D_0}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
ξ_κ	0.57	0.51	0.45	0.38	0.32	0.26	0.20	0.13	0.07	0.0

305. Gradual Contraction. If the section is decreased evenly in a sufficient distance, loss due to the contraction is practically eliminated. Abrupt contractions, diaphragms, or bushings should be avoided if possible.

306. Head Lost in Bends. Between the beginning and end of a bent or curved portion of a pipe, the loss in head will be greater than for an equal length of a straight portion of the same pipe. The reasons are many, but important among them are the increase in maximum velocity and redistribution of velocities. The increased resistance due to the effect of the bend may be computed as a separate additional loss and added to the friction head. Let h_b denote the additional head lost due to the bend

$$h_b = \xi_b \frac{V^2}{2g},$$

and let ξ_b denote factor for additional head lost in bend. The experimental data available for computing ξ_b are meager and somewhat discordant.

[1] Gibson, A. H., *Hydraulics and its Applications*, Van Nostrand, 1915.

307. For Sharp Bends or Elbows. Weisbach [1] deduced from experiments on a smooth iron pipe 3 centimeters (1.2 inches) in diameter the following formula (see Fig. 87):

$$\xi_b = 0.95 \sin^2 \frac{\theta}{2} + 2.05 \sin^4 \frac{\theta}{2}. \tag{32}$$

By formula (32) Weisbach calculated the following values:

TABLE XXI

θ	20°	40°	60°	80°	90°	100°	120°	140°
ξ_b	0.05	0.14	0.36	0.74	0.98	1.26	1.86	2.43

For smaller diameters he found ξ_b to be greater; for a diameter of 1 centimeter (0.4 inch) and $\theta = 90°$, $\xi_b = 1.54$.

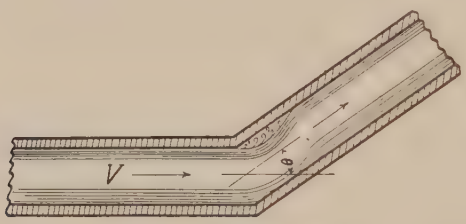


FIG. 87. — SHARP BEND

308. Curved Bends. Curved bends offer less resistance than elbows. Weisbach [2] from his own and Du Buats's experiments deduced a formula for circular pipes. Later experiments have shown that his formula is not strictly true, since the friction loss appears to increase with the radius of curvature beyond a certain point. Weisbach's experiments were limited in range and made only on very small pipes.

309. Williams, Hubbell, and Fenkell Experiments [3] on Curves. In 1897–1901 Williams, Hubbell, and Fenkell made an elaborate set of experiments on the flow in new cast-iron pipes, coated with coal tar, of 12, 16, and 30 inches diameter. The results given below are taken from a curve plotted by them, and are based chiefly on the 30-inch pipe. They show the increase in resistance caused by curves in portions of the pipe 80 diameters long,

[1] *Mechanics*, Coxe's translation, p. 895.
[2] *Mechanics*, Coxe's translation, p. 897.
[3] TRANSACTIONS A. S. C. E., vol. 47, 1902, pp. 1–335.

which included the curve, in comparison with an equal length of all straight pipe. The curves all had 90° central angle. In the following table D is the diameter of the pipe and ρ the radius of curvature on the center line, given in inches.

There were only two experiments on the 16-inch pipe, a 5-foot radius showing an excess of 25 per cent, and a compound curve showing an excess of 28 per cent over an equal length of straight pipe.

TABLE XXII

RELATIVE EXCESS LOSS OF HEAD DUE TO CURVES IN A LENGTH OF 80 DIAMETERS, INCLUDING THE CURVES

For the 30-inch pipe:

$\frac{\rho}{D}$	2	2½	3	4	5	10	15	20	25
Per cent increase due to curve:	14	13	14	18	24	50	67	80	93

For the 12-inch pipe the percentage increases due to curves for five experiments were as follows:

$\frac{\rho}{D}$	1.08	2	3	4
Per cent increase due to curve:	24.6	10.8	17.1	19.1
			30.	

The minimum increase observed was when $\rho/D = 2\frac{1}{2}$. For greater ratios the resistance due to curvature was observed to increase with the length of curve, that is, with increase of radius for a given central angle, which appeared contrary to all previous experiments; but Davis's experiments (§ 312), and Brightmore's (§ 313), indicate a similar increase though from different minima.

310. Saph and Schoder's Experiments. Saph and Schoder[1] made experiments on a 2-inch seamless drawn brass pipe. Their conclusion is: "The total excess losses of head due to curves showed no tendency to follow any simple law." The results are too voluminous to be reproduced.

They also made a set of experiments[2] on 6-inch pipes, from which Table XXIII is taken.

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[1] TRANSACTIONS A. S. C. E., vol. 47, pp. 295–323.
[2] TRANSACTIONS A. S. C. E., vol. 62 (1909), pp. 67–96.

TABLE XXIII

EXCESS LOSSES OF HEAD DUE TO CURVES, EXPRESSED IN TERMS OF VELOCITY HEADS; $h_b = \xi_b \frac{V^2}{2g}$

No. of Curve	Material	Diameter, Mean of Four End Calipers	Radius in Feet ρ	Radius in Pipe Diameters $\frac{\rho}{D}$	Length on Center Line in Feet	Values of ξ_b Velocity, in feet per second			
						3	5	10	16
1	Wrought Iron	6.09 in.	10.00	20	16.77	0.34	0.27	0.19	0.14
2	Wrought Iron	6.18 in.	7.50	15	12.84	0.15	0.09	0.05	0.02
3	Wrought Iron	6.16 in.	5.00	10	9.01	0.21	0.16	0.11	0.08
4	Wrought Iron	6.11 in.	4.00	8	7.34	0.28	0.22	0.17	0.13
5	Wrought Iron	6.11 in.	3.00	6	5.89	0.28	0.21	0.17	0.14
6	Wrought Iron	6.09 in.	2.50	5	5.08	0.14	0.12	0.11	0.11
7	Cast Iron	5.91 in.	2.00	4	3.64	0.24	0.18	0.16	0.12
8	Cast Iron	5.95 in.	1.50	3	2.86	0.21	0.18	0.16	0.12
9	Cast Iron	5.91 in.	1.08	2.16	2.54	0.22	0.19	0.17	0.13
10	Cast Iron	5.91 in.	0.95	1.90	1.75	0.25	0.21	0.19	0.16
11	Cast Iron	5.95 in.	0.88	1.76	3.62	0.24	0.24	0.23	0.22
12	Cast Iron	5.93 in.	0.67	1.34	1.05	0.39	0.33	0.29	0.26

311. Alexander's Experiments.[1] Alexander's experimental results on varnished wooden pipes $1\frac{1}{4}$ inches internal diameter are expressed as follows:

Where $\frac{D}{2\rho}$ is not greater than 0.2,

$$h_b = 1.66 \left(\frac{D}{2\rho}\right)^{0.83} L(0.00498)\, V^{1.777}, \tag{33}$$

and where $\frac{D}{2\rho}$ is from 0.2 to 0.5,

$$h_b = 25.0 \left(\frac{D}{2\rho}\right)^{2.5} L(0.00498)\, V^{1.777}, \tag{34}$$

where h_b is the loss of head due to bends in excess of the friction head for an equal length (L) of straight pipe. For the straight pipe Alexander determined the equation $S = 0.0407\, V^{1.77}$.

Alexander also derived the following equation from a computation of Saph and Schoder's 2-inch brass pipe experiments:

$$h_b = 3.27 \left(\frac{D}{2\rho}\right)^{0.83} L(0.00262)\, V^{1.75}. \tag{35}$$

[1] C. W. L. Alexander, *Proceedings Inst. of Civil Eng.*, vol. 159, p. 363.

312. Davis's Experiments on Bends and Elbows. G. J. Davis reported [1] the results of experiments made at the University of Wisconsin, part of which are here given. They are of especial interest because they include experiments on commercial elbows and tees. Figure 88 shows several of those tested.

All the curves experimented on made a turn of 90°, and had radii of curvature as given in the following table, XXIV.

Numbers 1 to 6, inclusive, were standard fittings purchased from the Crane Company. Number 7 was a special casting. Numbers 8 and 9 were bent from 2-inch pipe.

Drawings of Numbers 1 to 6, inclusive, are shown in Fig. 88. Number 7 was similar to Nos. 2 and 3, the only difference being in the longer radius of curvature. Numbers 8 and 9 were fitted with ordinary flanges, no attempt being made to obtain a flush joint.

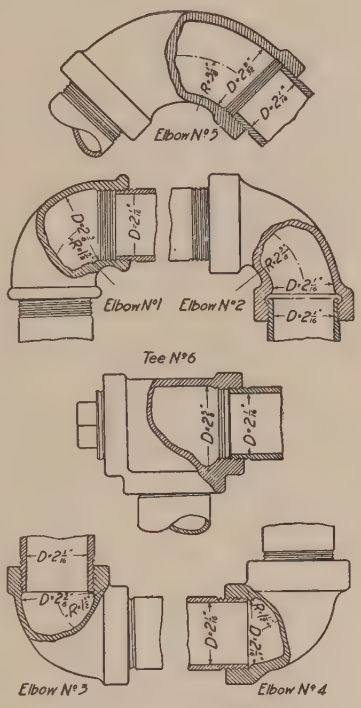

FIG. 88. — ELBOWS AND TEE

Table XXIV is taken from Davis's paper. The elbows were preceded and followed by straight pipe. The head was read at several piezometers; one (B) was located 1 foot upstream, another (E), 17 feet downstream from the elbow.

The excess loss of head due to curvature is taken as the difference between the total loss of head between B and E as shown by piezometers, and the head that would be obtained by multiplying the actual center-line length on the curve by the friction head per foot of straight pipe. The straight pipe was a nominal 2-inch wrought iron pipe; experimental results on the straight pipe are given in Table XXVI.

[1] TRANSACTIONS A. S. C. E., vol. 62, pp. 97–112.

TABLE XXIV

No.	Description	Internal Diameter of Elbow in Inches	$\frac{\rho}{D}$
1	Malleable iron elbow	$2\frac{3}{8}$	0.788
2	Cast iron, long-turn, drainage elbow	$2\frac{1}{16}$	1.150
3	Cast iron drainage elbow	$2\frac{3}{8}$	0.728
4	Cast iron, short turn	$2\frac{3}{8}$	0.728
5	Cast iron, long sweep	$2\frac{9}{32}$	1.520
6	Cast iron tee		0.
7	Cast iron	$2\frac{1}{16}$	2.500
8	Wrought iron or steel	$2\frac{1}{16}$	5.000
9	Wrought iron or steel	$2\frac{1}{16}$	10.000

Velocity in Feet per Second	Excess Loss of Head in Feet, Due to Curves					
	$\frac{\rho}{D} = 0.00$	$\frac{\rho}{D} = 0.728$	$\frac{\rho}{D} = 1.15$	$\frac{\rho}{D} = 2.50$	$\frac{\rho}{D} = 5.00$	$\frac{\rho}{D} = 10.00$
2	0.101	0.040	0.022	0.0086	0.0043	0.017
3	0.230	0.092	0.055	0.023	0.014	0.043
4	0.405	0.165	0.104	0.048	0.032	0.083
5	0.640	0.260	0.175	0.086	0.062	0.142
6	0.860	0.370	0.260	0.140	0.105	0.216
8	1.650	0.660	0.500	0.280	0.240	0.420
10		1.020	0.840	0.500	0.470	0.700
15		2.35	2.10	1.40	1.45	1.80

Taken from a curve given by Davis.

313. Brightmore's Experiments on Elbows and Bends. Brightmore [1] experimented on the effects of elbows and bends in 3 and 4-inch pipes. The elbows and bends were all rusted by sal ammoniac, and also the 4-inch straight pipe which was coupled with the 4-inch bends; but the 3-inch pipe was new and galvanized. The loss due to the bend was found not to take place entirely in the bend, but also in the straight pipe following. A length of straight pipe $6\frac{2}{3}$ feet long for the 3-inch, and 5 feet, or 6 feet $7\frac{1}{4}$ inches for the 4-inch, was, therefore, included with the bend in measuring the loss of head. From the loss of head in these combined bent and straight pieces, the friction head for an

[1] A. W. Brightmore, *Proceedings Inst. of Civil Engrs.*, vol. 169, pp. 315 *et seq.*

equal length of straight pipe was subtracted, giving the loss of head due to the bend. The following values were scaled from curves given by Brightmore.

TABLE XXV

EXCESS LOSS OF HEAD IN FEET, DUE TO CURVES

3-INCH VELOCITIES	ELBOW	$\frac{\rho}{D} = 2$	$\frac{\rho}{D} = 4$	$\frac{\rho}{D} = 6$	$\frac{\rho}{D} = 8$	$\frac{\rho}{D} = 10$	$\frac{\rho}{D} = 14$
3	.140	.0667	.05	.05	.05	.025	.0167
5	.450	.167	.125	.142	.125	.075	.0500
7	.950	.317	.242	.275	.242	.142	.0917
10	1.825	.642	.475	.63	.55	.283	.217
4-INCH							
3	.167	.0667	.0417	.050	.0333	.0250	
5	.433	.158	.108	.133	.108	.0833	
7	.925	.275	.217	.250	.208	.167	
10	1.833	.600	.458	.508	.458	.358	

314. Losses in Gates and Valves. There is comparatively little experimental data on losses through gates and valves. A wide-open gate usually offers very little obstruction to the flow. Williams, Hubbell, and Fenkell [1] found the loss through a wide-open 30-inch gate valve to be 0.07 ft. at a velocity of about 2.5 ft./sec.

A recent bulletin (1922) of the University of Wisconsin [2] gives the results of a series of tests on commercial gate valves of various sizes. The following table gives the average results of the various makes and types tested.

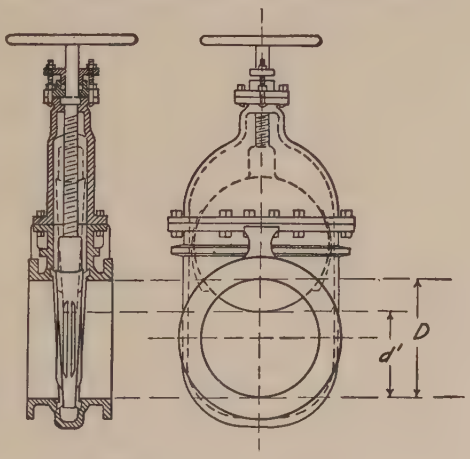

FIG. 89. — GATE VALVE

[1] TRANSACTIONS A. S. C. E., vol. 47, 1902.

[2] Corp and Ruble, BULLETIN OF UNIV. OF WIS., vol. 9, No. 1. *Experiments on loss of head in valves.*

VALUES OF ξ IN THE FORMULA $h = \xi \frac{V^2}{2g}$

DIAMETER OF VALVE IN INCHES	AMOUNT OF OPENING					
	1/8	1/4	3/8	1/2	3/4	FULL
$\frac{1}{2}$	374	53.6	18.3	7.7	2.2	.81
$\frac{3}{4}$	308	34.9	9.9	4.2	.92	.28
1	211	40.3	10.1	3.5	.88	.23
$1\frac{1}{2}$	111	19.7	6.5	2.9	.58	.20
2	146	22.5	7.1	3.2	.74	.17
4	67	13.0	4.6	1.9	.41	.16
6	87	17.1	6.1	2.6	.52	.14
8	66	13.5	4.9	2.2	.46	.10
12	96	17.4	5.6	2.3	.41	.05

315. Compound Pipes. Systems of pipes which branch at one point and reunite at another, forming several routes by which the water may be delivered not only to scattered consumers, but also by which in case of a break in one line users may be supplied by the other lines, are now considered essential in water supply systems, especially if they are to afford suitable fire protection.

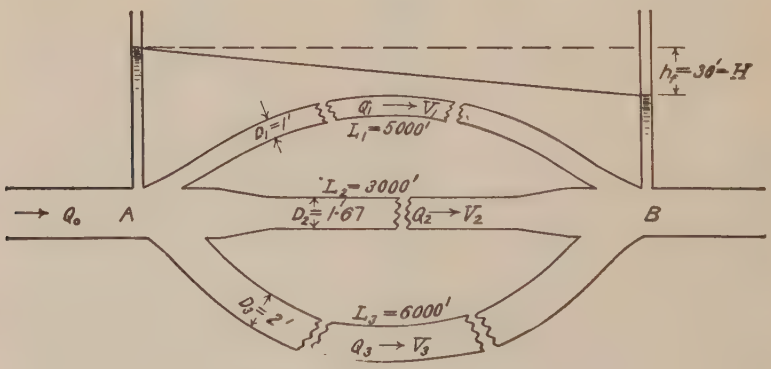

FIG. 90. — COMPOUND PIPE LINE

Let it be assumed that piezometers are set at A and B in Fig. 90; then during flow the total head H equals the difference in water levels between the two piezometers. Assuming that all

the head H is used in friction, we may compute the discharge through each pipe as through a simple pipe, viz.:

$$Q_1 = 6.3\left(\frac{30 \times 1^5}{0.0216 \times 5000}\right)^{1/2} = 3.32 \quad \text{cubic feet per second.}$$

$$Q_2 = 6.3\left(\frac{30 \times 1.67^5}{0.021 \times 3000}\right)^{1/2} = 15.59 \quad \text{cubic feet per second.}$$

$$Q_3 = 6.3\left(\frac{30 \times 2^5}{0.0207 \times 6000}\right)^{1/2} = 17.92 \quad \text{cubic feet per second.}$$

The total discharge through the main is

$$Q_0 = 3.32 + 15.59 + 17.92 = 36.83$$

If, on the other hand, the total discharge Q_0 is given, the head at A, the discharge of each individual pipe, and the head at B may be computed by trial by assuming various values of the friction head until the sum computed for the assumed head, $Q_1 + Q_2 + Q_3$, equals Q_0.

Pipe Experiments. On the following pages are given the results of a considerable number of experiments made upon flow in pipes. The size of the pipes included varies from $\frac{1}{10}$ inch to 18 feet and the material ranges from brass to concrete. This table attempts to give examples of various kinds and sizes of pipes in order that an idea may be obtained of how the loss in a pipe varies with its size, kind and condition, and to furnish a ground upon which to base estimates of the probable performance of other pipes.

The table gives:

a. The nominal or actual diameter of the pipe;
b. Its age in service;
c. The kind of coating on the inner surface;
d. Condition of the inner surface at the time of the experiments;
e. Length in feet between piezometers, which was straight unless noted;
f. The kind of joints ("B. & S." meaning modern bell and spigot);
g. The method of determining the discharge; and
h. The source from which the information was obtained.

Velocities are, or are assumed to be, higher than the critical velocity.

NOTE. It is worth recording, as corroborating the experience of others who have made similar studies of pipe experiments, that the prevailing weakness in the majority of the experiments here referred to was in the measurements of the heads on account of faulty piezometers or piezometric couplings, and not in the accuracy of measuring the volumes of flow.

TABLE XXVI

1. SMALL BRASS PIPES

New, seamless drawn, straight, 6 to 57 feet, preceded and followed by a straight piece; special screw piezometric couplings. Measurement by weight. Temperature, 40° F. Results taken from Saph and Schoder, TRANSACTIONS A. S. C. E., vol. 51 (1904), p. 253. The values given for H_f are the loss of head per 1000 feet of pipe.

D = ACTUAL DIAMETER IN FEET		VELOCITY IN FEET PER SECOND							
		1	2	3	4	5	6	8	10
D = .00893	C			74	76	78	79	82	
	H_f			739	1238	1850	2563	4296	
D = .01261	C			76	78	80	82	85	87
	H_f			495	824	1223	1688	2811	4173
D = .01498	C			75	78	80	82	85	88
	H_f			426	703	1037	1424	2351	3467
D = .01846	C			78	81	83	85	89	91
	H_f			319	527	777	1068	1759	2601
D = .02176	C		75	80	83	85	87	91	94
	H_f		129	261	430	632	867	1427	2099
D = .02347	C	70	77	81	84	87	89	92	95
	H_f	34.5	116	234	387	570	784	1294	1909
D = .02679	C	71	78	82	85	88	90	93	96
	H_f	29.4	98.3	199	329	484	666	1073	1619
D = .03137	C	73	80	84	87	90	92	95	98
	H_f	24.2	80.8	164	270	398	547	902	1330
D = .04149	C	74	81	85	88	91	93	96	99
	H_f	17.4	61.3	120	199	294	404	672	994
D = .05251	C	75	83	88	92	95	97	101	105
	H_f	13.4	44.0	88.7	145	212	290	475	697
D = .06831	C	82	89	94	98	101	103	107	110
	H_f	8.8	29.4	59.4	97.9	144	198	327	481
D = .08785	C	83	90	95	99	101	103	107	
	H_f	6.6	22.3	45.4	75.2	111	153	254	
D = .10311	C	84	92	97	101	104	106		
	H_f	5.45	18.3	37.0	61.2	90.3	124		
D = .12484	C	87	95	100	104	106	109	113	
	H_f	4.20	14.2	28.9	47.7	70.7	97.5	162	
D = .1742	C	90	98	104	107	110			
	H_f	2.82	9.49	19.3	31.9	47.1			

2. Lap-Welded Steel and Wrought-Iron Pipes

¼ Inch (Actual .35)

New; galvanized; clean; 10.064 feet; screw; by weight; Saph and Schoder.*

V	1.3	2	3	4	5	6	8	10	11.4
C	63	63	64	64	64	65	65	65	65
H_f	58.9	137	304	534	827	1182	2077	3218	4160

⅜ Inch (Actual .49)

New; galvanized; clean; 17.032 feet; screw; by weight; Saph and Schoder.*

V	1.7	2	3	4	5	6	8	10	10.8
C	76	76	78	79	79	80	81	82	82
H_f	49.9	68.0	148	256	392	555	961	1471	1705

½ Inch (Actual .63)

New; galvanized; clean; 18.119 feet; screw; by weight; Saph and Schoder.*

V	2.5	3	4	5	6	8	10	12.8
C	81	82	84	85	86	88	89	91
H_f	72.8	102	175	265	372	635	963	1525

¾ Inch (Actual .82)

New; galvanized; clean; 17.496 feet; screw; by weight; Saph and Schoder.*

V	2.6	3	4	5	6	8	10	11.4
C	87	88	91	93	94	97	99	101
H_f	50.6	65.5	110	165	229	384	574	726

1 Inch (Actual 1.04)

New; galvanized; clean; 18.751 feet; screw; by weight; Saph and Schoder.*

V	2.8	3	4	5	6	8	10	10.1
C	78	78	79	79	80	81	81	81
H_f	59.9	68.4	119	184	261	455	700	713

2 Inches (Actual 2.07)

New; uncoated; clean; 62 feet; screw; by volume; G. J. Davis, Jr.†

V	1.5	2	3	4	5	6	8	10	14.7
C	102	104	107	109	111	113	115	117	121
H_f	5.08	8.65	18.3	31.2	47.1	66.0	113	170	347

2 Inches

Old; wrought iron; uncoated; rusted; 12.26 feet; screw; by volume; Schoder and Gehring, 1904–1908.‡

V	1	2	3	4	5	6	8	10	Limits
C	86	86	87	87	87	87	88	88	of V not
H_f	3.15	12.4	27.7	49.0	76.3	109	193	301	stated

* Transactions A. S. C. E., vol. 51 (1903), p. 253.
† Transactions A. S. C. E., vol. 97 (1908), p. 97.
‡ Engineering Record, vol. 58, p. 241.

2. Lap-Welded Steel and Wrought-Iron Pipes — *Continued*

3 Inches (Actual 3.07)

Old; wrought iron; uncoated; rusted; 19.97 feet; screw; by volume; Schoder and Gehring, 1904–1908.*

V	1	2	3	4	5	6	8	10	Limits
C	97	101	102	104	105	106	107	108	of V not
H_f	1.62	6.09	13.2	22.9	35.0	49.6	86.0	132	stated

4 Inches (Actual 4.03)

Old; wrought iron; uncoated; rusted; 22.15 feet; screw; by volume; Schoder and Gehring, 1904–1908.*

V	1	2	3	4	5	6	8	10	Limits
C	100	102	103	103	104	104	105	106	of V not
H_f	1.18	4.56	10.1	17.6	27.2	38.8	68.1	105	stated

5 Inches (Actual 5.05)

Old; wrought iron; uncoated; rusted; 48.04 feet; flange; by volume; Schoder and Gehring, 1904–1908.*

V	1	2	3	4	5	6	8	10	Limits
C	98	101	103	104	105	106	107	109	of V not
H_f	0.99	3.72	8.07	14.0	21.4	30.3	52.5	80.5	stated

6 Inches (Actual 6.06)

Old; wrought iron; uncoated, but glazed by use as steam pipe; 46.1 feet; flange; by volume; Schoder and Gehring, 1904–1908.*

V	1	2	3	4	5	6	8	10	Limits
C	115	117	119	120	120	121	122	123	of V not
H_f	0.6	2.30	5.06	8.83	13.6	19.4	33.9	52.3	stated

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3. Cast-Iron Pipes

$3\frac{1}{8}$ Inches

Old; uncoated; with deposits; 328.1 feet; socket; by volume; Darcy.†

V	1	2	3	3.7
C	66	68	69	69
H_f	3.56	13.5	29.3	43.9

$3\frac{5}{32}$ Inches

Old; uncoated; cleaned; 328.1 feet; socket; by volume; the preceding pipe after cleaning; Darcy.‡

V	1	2	3	4	5
C	90	91	91	91	92
H_f	1.87	7.38	16.5	29.1	45.3

* Engineering Record, vol. 58, p. 241.

† Darcy, *Recherches Expérimentales relatives au Mouvement de l'Eau dans les Tuyaux*, 1850, Series 14, pp. 12, 56.

‡ Darcy, loc. cit., Series 15, pp. 12, 56.

3. CAST-IRON PIPES — *Continued*

3 7/32 Inches

New; uncoated; clean; 328.1 feet; socket; by volume; Darcy.*

V	1	2	3	4	5	6	8	10	10.7
C	90	93	95	97	98	99	100	101	102
H_f	1.83	6.83	14.8	25.5	39.0	55.1	95.1	145	165

5 25/64 Inches

New; uncoated; clean; 328.1 feet; socket; by volume; Darcy, 1850.†

V	1	2	3	4	5	6	8	10	15.4
C	102	106	108	109	110	111	112	114	116
H_f	0.849	3.19	6.92	12.0	18.4	26.0	45.1	69.0	157

6 Inches

4 years; coal tar; unknown; 1170 feet; *B. & S.*; by nozzle; E. B. Weston, 1876–1877.‡

V			4.7	5	6	8	9.3
C			99	99	101	103	105
H_f			18.2	20.3	28.4	48.1	63.4

6 Inches

38 years; uncoated; with tubercles 3/8 inch to 1 1/4 inches thick; service pipe in street; *B. & S.*; by Deacon meter; Brackett, 1887.§

V	0.38	0.57	0.95	1.13	1.32	1.51	1.70
C	30	32	32	27	27	27	26
H_f	1.30	2.50	6.90	14.4	19.2	25.4	33.8

6 Inches

Same pipe as preceding, after it was cleaned by scraping, Brackett.§

V		0.76	0.95	1.13	1.51	1.70	2.08	2.46
C		42	69	83	69	74	73	72
H_f		2.70	1.50	1.50	3.80	4.20	6.50	9.40

7 13/32 Inches

New; uncoated; clean; 328.1 feet; socket; by volume; Darcy, 1850.¶

V	1	2	3	4	5	6	8	10	16
C	102	104	105	106	106	107	107	108	109
H_f	0.623	2.41	5.31	9.30	14.4	20.5	35.9	55.5	139

* Darcy, loc. cit., Series 16, pp. 12, 56.
† Darcy, loc., cit., Series 17, pp. 12, 58.
‡ TRANSACTIONS A. S. C. E., vol. 22, p. 18.
§ JOURNAL OF THE NEW ENGLAND WATERWORKS ASSOCIATION, vol. 13, p. 343.
¶ Darcy, loc. cit., Series 18, pp. 12, 58.

3. Cast-Iron Pipes — *Continued*

8 Inches

3 years; asphalt; not stated; 105 feet; *B. & S.;* by Venturi meter; E. W. Schoder, 1907.*

V	1	2	3	4
C	101	104	106	107
H_f	0.59	2.22	4.81	8.33

9$\frac{37}{64}$ Inches

Old; uncoated; with deposits; 328.1 feet; flange; by volume; Darcy, 1850.†

V	1	2	3	4	5	6	8	10	12.6
C	75	75	75	75	75	75	75	75	75
H_f	0.904	3.59	8.05	14.3	22.2	32.0	56.7	88.4	140

9$\frac{1}{4}$ Inches

Old; uncoated; cleaned; 328.1 feet; flange; by volume; the preceding pipe after cleaning; Darcy, 1850.‡

V	1	2	3	4	5	6	8	10	14.7
C	94	95	96	96	97	97	98	98	99
H_f	0.566	2.20	4.87	8.57	13.3	19.0	33.3	51.6	110

11$\frac{11}{16}$ Inches

Old; uncoated; very well cleaned; 328.1 feet; socket; by volume; Darcy, 1850.§

V	1	2	3	4	5	6	8	10	10.4
C	102	103	103	103	104	104	104	104	104
H_f	0.395	1.56	3.48	6.15	9.56	13.7	24.3	37.7	40.8

12 Inches

New; coal tar; clean; various lengths, straight; *B. & S.;* Pitot tube; Williams, Hubbell, and Fenkell, 1898–1901.¶

V	1	2	3	4	5
C	102	110	115	118	121
H_f	0.388	1.33	2.74	4.58	6.81

12$\frac{3}{4}$ Inches

New; asphalt; clean; about 3 miles, with easy bends; in reservoir; Iben, 1874–1879.**

V	1.6	2	3	3.1
C	91	95	103	103
H_f	1.24	1.77	3.40	3.59

* Transactions A. S. C. E., vol. 62, pp. 67–96; and Engineering Record, vol. 58, p. 241.

† Darcy, loc. cit., Series 19, pp. 12, 58.

‡ Darcy, loc. cit., Series 20, pp. 12, 58.

§ Darcy, loc. cit., Series 21, pp. 12, 58.

¶ Transactions A. S. C. E., vol. 47, p. 187.

** See H. Smith, Jr., *Hydraulics*, p. 231.

3. CAST-IRON PIPES — *Continued*

12 Inches

Old; unknown; unknown; 8160 feet, six bends; *B. & S.*; unknown; Thomas Duncan.*

V	1	2	3.5
C	98	87	84
H_f	0.42	2.11	3.55

16 Inches

New; coal tar; clean; various lengths, straight; *B. & S.*; Pitot tube; Williams, Hubbell, and Fenkell, 1898–1901.†

V	1	2	3	4	5
C	119	126	129	132	134
H_f	0.211	0.767	1.63	2.78	4.22

16$\frac{15}{16}$ Inches

New; varnish; clean; 4½ to 6 miles, with easy bends; *B. & S.*; by volume; Lampe, 1869–1871.‡

V	1.6	2	3	3.1
C	109	112	116	116
H_f	0.609	0.914	1.91	2.03

16 Inches

18 years; coal tar; tuberculated; 13,350 feet, **four curves**, three gates; *B. & S.*; by pumps; Forbes.§

V	1	2
C	94	90
H_f	0.342	1.47

19$\frac{11}{16}$ Inches

New; uncoated; clean; 328.1 feet; socket; by volume; Darcy, 1850.¶

V	1.4	2	3	3.7
C	106	110	114	116
H_f	0.422	0.808	1.69	2.48

20 Inches

1 to 5 years; coal tar; not tuberculated; 75,000 feet; alignment very irregular; *B. & S.*; by pumps; Brush, 1882–1887.**

V		2	3
C		115	110
H_f		0.729	1.80

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* PROCEEDINGS INST. C. E., 1852, p. 499.
† TRANSACTIONS A. S. C. E., vol. 47, p. 187.
‡ See H. Smith, Jr., *Hydraulics*, pp. 233–234.
§ JOURNAL, N. E. W. W., vol. 6 (1891), p. 164.
¶ Darcy, loc. cit., Series 22, pp. 12, 58.
** TRANSACTIONS A. S. C. E., vol. 19, p. 92.

3. Cast-Iron Pipes — *Continued*

28 Inches

New; coated; clean; 2006 feet, 2 curves; *B. & S.;* by weir; I. W. Smith.*

$V = 7.51$; $C = 123$; $H_f = 6.34$

30 Inches

New; coal tar; clean; various lengths, straight; *B. & S.;* Pitot tube; Williams, Hubbell, and Fenkell, 1898–1901.†

V	1	2	3
C	119	122	124
H_f	0.113	0.429	0.937

30 Inches

Coal tar; 5740 feet, several curves; *B. & S.;* by weir; C. W. Sherman.‡

1 year old, $C = 111$. **10 years old,** V = from 0.75 to 1.5, $C = 103$

32 Inches

New; coated; clean; 21,460 and 9613 feet, with 17 curves; *B. & S.;* by weir; I. W. Smith, 1894.*

$V = 5.75$; $C = 126$; $H_f = 3.10$

36 Inches

Asphalt; about 1000 feet, with curves; *B. & S.;* by use of water in a reservoir; Kuichling, 1895–1899.§

Age, years	1.28	1.30	3.06	3.37	3.91	4.47	5.04	5.34
V	4.20	4.24	4.13	4.02	4.03	4.03	4.08	4.08
C	129	125	99	66	70	75	80	82
H_f	1.39	1.51	2.28	4.85	4.34	3.76	3.45	3.26

In September, 1898, when the pipe was about 4.2 years old, it was entered and examined; extensive organic growths were found on the top and sides, the lower part was considerably tuberculated.

36 Inches

Laid in 1894; coal tar; 5500 feet, several curves; *B. & S.;* by weir; C. W. Sherman.¶

	1895	**1896**	**1897**
V	4.7	0.5 to 4.5	3.2
C	136	113	114

In 1896 the pipe was examined; only a small amount of tuberculation was found, but the whole interior surface was covered with a slimy substance (*Polyzoon Fredericella*).

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* Transactions A. S. C. E., vol. 36 (1894), p. 203.
† Transactions A. S. C. E., vol. 47, pp. 187–188.
‡ *Report of the Water Commissioner*, Boston, 1898, p. 123.
§ Transactions A. S. C. E., vol. 44, p. 56.
¶ *Report of the Water Commissioner*, Boston, 1897, p. 55; 1898, p. 123.

3. CAST-IRON PIPES — *Continued*

48 Inches

⅓ year; "pitch"; clean; 3557 feet, with several curves; *B. & S.;* by volume; A. F. Bruce.*

V	1.8	2	3	3.6
C	136	136	137	138
H_f	0.176	0.216	0.479	0.685

48 Inches

2 years; Angus Smith coating; clean; 1747 feet, 2 easy curves; *B. & S.;* by weir; Stearns.†

V	1‡	3.7	4	5	6	6.2	
C	130	140	141	142	144	144	
H_f	0.059	0.699	0.811	1.24	1.74	1.86	

48 Inches

17 years; Angus Smith coating; tuberculated; 1747 feet; by weir; same pipe as the one preceding, 15 years later; Fitzgerald, 1894–1895.§

V	1	2	3	3.5
C	107	106	105	105
H_f	0.088	0.359	0.818	1.12

48 Inches

17 years; Angus Smith coating; tuberculated; 1747 feet; by weir; a pipe laid parallel to the last, similar in every way; Fitzgerald, 1894–1895.§

V	1	2	3	3.5	4	5	5.8
C	110	110	109	109	109	109	109
H_f	0.082	0.333	0.754	1.03	1.35	2.12	2.86

48 Inches

17 years; Angus Smith coating; cleaned; 1747 feet; by weir; same pipe as Stearns's 48-inch, after cleaning; see above. Fitzgerald, 1895.§

V	1	2	3	3.5	4	5	6	7.2
C	132	136	139	140	140	142	143	144
H_f	0.0575	0.216	0.469	0.629	0.812	1.24	1.76	2.50

48 Inches

8 years; Angus Smith coating; partly tuberculated; 14,124 feet, with curves; *B. & S.;* by weir; J. W. Gale. ¶

$$V = 3.46; \qquad C = 112; \qquad H_f = 0.947$$

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* PROCEEDINGS INST. C. E., vol. 123, p. 414.
† TRANSACTIONS A. S. C. E., vol. 14 (1880), p. 4.
‡ Below the limits of experiments.
§ TRANSACTIONS A. S. C. E., vol. 35, pp. 241 *et seq.*
¶ INSTITUTE OF ENGINEERS AND SHIPBUILDERS IN SCOTLAND, vol. 12 (1869), 136.

3. Cast-Iron Pipes — *Continued*

61⅛ Inches

New; very smooth coating; clean; 1800 feet, with curves; *B. & S.;* not stated; Fitzgerald.*

V	4.7	5	6	6.5
C	150	151	153	154
H_f	0.773	0.865	1.21	1.39

60 Inches

8 years; coated; tuberculated; 8215 feet, with curves; *B. & S.;* by pumps; Fenkell.†

V	0.2	0.6	1.0	1.1
C	107	101	99	99
H_f	0.00279	0.028	0.082	0.100

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4. Riveted Steel or Sheet-Iron Pipes

3 Inches

New; galvanized; clean; 24.9 feet; taper; by weight; Giltner and Ketchum, 1899–00; reported by G. S. Williams.‡

V	1	2	2.3
C	80	83	84
H_f	2.49	9.23	12.0

3¼ Inches

New; asphalt; clean; 328.1 feet; screw coupling; by volume; Darcy.§

V	1	2	3	4	5	6	8	10	12.8
C	97	104	109	112	114	117	120	123	126
H_f	1.56	5.42	11.2	18.9	28.2	39.1	65.7	98.1	153

4 Inches

New; asphalt; clean; standard spiral riveted; 80.1 feet; flange; by volume; Schoder and Gehring.¶

V	1	2	3	4	5	6	8	10 Limits
C	94	99	102	104	106	107	109	111 of V not
H_f	1.34	4.83	10.2	17.4	26.3	36.9	62.8	94.9 stated

6 Inches

New; asphalt; clean; standard spiral riveted; 60 feet flange; by volume; Schoder and Gehring.¶

V	1	2	3	4	5	6	8	10 Limits
C	100	106	109	111	113	115	117	119 of V not
H_f	0.80	2.88	6.11	10.4	15.7	22.0	37.5	56.6 stated

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* Transactions A. S. C. E., vol. 44 (1900), p. 88.
† Transactions A. S. C. E., vol. 51 (1903), p. 325.
‡ Jour. Assn. Eng. Soc., vol. 26, p. 181.
§ Darcy, loc. cit., Series 8, pp. 12, 52.
¶ Engineering Record, vol. 58 (1908), p. 241.

4. Riveted Steel or Sheet-Iron Pipes — *Continued*

6 Inches

New; asphalt; clean; double extra heavy spiral riveted; 73.55 feet; flange; by volume; Schoder and Gehring.*

V	1	2	3	4	5	6	8	10	Limits
C	92	94	96	97	98	99	100	101	of V not
H_f	0.97	3.67	7.99	13.9	21.3	30.3	52.6	80.7	stated

7 2/32 Inches

New; asphalt; clean; 328.1 feet; screw coupling; by volume; Darcy.†

V	1	2	3	4	5	6	8	10	19.7
C	101	110	115	119	122	125	129	132	143
H_f	0.605	2.06	4.22	7.04	10.5	14.4	24.0	35.6	118

10 3/64 Inches

5 years; uncoated; clean; about 700 feet, with curves; taper; by weir; H. Smith, Jr.‡

V			4.7	5	6	8	10
C			108	108	110	113	115
H_f			8.38	9.38	13.1	22.1	33.1

11 7/32 Inches

New; asphalt; clean; 328.1 feet; screw coupling; by volume; Darcy.§

V	1.3	2	3	4	5	6	8	10	10.5
C	105	110	115	118	121	124	128	131	132
H_f	0.660	1.42	2.93	4.88	7.26	10.1	16.8	24.9	27.2

12 3/64 Inches

5 years; uncoated; clean; about 700 feet, with curves; taper; by weir; H. Smith, Jr.‡

V			4.6	5	6	8	10	10.7
C			110	111	112	113	114	114
H_f			6.58	7.72	10.9	19.0	29.2	33.2

14 7/64 Inches

5 years; uncoated; clean; about 700 feet, with curves; taper; by weir; H. Smith, Jr.‡

V			4.4	5	6	8	10	12
C			114	115	116	119	120	122
H_f			4.88	6.19	8.69	14.8	22.5	31.5

16 Inches

New; asphalt; clean; 16,416 feet, with curves; cylinder; in reservoir; Adams.¶

$$V = 4.58; \qquad C = 112; \qquad H_f = 5.00.$$

* Engineering Record, vol. 58 (1908), p. 241.
† Darcy, loc. cit., Series 9, pp. 12, 52.
‡ H. Smith, Jr., *Hydraulics*, pp. 241, 302.
§ Darcy, loc. cit., Series 10, pp. 12, 54.
¶ Transactions A. S. C. E., vol. 36 (1895), p. 25.

4. Riveted Steel or Sheet-Iron Pipes — *Continued*

30 Inches

New; asphalt and coal tar; clean; 308 miles, with curves; cylinder; by pumps.*

V	1.89	2.12
C	116	116
H_f	0.426	0.530

33 Inches

New; asphalt; clean; 34,176 feet, with curves; taper; by weir and reservoir; I. W. Smith.†

$$V = 6.12; \qquad C = 123; \qquad H_f = 3.57.$$

35 Inches

New; asphalt; clean; 39,829 feet, with curves; same as 33-inch.

$$V = 5.44; \qquad C = 127; \qquad H_f = 2.53.$$

36 Inches

New; asphalt; clean; 25,000 feet; cylinder joints; by weir; Herschel, 1892.‡

							Same, 4 years old	
V	1	2	3	4	5	5.7	**1896**	4.93
C	95	103	107	112	115	117		106
H_f	0.149	0.505	1.03	1.71	2.53	3.19		2.87

38 Inches

Built in 1893–1894; asphalt and Sabin's varnish; 46,339 feet, with curves; cylinder; in reservoir; Kuichling.§

June 9-15, 1896

V	0.490	0.612	0.758	0.888	0.965	1.06	1.13	1.19	1.27
C	95	95	110	111	97	106	111	112	110
H_f	0.0337	0.0508	0.0605	0.0810	0.125	0.126	0.131	0.142	0.166

Oct. 1896	July 6-17, 1897								
1.22	0.505	0.637	0.814	0.897	1.02	1.11	1.15	1.22	1.24
102	113	114	117	112	112	115	117	115	109
0.181	0.0254	0.0397	0.0615	0.0819	0.105	0.117	0.123	0.142	0.163

Another part of the same line, 91,641 feet.

	Oct. '95	Dec. '95	July '97	Nov. '97
V	3.27	3.23	3.25	3.15
C	117	114	116	112
H_f	0.994	1.01	0.999	0.997

* Unwin, *Hydraulics*, p. 193.
† Transactions A. S. C. E., vol. 36 (1894), p. 203.
‡ 115 *Experiments*, p. 28.
§ *Report of the Executive Board*, Rochester, N. Y., 1897, pp. 40–42.

4. Riveted Steel or Sheet-Iron Pipes — *Continued*

Another part of the same line, 45,394 feet. This piece is connected directly with Kuichling's 36-inch cast-iron pipe; gaging was simultaneous; compare them.

Age in years	1.28	1.30	3.06	3.37	3.91	4.47	5.04	5.34
V	3.88	3.91	3.81	3.71	3.72	3.71	3.77	3.76
C	109	109	106	107	106	106	108	108
H_f	1.59	1.61	1.63	1.53	1.57	1.55	1.54	1.54

The pipe was examined when it was about 4.2 years old; only slight tuberculation was found.

42 Inches — **New**; asphalt; clean; 50,965 feet, with curves; taper; by weir and reservoir; I. W. Smith.*

$$V = 3.78; \qquad C = 116; \qquad H_f = 1.21$$

42 Inches — **New**; asphalt; clean; 49,833 to 81,139 feet, with curves; taper; by Venturi meter; Herschel, 1896.†

V	1	2	3	4	5
C	101	105	107	108	109
H_f	0.111	0.417	0.905	1.57	2.40

48 Inches — **New**; asphalt; clean; 24,648 feet, with curves; taper; by Venturi meter; Herschel, 1896.‡

V	3	4	4.7
C	101	104	105
H_f	0.886	1.50	2.01

48 Inches — **New**; asphalt; clean; 24,630 to 74,396 feet, with curves; cylinder; by Venturi meter; Herschel, 1892.§

V	2	3	4	5	5.5
C	109	111	112	113	114
H_f	0.339	0.738	1.28	1.97	2.37

48 Inches — **4 years**; part of same line; Herschel, 1896; below Pompton Notch; by Venturi meter.

V	3	4	5	6
C	100	103	105	107
H_f	0.910	1.53	2.28	3.17

* Transactions A. S. C. E., vol. 36 (1894), p. 203.
† 115 *Experiments*, pp. 29, 53.
‡ 115 *Experiments*, pp. 28, 53.
§ 115 *Experiments*, pp. 27, 52.

4. Riveted Steel or Sheet-Iron Pipes — *Continued*

48 Inches

4 years; another part of same; Herschel, 1896; above Pompton Notch; by Venturi meter.

V	3	4	4.6
C	93	96	97
H_f	1.05	1.77	2.28

72½ Inches

New; asphalt; unknown, about 4400 feet, with curves; butt joint; by Venturi meter; Marx, Wing, and Hoskins.*

V	1	2	3	3.8
C	108	108	108	108
H_f	0.0563	0.227	0.512	0.824

72½ Inches

2 years; asphalt; unknown; 4427 feet, with curves; butt; same pipe as last; by Venturi meter; Marx, Wing, and Hoskins.†

V	1	2	3	4	5	5.3
C	84	92	98	102	105	106
H_f	0.0941	0.310	0.623	1.02	1.50	1.66

108 Inches

5 years; uncoated; rusty, not tuberculated; 153 feet; cylinder; by weir; Herschel, 1887.‡

V	1	2	3	4	4.5
C	117	111	108	106	105
H_f	0.0342	0.151	0.359	0.664	0.855

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5. Wooden Stave Pipes

14 Inches

New; redwood; clean; 3436 to 9912 feet, with curves; by weir and volume; Adams.§

V	1	1.5
C	107	113
H_f	0.300	0.607

18 Inches

New; yellow fir; clean; 23,310 feet, with curves; in reservoir; Adams.¶

$$V = 3.61; \qquad C = 133; \qquad H_f = 1.96$$

* Transactions A. S. C. E., vol. 40 (1897), p. 493.
† Transactions A. S. C. E., vol. 44 (1899), p. 46.
‡ 115 *Experiments*, p. 29.
§ Transactions A. S. C. E., vol. 40 (1898), p. 545.
¶ Transactions A. S. C. E., vol. 36 (1895), p. 25.

5. Wooden Stave Pipes — *Continued*

44½ Inches — **Less than 1 year;** clean; 4041 feet, with curves; by 7½-inch Haskell current meter; T. A. Noble.*

V	3.5	4	4.8
C	111	113	116
H_f	1.07	1.37	1.87

56 3/16 Inches — **Less than 1 year;** some growths (spongilla); 2679 feet, with curves; 7½ Haskell meter; T. A. Noble.*

V	2.3	3	4	4.7
C	117	121	126	129
H_f	0.343	0.541	0.888	1.17

72½ Inches — **New;** Douglas fir; 2,710 feet, with bends; by Venturi meter; Marx, Wing, and Hoskins.†

V	1.2	2	3	3.6
C	106	114	121	124
H_f	0.0851	0.204	0.408	0.557

72½ Inches — **2 years;** same pipe as last; 22,672 feet, with bends; by Venturi meter; Marx, Wing, and Hoskins.‡

V	1.2	2	3	4	5	5.3
C	119	120	121	122	122	122
H_f	0.0676	0.184	0.407	0.716	1.11	1.24

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6. Concrete Pipe

12 Inches — **30 years;** poor joints and interior; 621 feet long; color velocity.§

$$V = 1.60; \qquad C = 74.7$$

12 Inches — **4 years;** good joints and interior; 218 feet long; floats.§

$$V = 5.85; \qquad C = 95.9$$

36 Inches — **3 years;** smooth joints; 7,282 feet long; color velocity.§

$$V = 3.48; \qquad C = 117.6$$

Same, two years later.

V	5.45	7.12	7.44	8.22	8.66	9.06
C	138.3	136.5	145.2	130.1	137.2	143.8

* Transactions A. S. C. E., vol. 40 (1902), p. 143.
† Transactions A. S. C. E., vol. 40 (1897), p. 512.
‡ Transactions A. S. C. E., vol. 44 (1899), p. 48.
§ Department of Agriculture Bulletin No. 852.

6. Concrete Pipe — *Continued*

42 Inches

1 year; perfect joints; 1,336 feet long; color velocity.*

V	1.01	1.19	1.41	1.66	1.75	1.96	2.24	2.53	2.91
C	123.9	129.2	129.2	132.7	128.5	131.7	134.9	132.8	135.1

46 Inches

5 years; 9,774 feet long; color velocity.*

V	1.41	2.18	3.17	3.98	4.00	4.17	4.21
C	147	139.7	139	145	138	130	135

54 Inches

New; smooth, with perfect joints; 21,047 feet long; color velocity.†

$$V = 2.63; \qquad C = 146.9$$

60 Inches

New; smooth, with perfect joints; 34,788 feet long, including 29 bends (456° total); Venturi meter.†

$$V = 2.25; \qquad C = 140.5$$

Same; 80,898 feet long, sinuous.

$$V = 2.25; \qquad C = 146.8$$

174 Inches (14.5 feet)

New; average good concrete; 8,419 feet long; Venturi meter.*

V	1.58	2.09	2.84	2.86	3.76	3.78	4.58	4.67
C	150	155	140.5	159	159	137	135	154

216 Inches (18 feet)

4 years; exceptionally smooth, hand-rubbed throughout; 6,466 feet long; color velocity.*

V	4	8	12	16	20
C	181	178	180	183	193

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7. Corrugated Culvert Pipe

24 Inches

New; 24-inch minimum, 25-inch maximum diameter; corrugation pitch, 2½ inches; carrying sewage; weir.‡

V	1.55	1.57	1.58	1.60
S	.00155	.00156	.00132	.00143
L	2808	2808	1038	1038
C	55.7	56.3	61.5	59.6

* Department of Agriculture Bulletin No. 852.
† Engineering News-Record, May 28, 1925.
‡ Engineering News, April 17, 1913.

PROBLEMS

1. The mean velocity of flow in a new 18-inch cast-iron pipe is 6 feet per second. Determine slope of the hydraulic grade line.

2. The discharge through a 12-inch cast-iron pipe is 1500 gallons per minute. Compute the head lost in friction in 2000 feet.

3. What head is necessary to deliver 1,330,000 gallons per day through a 1-foot cast-iron pipe, 17,684 feet long? (*a*) When the pipe is new. (*b*) When the pipe is 20 years old.

4. Water is raised by a pump through an actual vertical distance of 100 feet, and delivered at the rate of 8.124 million gallons per day through a 42-inch cast-iron pipe 5 miles long. The work of the pump is equivalent to raising the water through what height?

5. The elevation of the water surface in a storage reservoir, A, is 78 feet. In a distribution reservoir, B, it is 60 feet. Distance between them is 3 miles. They are connected by a 2.5-foot new cast-iron pipe. What will be the discharge in cubic feet per second? Compute the elevation of water in a piezometer tube 10,000 feet distant from A.

6. Two reservoirs are connected by a 15-inch pipe 4 miles long. In each reservoir the center of the pipe is 20 feet below the water surface. For the first $\frac{1}{4}$ of distance, the pipe is laid on a grade of 1 in 660; the next $\frac{1}{4}$, 1 in 330; for the last $\frac{1}{2}$, level. Determine the discharge in gallons per minute. Compute height of piezometer reading at each change of grade and reduce to pounds per square inch.

7. A tank of 15,000 gallons capacity is 30 feet above the street. The pressure on the street main is 30 pounds per square inch. Between the main and the tank is a pipe 1000 feet long. What diameter must it have to fill the tank in 60 minutes? With this diameter what head would be needed to fill tank in 10 minutes?

8. A riveted steel pipe 60 inches in diameter is 25,000 feet long. At 5000 feet from the inlet 1,000,000 gallons per day are taken off; at 10,000 feet, 2,000,000 gallons per day; at 15,000 feet, 4,000,000 gallons per day; at 20,000 feet, 3,000,000 gallons per day; and from the end 6,000,000 gallons per day. (*a*) Compute and plot the hydraulic grade line. (*b*) Compute also for a wooden stave pipe.

9. Water is raised by a pump through an actual vertical distance of 100 feet, and is to be delivered at the rate of 8,124,000 gallons per day through: (*a*) a 4-foot cast-iron pipe; or (*b*) a 2-foot cast-iron pipe; in each case 10 miles long. The work of the pump is equivalent to lifting the water to what height in each case?

10. What diameter of cast-iron pipe will be required to discharge 7,000,000 gallons per day under a head of 100 feet, through a length of 52,800 feet, making a reasonable allowance for deterioration in use?

11. What will be the velocity through a riveted steel pipe 40,000 feet long, 8 feet in diameter, under a head of 50 feet? (*a*) New. (*b*) 15 years old.

12. What pressure in pounds per square inch would be required at the street end of a house-service pipe of galvanized welded steel pipe $1\frac{1}{2}$ inches in

diameter, to deliver 8 gallons per minute at a faucet, 400 feet distant? Assume five elbows.

13. What is the loss of head at entrance for an 18-inch cast-iron pipe having the inlet flush with the wall of a dam, if discharging 5000 gallons per minute?

14. Determine the loss of head in a bend of 45 degrees with a radius of 50 feet, in a 42-inch pipe discharging 10,000 gallons per minute.

15. The slope of a hydraulic grade line is 0.001. Compute the diameter of a galvanized steel pipe to deliver 470 cubic feet per minute.

16. 7480 gallons per minute are pumped through a pipe line composed of 3000 feet of 36-inch, 1000 feet of 30-inch, and 2000 feet of 24-inch cast-iron pipes. Compute the friction head in each length.

17. Two reservoirs, having a difference in elevation of 25 feet, are connected by a 4-foot pipe 10,000 feet long. At 1000 feet from the upper reservoir, a pipe 2 feet in diameter and 4900 feet long leads to another reservoir, at an elevation 13 feet lower than the upper reservoir. What is the discharge into each reservoir and what is the piezometer reading at the point of divergence?

18. A distribution system of cast-iron pipes is laid out in the form of a gridiron, measuring 300 feet each way to points of intersection.

The pipes are of two sizes, being 8-inch in one direction and 6-inch in the other, which is at right angles to that first mentioned. These pipes are connected throughout and there are no dead ends.

Assuming, as in the case of a fire, that it is required to deliver 1500 gallons per minute under a hydrant pressure of 70 pounds per square inch at point B, whose coordinates are 1800 feet by 1800 feet from the point of supply A, determine the necessary pressure at the point A, if A and B are at the same elevation.

NOTE. A most exhaustive study by the late Hiram F. Mills upon flow in pipes has recently appeared in the PROCEEDINGS OF THE AMERICAN ACADEMY OF ARTS AND SCIENCES (vol. 15, No. 2, 1924). This is well worth careful study.

CHAPTER XVI

THE FLOW OF WATER IN OPEN CHANNELS

316. Open Channels. Open channels are natural or artificial waterways of any form in which the flowing water has its upper surface exposed to the air. Brooks, rivers, canals, troughs, and also, if only partly filled, covered conduits or pipes are open channels.

317. Uniform Flow. Uniform, steady flow in open channels will exist when the following conditions occur simultaneously: if the volume of flow is constant for successive equal intervals of time, if the dimensions and outlines of the channel are constant from place to place, if the lining is of uniform material and smoothness, and if the slope of the bottom is constant. Then in successive sections the cross-sectional areas of the stream and the mean velocity will be constant, and the slope of the water surface will be parallel to the slope of the bottom; and the slope S will be

$$S = \frac{h_f}{L},$$

where h_f denotes the loss of head in friction, which in this case is the difference in elevation of the surface of the water at two cross sections of a stream separated by a distance L.

The simultaneous occurrence of all conditions requisite for uniform steady flow is in reality rare. A change from a steady flow of a given volume to that of a different volume in the same channel will alter (perhaps materially) the slope of the water surface, which will no longer be parallel to the slope of the bottom. Hence the areas of, and the mean velocity in successive areas of, the stream will no longer be constant. Apparently slight variations in the character of the lining, the shape of the channel, or the slope of the bottom may produce marked changes in the resistance to flow, and the conditions which depend upon it.

More or less turbulence almost always exists in the flow, except, perhaps, in the case of large, very slow-moving rivers. As the velocity of flow becomes higher, the flow is always turbulent.

As the surface of the water in an open channel is free to rise or fall, the stream, in adapting itself to variations or changes, produces a great variety of combinations of the factors causing flow. To correlate experimental results is often intricate and difficult. Except for channels of regular form, and very uniform linings, the formulation and reapplication of experimental results are not always trustworthy. Empirical formulas must be used, however, together with direct recourse in important cases to experimental data. The accuracy of the result will depend largely upon the closeness with which a case under consideration fits the available experimental data.

318. The Chezy Formula. If it be assumed that the loss of head varies as the square of the mean velocity, Chezy's formula, or some equivalent expression, is most commonly used in calculating discharge, namely:

$$V = C(RS)^{1/2}, \tag{1}$$

where V is mean velocity of flow in a stream cross section, in feet per second; R is mean hydraulic radius, in feet; S is the slope of water surface; and C is an empirical coefficient depending upon V, upon R, upon the degree of roughness of the lining of the channel, and to some slight extent upon the slope.

319. Exponential Formulas. The resistance to flow has repeatedly been proved to be not always proportional to the square of the mean velocity, but to some other power of V, varying from 1.75 to 2.10 or higher; nor can the relation between velocity, slope, and the mean hydraulic radius be represented as a definite general law.

By classifying channels according to the roughness of their linings, it may be shown that if accurate experimental data covering the required conditions are available, the following expression, or some equivalent form, will give the true relation between S, V, and R for a particular channel:

$$S = \kappa \frac{V^n}{R^m}, \tag{2}$$

where κ denotes a coefficient of roughness; n is the exponent of V, and m is the exponent of R. The values of κ, m, and n will depend upon the roughness of the lining and to a limited extent upon the form of the channel.

Such a formula requires a separate determination of κ, m, and n for many classes of channels, and will be most accurate if based upon a large number of observations of nearly equal precision for each class; and means, in effect, a separate formula for each class. Such a procedure is logical and may ultimately be the prevailing practice; but it will require a great increase in existing data. At the present time the formula of this type most generally used is the Hazen-Williams formula given in § 288.

320. Coefficient C for the Chezy Formula. The coefficient C is a variable, usually assumed to depend on the degree of roughness or smoothness of the channel lining, the mean hydraulic radius, the mean velocity of flow, and the slope. How much C depends upon, or is independent of, the slope, is an open question. The value of C may be determined directly from experiments, or by the use of an empirical formula.

321. Experimental Determination of C. An experimental determination of C comprises a measurement of the slope, the discharge, the dimensions of a stream and of the channel in which it is contained. The following computation gives C for a given flow:

$$\frac{Q}{A} = V; \qquad \frac{\text{Area}}{\text{Wetted Perimeter}} = R; \qquad \frac{h_f}{L} = S.$$

Then since

$$(3) \qquad V = C\left(R\frac{h_f}{L}\right)^{1/2}, \qquad C = V\left(\frac{L}{Rh_f}\right)^{1/2}.$$

The results of observations on a series of steady flows, if made with accuracy, and plotted on coordinate paper, give a curve or curves which will be a graphic representation of the law of flow in the experimental channel. From such a curve values of C may be taken within the range of the experiments, with as high a degree of precision as the experiments themselves warrant.

Such experimental results are the most trustworthy guides for obtaining accurate coefficients for other similar channels; but very close similarity is infrequent. To be of use, the results of experiments must, in general, be formulated. The formula most widely used is Kutter's; but for smooth channels of small hydraulic radius Bazin's is good.

322. Kutter's Formula for Calculating *C*. E. Ganguillet and W. R. Kutter, two Swiss engineers, published,[1] as the result of their studies of all the then (1869) available gagings of the flow of water in channels, a general formula for computing the values of C, intended to be applicable to any kind of channel closed or open. The formula proposed by them is:

$$C = \frac{41.66 + \frac{1.811}{n} + \frac{0.00281}{S}}{1 + \left(41.66 + \frac{0.00281}{S}\right)\frac{n}{(R)^{1/2}}}, \tag{4}$$

where S is the slope; R is the mean hydraulic radius; and n is a coefficient of roughness determined by experiment, or estimated. The more common values given to the coefficients of roughness n are as follows:

n	Character of the Lining of a Channel
.009	very smooth boards carefully joined; very smooth glass or enameled tubes.
.010	planed boards; neat cement plaster.
.011	cement mortar $\frac{1}{3}$ sand.
.012	common boards unplaned, well joined; new best quality brick masonry.
.013	new ashlar; ordinary, good brick masonry; well-laid vitrified sewer pipes.
.015	dirty cement, brick and vitrified pipe sewers; or poorly laid vitrified pipes; canvas lining on frames.
.017	good rubble masonry and rough ashlar with cement joints; rough brick masonry.
.02	clean canals in firm gravel; rough rubble (dry or mortar).
.025	natural or artificial channels in earth of regular form and free from weeds or detritus.
.03–.04	channels in earth, with detritus or aquatic plants, varying from good to bad condition.
.04–.05	natural streams or irregular form with flood flow.

A much more complete table of values of "n" has been compiled by Robert E. Horton.[2] This much more nearly fits present day requirements than do Kutter's original values.

[1] *Flow of Water in Rivers and Other Channels*, translated by Hering and Trautwine. Wiley & Sons.

[2] Engineering News, Feb. 24 and May 4, 1916.

Surface	Perfect	Good	Fair	Bad
Uncoated C. I. pipe	.012	.013	.014	.015
Coated C. I. pipe	.011	**.012***	**.013***	
Commercial W. I. pipe, black	.012	.013	.014	.015
Commercial W. I. pipe, galv.	.013	.014	.015	.017
Smooth brass and glass pipe	.009	.010	.011	.013
Smooth lockbar and welded "OD" pipe	.010	**.011***	**.013***	
Riveted and spiral steel pipe	.013	**.015***	**.017***	
Vitrified sewer pipe	.010–.011	**.013***	.015	.017
Glazed brick work	.011	.012	**.013***	.015
Brick in cement mortar: brick sewers	.012	.013	**.015***	.017
Neat cement surfaces	.010	.011	.012	.013
Cement mortar surfaces	.011	.012	**.013***	.015
Concrete pipe	.012	.013	.015	.016
Wood stave pipe	.010	.011	.012	.013
Plank Flumes:				
Planed	.010	**.012***	.013	.014
Unplaned	.011	**.013***	.014	.015
With battens	.012	**.015***	.016	
Concrete lined channels	.012	**.014***	**.016***	.018
Cement rubble surface	.017	.020	.025	.030
Dry rubble surface	.025	.030	.033	.035
Dressed ashlar surface	.013	.014	.015	.017
Semicircular metal flumes, smooth	.011	.012	.013	.015
Semicircular metal flumes, corrugated	.0225	.025	.0275	.030
Canals and Ditches:				
Earth, straight and uniform	.017	.020	**.0225***	.025
Rock cuts, smooth and uniform	.025	.030	**.033***	.035
Rock cuts, jagged and irregular	.035	.040	.045	
Winding sluggish canals	.0225	.025	.0275	.030
Dredged earth channels	.025	**.0275***	.030	.033
Canals with rough stony beds weeds on earth banks	.025	.030	**.035***	.040
Earth bottom, rubble sides	.028	**.030***	**.033***	.035
Natural Stream Channels:				
(1) Clean, straight bank, full stage, no rifts or deep pools	.025	.0275	.030	.033
(2) Same as (1), but some weeds and stones	.030	.033	.035	.040
(3) Winding, some pools and shoals, clean	.033	.035	.040	.045
(4) Same as (3), lower stages, more ineffective slope and sections	.040	.045	.050	.055
(5) Same as (3), some weeds and stones	.035	.040	.045	.050
(6) Same as (4), stony sections	.045	.050	.055	.060
(7) Sluggish river reaches, rather weedy or with very deep pools	.050	.060	.070	.080
(8) Very weedy reaches	.075	.100	.125	.150

* Values commonly used in design.

323. Manning's Formula. A formula published by Robert Manning in 1890 gives results quite as accurate as Kutter's for small and medium sized channels, and is much less complicated to use.

$$V = \frac{1.486}{n} R^{0.67} S^{0.5}, \tag{5}$$

or for C,

$$C = \frac{1.486}{n} R^{1/6}. \tag{6}$$

The value of n is not quite the same as in Kutter's formula, but over the range met with in common practice it is so nearly the same that Kutter's n is used.

324. Bazin's Formula for Calculating C. Bazin proposed [1] a formula for computing the flow in open channels, which in distinction to an earlier one is called Bazin's "new formula." The formula itself is for computing C for use in the Chezy formula.

$$V = \frac{157.6}{1 + \frac{m}{R^{1/2}}} (RS)^{1/2}, \tag{7}$$

$$C = \frac{157.6}{1 + \frac{m}{R^{1/2}}} = \frac{157.6\, R^{1/2}}{(R)^{1/2} + m}, \tag{8}$$

where m is the coefficient of roughness; R is the mean hydraulic radius; and S is the slope. This formula eliminates the variation

Values of m

Class I	$m = 0.109$.	Very smooth surfaces; neat cement; planed wood.
Class II	$m = 0.290$.	Smooth surfaces; planks, bricks, ashlar.
Class III a	$m = 0.833$.	Rough surfaces; rubble masonry.
b	$m = 1.54$.	Canals with mixed linings; very regular earth or paved with stones.
Class IV	$m = 2.35$.	Earth canals in ordinary condition.
Class V	$m = 3.17$.	Earth canals in bad condition.

[1] Annales des Ponts et Chaussées, Mém. et Doc., 1897, 4e trimestre, p. 20 *et seq.*

in C with changes in S (in which it differs from the Kutter formula), and makes the variations in C depend only upon variations in the mean hydraulic radius, and in the coefficient of roughness. This formula represents the results of a very careful study of existing data and is extremely valuable in designing relatively small channels and those with very smooth linings.

King gives a more complete list of values of m, based largely on Horton's values of n in the Kutter formula.[1]

	Best	Good	Fair	Bad
Vitrified sewer pipe	0.10	0.40	0.60	0.90
Common clay drain tile	0.20	0.30	0.50	0.90
Glazed brickwork	0.10	0.25	0.40	0.60
Brick in cement mortar	0.25	0.40	0.60	0.90
Neat cement surfaces	0.00	0.10	0.25	0.40
Cement-mortar surfaces	0.10	0.20	0.40	0.60
Concrete pipe	0.25	0.40	0.60	0.75
Plank flumes, planed	0.00	0.25	0.40	0.50
Plank flumes, unplaned	0.10	0.40	0.50	0.60
Plank flumes, with battens	0.25	0.60	0.75	1.00
Concrete-lined channels	0.25	0.50	0.75	1.00
Rubble masonry	0.90	1.25	1.90	2.50
Dry rubble	1.90	2.50	2.90	3.15
Ashlar	0.40	0.50	0.65	0.90
Smooth metal flumes	0.10	0.25	0.40	0.60
Corrugated metal flumes	1.60	1.90	2.20	2.50
Earth canals in good condition	0.90	1.25	1.60	1.90
Earth canals in weeds, rocks, etc.	1.90	2.50	3.15	3.80
Canals excavated in rock	2.50	3.15	3.70	4.20
Natural streams in good condition	1.90	2.50	3.15	3.80
Natural streams with weeds, rocks, etc.	3.15	4.40	6.30	8.80

325. Comparison of Formulas. While there are a great many other formulas for determining flow in open channels, Kutter's, Manning's, and Bazin's are the only ones we need concern ourselves with at present. The essential point of difference found in Kutter's formula is the slope correction. It is generally admitted that the slope has some effect upon the value of C. This is, however, important only in the case of large rivers with very flat slopes. In most cases the formula will give essentially the same results if the terms involving the slope are left out. In general, Kutter's formula gives excellent results for almost any case, and gives results nearer the truth than the other two for

[1] H. W. King, *Handbook of Hydraulics*, McGraw-Hill, 1918.

large rivers with flat slopes. Its chief disadvantage is its complicated form. The Miami Conservancy District considers Kutter's formula, while not ideal, to be the best equation available at the present time.[1]

Manning's formula gives results practically the same as Kutter's for channels with a hydraulic radius of from one to ten, except for the flattest slopes.[2] For hydraulic radii higher than ten it will give velocities somewhat higher than Kutter's for the same value of n. The agreement between the two formulas over the usual working range is very close, and the much simpler form of Manning's formula has much to recommend it.

Bazin's formula is also much simpler than Kutter's, and has no slope correction. It is based upon practically the same experiments as Kutter's formula, and will give nearly the same results except when the slope is very flat.

To show the agreement of these three formulas, the value of C has been worked out by each for two examples, one a small artificial channel, the other a natural river, for which the value of C had been determined by experiment. Values of n and m were chosen after a careful examination of the description of the channel and of the various values of n and m found in other experiments. The experimental values for the two channels were, however, not known until after the results by formula had been obtained. All the procedure of using the formulas was as it would be for an unknown channel whose superficial characteristics only were known.

A Rectangular Cement Plastered Channel

4.5′ wide, 3.1′ deep, slope = 0.00126. Say $n = 0.013$, then the value of C is:

By Kutter	122.1
By Kutter, slope correction left out	122.7
By Manning	119.6
By Bazin, m taken as 0.20	134.1
By Bazin, m taken as 0.30	123.8
Actual by experiment	123.3

[1] Miami Conservancy District Technical Reports, Part IV.
[2] H. W. King, *Handbook of Hydraulics*, McGraw-Hill, 1918.

A Natural River (Miami at Tadmor)

Stream bed composed of clay, sand, and gravel, no rocks or boulders; edges of channel partly obstructed by bushes and trees.

202′ wide, 5.8′ R, slope = 0.00026. Say $n = 0.035$, then the value of C is:

By Kutter	59.3
By Kutter, slope correction left out	57.8
By Manning	56.9
By Bazin, m taken as 3.15	68.0
Actual by experiment	66.5

If n had been taken as 0.030

By Kutter	68.2
By Kutter, slope correction left out	63.5
By Manning	66.3

Thus it will be seen that the results obtained by any one of the formulas will largely depend upon the choice of n and m. Because of this Kutter's and Manning's formulas have a decided advantage over Bazin's, in that the values of n have been more completely determined and tabulated. When correctly used any one of these three formulas will give good results. While Bazin's formula has many advantages, it would appear that the better knowledge of the value of n in Kutter's and Manning's formulas more than overbalances them, and that these two are to be preferred, Manning's for general use, and Kutter's for special cases involving large rivers and flat slopes. None of the three are recommended for use with pipes. In many cases, since a value of n or m must be picked anyway, it is simpler and sufficiently accurate, particularly for rough approximations and preliminary design, to pick a value of C directly, as is customarily done in the case of pipes, without going through the mechanism of solving a formula which itself is based upon an assumed value. For more accurate work, however, the use of one of these three formulas is to be recommended.

326. Experimental Results. Table XXVIII at the end of this chapter contains some experimental results on open channels. This list contains examples of flow in pipes running partly full, in ditches and small canals, and in natural rivers. The table is

not intended to cover all cases, but it does give an idea of the approximate hydraulic characteristics that may be expected for various types of open channels.

327. Forms of Channel Cross Sections. The most efficient form of channel cross section, from a hydraulic standpoint, assuming the Chezy formula to represent the law of flow, is that which with a given slope and a given area will have the maximum discharge, or what is equivalent, the maximum velocity for a given area. This condition occurs when the ratio of the cross-sectional area to the wetted perimeter is a maximum, which means that for a given area the hydraulic radius is a maximum.

A semicircular section most thoroughly fulfills the conditions for highest hydraulic efficiency, because for a given area the wetted perimeter is less than for any other form. It is probable also that for similar linings and with other conditions equal,

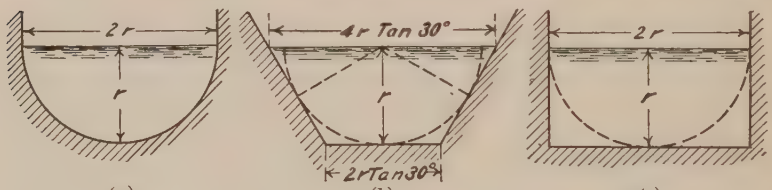


FIG. 91

the actual resistance to flow is less than for any other shape. The mean hydraulic radius of a semicircular section is one half the depth. See Fig. 91 *a*.

For (*a*) $A = \frac{\pi r^2}{2}$; $w.p. = \pi r$; $R = \frac{\pi r^2}{2\,\pi r} = \frac{r}{2}$;

Of regular polygons, one half a regular hexagon or one half a square are the best of their particular shapes for hydraulic efficiency, each having a mean hydraulic radius equal to one half the depth. See Figs. 91 *b* and 91 *c*.

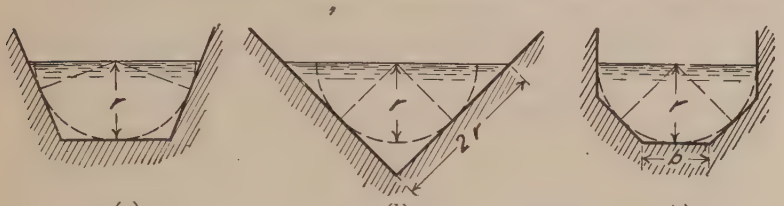
For (*b*) $A = 3\,r^2 \tan 30°$; $w.p. = 6\,r \tan 30°$; $R = \frac{3\,r^2 \tan 30°}{6\,r \tan 30°} = \frac{r}{2}$;

For (*c*) $A = 2r^2$; $w.p. = 4\,r$; $R = \frac{2\,r^2}{4\,r} = \frac{r}{2}$.

For trapezoidal sections having a level bottom line, and two inclined side walls, but at some angle of inclination other than

60°, like Fig. 92 *a*, the best form will be that in which all the faces are tangent to an inscribed semicircle with its center in the water surface. R can be proved to be equal in all such sections to $r/2$.

Any section formed as the lower half of a regular polygon of an even number of sides, and each of its faces tangent to a semicircle with its center in the water surface, will have its mean



(*a*) (*b*) (*c*)

FIG. 92

hydraulic radius equal to one half the radius of the inscribed circle. (See Figs. 92 *b* and 92 *c*.)

328. Factors Determining the Shapes of Channels. The shapes of channels are not infrequently fixed by many important considerations other than to secure best hydraulic efficiency.

The question of cost of construction and maintenance, the choice of materials available for supporting or lining the channel, the limits to the slope of the bottom or its depth imposed by the character of the material or its surroundings, the frequently necessary provision for cleaning or inspecting, the securing of a good form of section in a channel both for large and small flows, the prevention of scouring or silting, and if in rock whether to use a large rough section or a smaller lined one; these and many other considerations give rise to a great choice of cross sections. The best form for many channels can be settled only by a thorough study of the hydraulic efficiency in connection with cost and permanency.

329. The Effect of Ice Covering on the Flow in an Open Channel. In northern climates, in the winter time, natural streams as well as power canals may be covered more or less continuously with ice which rests on the water surface. In such cases the ice adds to the wetted perimeter of the stream and increases the resistance to flow. The effect of ice covering may best be deter-

mined by gagings with a current meter when the ice is in place; if gagings are not feasible and computations of discharge by empirical formulas are necessary, the wetted perimeter should be increased by the total top width of the water surface. In addition to this, the area of the section is usually diminished by the thickness of the ice. In the design of channels for such conditions, the reduction should be provided for in fixing the dimensions.

330. The Effect of Minute Growths and Deposits. This question has been given considerable importance, due to the measurements in the Sudbury and Wachusett aqueducts, after being in commission for a number of years, when the carrying capacity of these aqueducts was found to be about 10 per cent less than when first used. This loss can be considerably reduced by cleaning, and this cleaning will probably be a part of the maintenance of every big aqueduct in the future.

331. The Effect of Anchor Ice and Frazil Ice. Anchor ice forms on the bottom of moderately shallow streams. It forms by radiation, usually on cold, clear nights, rising to the surface during the daytime, after the sun has come out. It does not form in channels with an ice cover, nor under low lying bridges.

Frazil ice is composed of small spicular crystals. When carried along in moving water it has the appearance of a mass of water-logged snow. Frazil or slush ice forms on the surface in streams where the velocity is too great for surface ice to form, and will settle and build upon anchor ice or upon the bottom, or be carried along by the current just about in suspension until it is caught by some obstruction and there builds up. The effect of anchor ice and frazil ice is to diminish the stream area, roughen the bed, and greatly retard if not stop all flow. To provide for this, ample allowance should be made by enlarging the channel to such an extent that the surface ice will form readily and protect the water.

332. Distribution of Velocities in the Cross Section of Open Channels. In straight channels without disturbing influences, it may be stated that in general the velocity is least at the perimeter, and increases toward the center from the sides, and from the bottom toward the top. The maximum velocity is usually found somewhere between mid-depth and the surface, about in the middle of the channel.

In channels with curved alignment, or so near curves as to feel their effects, the distribution of velocities as compared with that of straight channels is more or less modified; for example, the velocity may be a maximum at or near the outside of the curve. The discharge from submerged wheels, orifices, gates, or entering streams, either above or below a stream section, may cause an entire redistribution of velocities at the point. A great many measurements based upon the determination of a few velocities have been vitiated by the effect of such disturbances.

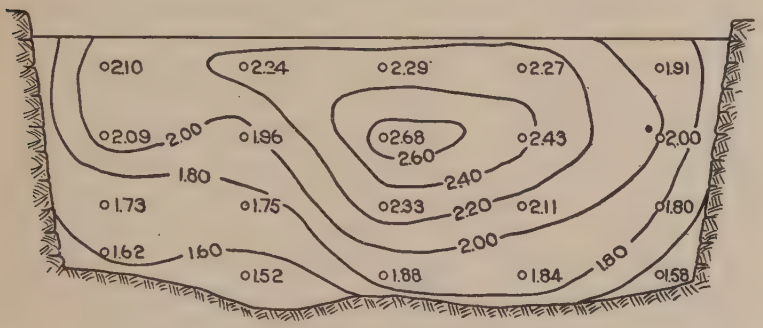


FIG. 93. — VELOCITY CONTOURS IN A CANAL

333. Diagram of Equal Velocities. The relation of velocities in any stream cross section may be shown by plotting on cross-section paper the outline of the section, and the velocities at the points where actual observations have been made. On this plot, by a process like that of interpolating contours on topographic maps, a series of lines may be drawn, each one representing approximately the locus of all points having the same velocity. Such lines may be called contours of equal velocity, their area determined by planimeter, and from these areas the discharge by ordinary rules of mensuration.

334. Vertical Velocity Curve. A vertical velocity curve (see Chapter XIV) is a curve drawn through plotted points of actual observation in a vertical plane parallel to the axis of flow, with proportional depths as ordinates, and velocities as abscissas.

335. Horizontal Velocity Curve. A horizontal velocity curve is a curve drawn through points of actual observations in a horizontal plane parallel to the axis of flow, with distances from one side as ordinates, and velocities as abscissas.

336. Use of Velocity Curves. From a study of such curves, it is possible to find some fairly definite relations between mean velocity and the velocity at given points in any particular channel, and for a series of steady flows. Although a thorough study of experimental results has yielded formulated relations between velocities at given points and mean velocities, presumably of general application, such formulas should be used with extreme reluctance, and only where actual measurements cannot be made by current meter or other apparatus. One actual traverse will establish this ratio better than it can be calculated in advance.

337. Stream Gaging. A rating curve may be constructed for a given river by which the flow may be determined when the gage height is known (Fig. 94). Such a curve is obtained by gaging the flow of the stream at several various stages, and observing the gage height for each stage. The gage need not be referenced to any definite datum as long as the gage heights are always read from the same gage. The gage heights are plotted as ordinates against the measured flows as abscissas, and a curve is then drawn through these points. Sometimes the points plot so irregularly that the curve is not clearly defined, but generally from six to a dozen observations will be sufficient to define the curve. Such a curve should be checked occasionally from year to year, not only to determine that no change has occurred, but also further to improve its accuracy.

The choice of a satisfactory gaging station is often difficult. The relation between gage height and discharge is determined by what is known as the control. This varies from a dam at some point downstream to a sand bar or some other such irregularity in the stream bed. In effect it is a weir. When the control is a dam and its dimensions are known, a rating curve can often be constructed by applying a suitable coefficient to the formula for weirs with an irregular crest, but for natural controls such as falls and rapids the rating curve can be obtained only by actual measurements.

It is desirable to have the gaging station as near the control as possible, preferably, when possible, in the pond formed by the control. Such a pond is not always formed; in the case of natural controls it seldom is, but the water directly above the control usually moves at a lower velocity. The actual stream gaging

measurements need not be made at the gaging station, they may be made either up or downstream over a considerable distance, provided no tributaries come into the stream in that distance.

If a tributary comes into the main stream directly above the control, very deceptive results may be obtained. A flood on the tributary may cause a considerable rise in the pond above the control and indicate a higher flow on the main stream than really occurs. A gaging station of this sort should always be avoided.

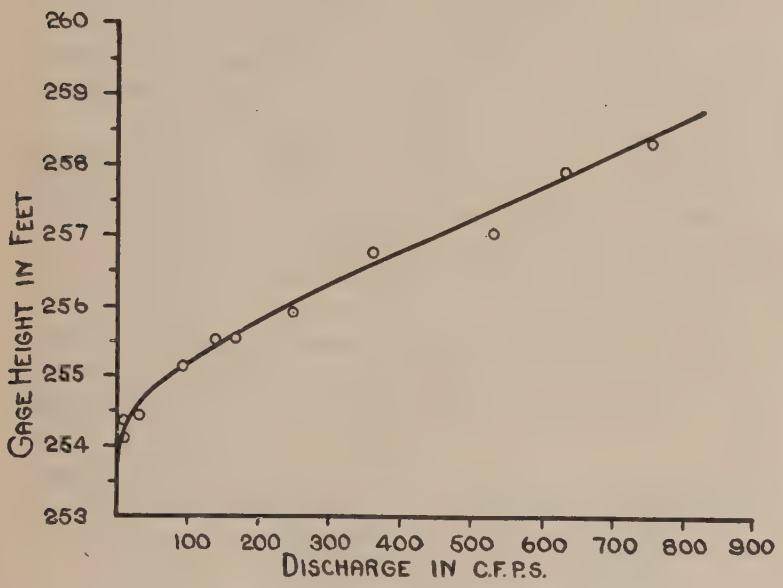


FIG. 94. — RIVER RATING CURVE

A dam with a crest at a definite elevation is a permanent control provided that all the water passes over the dam and none is passed around for water power purposes. If the crest of the dam is raised by flashboards, the control is changed. If part or all of the flow of the river is passed around the dam for power purposes and the amount used by the water wheels is known, the total flow may be obtained by adding this quantity to what is shown by the rating curve as going over the dam.

A temporary change in control may occur, particularly in winter when ice forms in such ways as to impede the flow. Ice jams often form in the spring when the river breaks up. In

rivers where lumbering operations are carried on, log jams often cause troubles. All these changes are temporary in nature, and when they have been removed, the former control will return.

A permanent change in control may occur naturally, through gradual erosion or through the deposit of silt, or it may be caused by the construction of a bridge, the building of levees or training walls, the dredging of the stream bed, or other such causes. When such a change occurs a new rating curve must be determined.

The effect of ice in winter often complicates the securing of continuous stream-flow records. An ice cover will cause the water to back up and record a gage height greater than the true height. This can be corrected for but the method is too involved to describe here.[1]

A hydrograph is a curve in which the stream flow is plotted as ordinates against time as abscissas. They are usually drawn on a daily or a monthly basis, and the flow is plotted either as the total flow, or, more usually, as the flow per square mile of drainage area.

Hydrographs show in graphic form the behavior of a river, and they are of great value in studying the characteristics of a river and its probable maximum and minimum flows. Stream-flow records should cover as long a period of time as possible and a study of a stream based on the records of a single year is apt to lead to very erroneous conclusions. Hydrographs covering a ten-year period will generally contain enough wet and dry years to give a good indication of the normal behavior of the river.

338. Variable Flow in Open Channels. If the cross-sectional area of an open channel changes from place to place, instead of being uniform, the problem of computing the flow will be modified, and may become, according to given conditions, complicated, more or less uncertain, or very difficult of solution.

339. Steady Non-Uniform Flow in Artificial Channels. Artificial channels, if not uniform, are, for the most part, either combinations of independent channels each of different area but within themselves uniform, and connected by an enlarging or reducing channel of relatively short length; or two or more lengths of the same uniform channel connected by bends.

[1] See Hoyt and Grover, *River Discharge*, 4th edition, Wiley, 1917.

340. The Loss of Head in Non-Uniform Artificial Channels may be computed as the sum of the friction heads in each of the parts which are straight and uniform within themselves, and of the losses of head due to enlargements, contractions, bends, or any other irregularity between the uniform parts which produce eddies or other disturbances.

341. Enlargements or Contractions. The losses of head caused by enlargements or contractions may be roughly estimated by methods similar to those indicated for pipes under pressure, or the losses may be provided for in assuming a sufficiently low value of C.

342. Bends. In open channels no satisfactory solution for the losses of head caused by bends can be offered. The loss is frequently assumed to be $0.5\ V^2/(2\ g)$.

343. The Hydraulic Jump. For water flowing in an open channel at any given velocity, there is a so-called ***critical depth.*** This occurs when the velocity head equals one-half the depth, that is, when $V^2 = gD$.

When water flows in a channel at a depth lower than the critical depth, it may suddenly change and flow at a stage of greater depth and lower velocity. This sudden change from a lower to a higher stage produces the so-called ***hydraulic jump.***

Let *abge*, Fig. 95, represent a mass of water moving through a hydraulic jump. In a short interval of time it is supposed to move to the position *cdfh*. The hydraulic jump has the following characteristics.

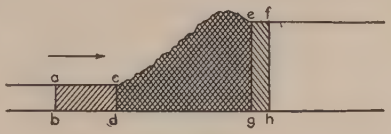


Fig. 95

The water entering at *ab* has a nearly uniform steady high velocity and is transparent.

The water leaving at *fh* has a fairly uniform but relatively low velocity and is transparent.

The water surface between *c* and *e* rises rapidly and is much disturbed by spraying and spattering.

The mass of water in passing from the high velocity to the low velocity must lose momentum. To cause such a loss of momentum a force is necessary. This is supplied by the unbalanced force of the static pressure of the greater depth of water below the jump.

Let us use the following notation:

D_1 = the depth of the stream before the jump (feet),

V_1 = the velocity of the stream before the jump (feet per second),

D_2 = the depth of the stream after the jump (feet),

V_2 = the velocity of the stream after the jump (feet per second),

Q = the quantity flowing (cubic feet per second).

Then we shall have

Mass of water flowing per sec. $= \frac{wQ}{g}$,

Change of velocity $= V_1 - V_2$,

Change of momentum per second $=$

$$\frac{wQ}{g}(V_1 - V_2) = \frac{wQ}{g}\left(V_1 - V_1\frac{D_1}{D_2}\right) = \frac{wQV_1}{gD_2}(D_2 - D_1),$$

Static pressure on $ab = \frac{wD_1^2}{2}$,

Static pressure on $fh = \frac{wD_2^2}{2}$,

Therefore

$$\frac{wQV_1}{gD_2}(D_2 - D_1) = \frac{w}{2}(D_2^2 - D_1^2).$$

Dividing both members by $w(D_2 - D_1)$

$$\frac{QV_1}{gD_2} = \frac{D_1 + D_2}{2},$$

from which we may obtain

$$D_2^2 + D_2D_1 = \frac{2\,QV_1}{g},$$

or, after substituting V_1D_1 for Q, and solving the quadratic equation,

$$D_2 = -\frac{D_1}{2} \pm \sqrt{\frac{2\,V_1^2D_1}{g} + \frac{D_1^2}{4}}.$$

This is known as *Unwin's momentum equation*. It appears to have been first worked out by Belanger in 1838. The most surprising point about this formula is that, although it is deduced

by pure mathematics and contains no modifying coefficients, the results obtained by its use check the observed heights after the jump with an almost amazing agreement, particularly when the ratio D_2/D_1 is not great.

The point of occurrence of the jump is more difficult of determination. When the channel is uniform, it is almost impossible to predict the point, but if, the bottom of the channel is roughened, or, if a small obstruction is placed there, the point of jump may be easily controlled.

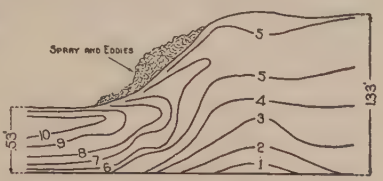


Fig. 96

Figure 96 shows lines of equal velocity observed in a hydraulic jump in a uniform channel.

$$D_1 = 0.53 \text{ ft.}, \quad D_2 = 1.33 \text{ ft.}, \qquad V_1 = 8.71 \text{ ft./sec.}$$

The value of D_2 obtained by the use of the momentum formula is 1.34 ft..

Note. The greater part of the above has been taken, either directly or slightly modified, from the technical reports of the Miami Conservancy District, Part III, *Theory of the Hydraulic Jump*, Sherman M. Woodward, and *The Hydraulic Jump*, Ross M. Riegel and John C. Beebe. Valuable information is also given in Transactions A. S. C. E., 1916, *The hydraulic jump*, Karl R. Kennison, and Engineering News, Nov. 25, 1920, *The hydraulic jump*, Julian Hinds. This last treats the practical application of the jump to the design of non-uniform channels.

344. The Backwater Curve. Steady Non-Uniform Flow in Natural Channels. Natural channels for the most part comprise a succession of varying cross sections, never wholly uniform, but more or less approximating uniformity. Coefficients derived from observations on uniform flow do not exactly fit variable flow, nor does a coefficient for one variable flow fit another; and though estimates must be made, they may be far afield unless some actual measurements are available for the stream under consideration. Often the problems concerning the carrying capacity of natural streams have to do with extreme conditions, which may readily be estimated, but with a vague knowledge of just how accurate the results may be. The following example will illustrate a problem common to river hydraulics, and shows a method by which this problem and similar problems may be solved.

345. An Example. *A dam was proposed to create a waterfall* for a power station at a certain point on a given river. Before constructing the dam certain questions concerning the effect on the river of building the dam had to be answered; among them (*a*) *how high would the water rise at the dam;* and (*b*) *how high at a point about three miles upstream* where there is an existing water-power plant with the running of which interference was not legally permissible.

The first question (*a*) could be answered by computing, by suitable weir formulas, the head of water on the crest of the dam necessary to discharge the maximum known or predictable freshet.

The second question (*b*) could be answered only by measuring actual cross sections, and computing and plotting the hydraulic grade line of the river as it probably would be after the dam was built.

The steps in this process are about as follows:

The location and elevation of all high-water marks, especially those of the greatest flood flow.

Profiles of the water surface at the greatest and ordinary yearly flood flows and a profile of the stream bed on its axis.

Cross sections on the bed of the river, taken wherever any marked changes in section occurred and at enough intervals between such places as would approximately give the mean area. Such cross sections should include the banks to the highest elevation to which the river could possibly rise after the dam was built.

Determination of flood flows from records.

Computations (from the accumulated information) of values of n for successive sections of the stream, to be used in computing the hydraulic grade line for the assumed conditions.

The maximum flood flow in this case was known to be about 22,700 cubic feet per second; such a flood was likely to occur only about two or three times in a century.

From a study of the profiles and cross sections it was found that the river from the proposed dam to the water power three miles upstream might be reasonably divided into 4 lengths.

By means of the profiles and the cross sections, the mean area in each length was determined.

From the profiles of the water surface the rise in water surface for each length was taken in this case for maximum flood flow.

From the cross sections the mean wetted perimeter was scaled for each length.

The mean velocity in each length was next computed for the greatest flood by dividing the mean area of each length into 22,700.

The mean hydraulic radius for each length was computed from the area and wetted perimeter $R = \frac{A}{w.\,p.}$.

The value of n for each length of river was then determined by the Chezy formula and Manning's formula.

$$V = C\sqrt{RS} \quad \text{or} \quad C = V/\sqrt{RS}$$

$$C = \frac{1.486}{n} R^{1/6} \quad \text{or} \quad n = \frac{1.486}{C} R^{1/6}.$$

The height over the crest of the dam necessary to pass a flood of 22,700 c.f.p.s. was then determined by a suitable weir formula,

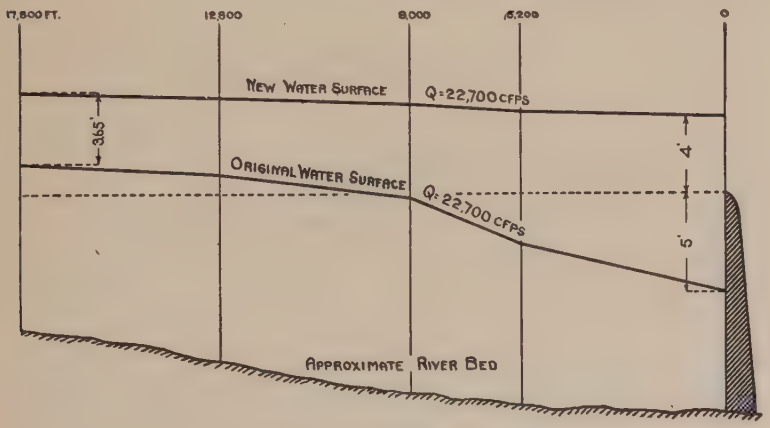


FIG. 97. — BACKWATER CURVE

and found to be 3.98 ft., or practically four feet. It was decided that the dam crest should be five feet higher than the original flood water surface, making a total raising of the water level at the dam site of nine feet. Using this as a starting point the increase in area, wetted perimeter, and hydraulic radius was determined for the first length. The value of n found for this length was then used in finding a new value of C for the

length, by which the loss of head was determined. This was added to the height of water at the dam site, 9.00 feet, and gave the starting point for the next section. In this way the loss for each section was found. Before the dam was built the water surface at the upper mill stood 6.55 feet above the level at the dam site. The computed height for a dam raising the water surface 9 ft. was found to be 10.20 ft. In other words, the water would stand 3.65 ft. higher at the point 17,800 ft. upstream after the dam was built, when the flow was 22,700 c.f.p.s.

Determination of n for Each Length

Section	Length	Area	h_λ	p	R	V	C	n
0–1	5,200	5,350	2.50	730	7.33	4.25	71.5	.029
1–2	2,800	4,170	2.38	459	9.10	5.44	61.8	.035
2–3	4,800	6,500	1.13	660	9.85	3.49	72.5	.030
3–4	5,000	10,700	.54	1,220	8.77	2.12	68.8	.031
	17,800		6.55					

Determination of Loss of Head
Water Surface Raised by Dam

Section	Length	Area	p	R	V	n	C	h_λ
0–1	5,200	11,650	798	14.6	1.95	.029	80.1	.21
1–2	2,800	7,270	482	15.1	3.12	.035	66.6	.41
2–3	4,800	9,630	684	14.1	2.36	.030	77.0	.32
3–4	5,000	14,340	1,413	9.74	1.58	.031	70.0	.26
								1.20

346. Transportation of Solids by Moving Water. Particles of solid matter heavier than water may be moved by a stream either by transporting them in suspension or by rolling or drifting them along the stream bed. This does not refer to the moving of large stones by floating ice. Suspended matter is largely clay, fine sand, or loam; and drifting matter is usually coarse sand, gravel, or large pebbles. The shifting of solid particles in water implies erosion in some places and silting or building up in others. A stream may carry in suspension material which it has not power to erode, and can drift material which it cannot carry. The one condition may occur continuously, the other only during

freshets. The eroding and transporting power of a stream depends chiefly upon its velocity of flow, and the diameter and density of the particles of material through which it flows.[1]

The question of silting, erosion and the transportation of solids by water is too involved to discuss at length. Kennedy[2] found

TABLE XXVII

Material	Velocity	Remarks
Soft earth	0.25	
Fine clay	0.25	
Soft clay	0.50	Very variable and depends upon the adhesion of the clay particles
Finest sand	0.50	Such sand as is left when clay is eroded
Fine sand	0.70	The usual fine sand of rivers
Coarser sand	0.80	
Gravel or coarse sand	1.0	Largely depends upon the shape of the grains. Round gravel of say 0.05 inch size is rolled along a smooth surface at 0.30 foot per second. Pea size at 0.60 foot per second, and bean size at 1.1 foot per second
Pebbles 1 inch in diameter	2.0	
Pebbles, egg size	3.0–3.3	At this velocity the shape of the individual masses ceases to have a very marked effect
Stones 3 inches in diameter	5.0	These values are probably not very accurate, and it is extremely doubtful whether the velocities given accurately represent the bottom velocities
Boulders, 6 inches to 8 inches in diameter	6.6	
Boulders, 1 foot to 18 inches in diameter	10.0	

that the velocity at which silt bearing waters just failed to deposit their burden was

$$V = Cd^{0.64},$$

where V is the mean velocity, d is the depth in feet, and C is a coefficient varying with the class of silt. The value of this for the silt carried by the Punjab (North India) canal system (Bari Doab) upon which the experiments were made was 0.84.

[1] Parker (*Control of Water*) gives a very thorough discussion of erosion and transportation of solids.

[2] Kennedy, R. G., *Prevention of silting in irrigation canals*, Proc. Institution of Civil Eng., vol. 119, p. 281.

Parker's [1] studies of this formula led him to the belief that the value of C increased in proportion to the volume of silt carried by each unit volume of water, and that the exponent, 0.64, should be decreased as the silt became coarser. Table XXVII is taken from Parker, and gives the bottom velocities which just produce motion in the materials mentioned.

347. Design of Earth Channels Modified by the Transporting Power of Water. This transportation of materials by the water often makes necessary lower velocities than other conditions require for power or other canals in earth, in order to keep the cross section, once dug through soft material, of the form designed. On the contrary, heavy silt-bearing rivers may make necessary higher velocities at some points to prevent deposits, where such deposits would be a disadvantage.

PROBLEMS

1. Compute the mean hydraulic radii (*a*) for a circular channel 2 feet in diameter when flowing at the following depths: $\frac{1}{2}$ foot; 1 foot; $1\frac{1}{2}$ feet; and full; and (*b*) for a channel 2 feet square when flowing at the same depths.

2. A wooden flume 2500 feet long, 6 feet by 6 feet, made of plank laid lengthwise; slope = 0.0002. (*a*) What is the discharge, when the mean hydraulic radius is 1.41? (*b*) What velocity?

3. What should be the maximum discharge through a 20-inch vitrified sewer pipe laid on a slope of 0.0003? What will be discharge when R is a maximum?

4. Compute the discharge in cubic feet per second through a brick-lined circular sewer, 18 inches in diameter, grade 1 in 200, when flowing at a depth of 15 inches.

5. What diameter of circular sewer, if of brick, would be required on a grade of 1 in 1000 to carry 150 cubic feet per second, flowing $\frac{1}{2}$ full?

6. (*a*) What grade should be given a 20-inch vitrified sewer pipe to discharge 68.8 cubic feet per minute when flowing $\frac{1}{2}$ full? (*b*) For the determined grade compute the maximum discharge.

7. Compute the discharge through a rectangular channel; bottom of gravel; side walls of rubble masonry. Depth of water, 10 feet; mean width 25 feet; bottom slope, 1 in 5000.

8. Assume a canal with bottom 80 feet wide in gravel, side slope, 2 to 1. Slope of bottom, 2 feet in a mile. Depth of water, 7 feet. What will be the discharge in cubic feet per second?

9. A canal is to be built to discharge 1000 cubic feet per second. Compute the best cross section having a bottom slope of 0.0002: (*a*) in gravel, (*b*) in sand, (*c*) rectangular with dry rubble bottom and sides, and (*d*) rectangular with concrete bottom and side walls. Determine whether, in (*b*) scouring will occur.

[1] Parker, *Control of Water*, Van Nostrand, 1915.

TABLE XXVIII

Experiments on Open Channels

Numbers 1 to 17 are experiments by Darcy and Bazin taken from their *Recherches Hydrauliques*. The symbol *R. H.* followed by a number refers to the pages in that book; the symbol D. & B. followed by a number indicates their serial number for the experiments.

(1) **Semicircular;** D = 4.101′; lining, neat cement; slope of bottom = 0.0015; D. & B. 24; *R. H.* 426.

V	3.02	3.72	4.16	4.60	4.87	5.12	5.29	5.51	5.75	5.91	6.06	6.11
R	.366	.503	.605	.682	.750	.809	.867	.915	.949	.992	1.029	1.034
C	129	136	138	144	145	147	147	149	152	153	154	155

(2) **Semicircular;** *D* = 4.101′; lining, $\frac{2}{3}$ cement, $\frac{1}{3}$ very fine sand; slope of bottom = 0.0015; D. & B. 25; *R. H.* 429.

V	2.87	3.43	3.87	4.30	4.51	4.80	4.94	5.20	5.38	5.48	5.55	5.66
R	.379	.529	.635	.706	.787	.839	.900	.941	.984	1.007	1.021	1.038
C	121	122	125	132	131	135	134	138	140	141	142	144

(3) **Semicircular;** *D* = 4.593′; lining, planks; slope of bottom = 0.0014; D. & B. 26; *R. H.* 432.

V	2.61	3.23	3.71	4.04	4.25	4.51	4.64	4.87	5.00	5.18	5.29	5.45	5.54
R	.390	.537	.632	.717	.796	.856	.921	.964	1.015	1.054	1.096	1.129	1.148
C	108	114	121	123	123	126	125	128	128	130	130	132	134

(4) **Semicircular;** *D* = 4.002′; lining, small gravel (1 to 2 cm.) set in mortar $\frac{1}{4}$ cement; slope of bottom = 0.0015; D. & B. 27; *R. H.* 435.

V	2.17	2.50	2.69	2.93	3.05	3.22	3.33	3.54	3.73	3.95
R	.454	.546	.619	.681	.731	.784	.826	.900	.968	1.012
C	78	82	82	84	84	85	84	85	85	88

(5) **Rectangular;** width, 5.943′; lining, neat cement; slope of bottom = 0.0049; D. & B. 2; *R. H.* 356.

V	3.34	4.39	5.04	5.68	6.51	6.83	7.12	7.41	7.63	7.86	8.07
R	.168	.251	.322	.375	.474	.518	.558	.595	.632	.665	.696
C	117	125	127	132	135	136	136	137	137	138	138

(6) **Rectangular;** width, 6.529′; lining, planks; slope of bottom = 0.0049; D. & B. 7; *R. H.* 373.

V	2.71	3.70	4.35	4.85	5.29	5.61	5.93	6.23	6.45	6.71	6.90	7.15
R	.188	.272	.342	.402	.453	.504	.547	.587	.628	.662	.698	.727
C	89	101	106	109	112	113	115	116	116	118	118	120

(7) **Rectangular;** width, 6.27′; lining, bricks laid flat; slope of bottom = 0.0049; D. & B. 3; *R. H.* 361.

V	2.75	3.66	4.18	4.72	5.10	5.33	5.68	6.01	6.15	6.47	6.60	6.72
R	.192	.284	.365	.424	.481	.540	.582	.620	.668	.697	.739	.779
C	90	98	99	104	105	104	106	109	107	111	110	109

(8) **Rectangular;** width, 6.01′; lining, small gravel (1 to 2 cm.) set in cement mortar; slope of bottom = 0.0049; D. & B. 4; *R. H.* 364.

V	2.16	2.95	3.40	3.84	4.14	4.43	4.64	4.88	5.12	5.26	5.43	5.57
R	.250	.357	.450	.520	.588	.644	.700	.746	.785	.832	.872	.910
C	62	71	73	76	77	79	79	81	83	82	83	83

(9) **Rectangular;** width, 6.106′; lining, gravel (3 to 4 cm.) set in cement mortar; slope of bottom = 0.0049; D. & B. 5; *R. H.* 367.

V	1.79	2.43	2.90	3.27	3.56	3.85	4.03	4.23	4.43	4.60	4.78	4.90
R	.291	.417	.510	.587	.656	.712	.772	.823	.867	.909	.946	.987
C	48	54	58	61	63	65	66	67	68	69	70	70

(10) **Trapezoidal;** bottom width, 3.281′, top width, 6.56; side batter, 1 to 1; lining, planks; slope of bottom = 0.0015; D. & B. 21; *R. H.* 417.

V	2.39	2.93	3.35	3.62	3.85	4.03	4.20	4.39	4.51	4.64	4.76	4.87
R	.334	.485	.586	.673	.744	.809	.864	.911	.959	1.002	1.047	1.097
C	107	109	113	114	115	116	117	119	119	120	120	120

(11) **Triangular;** side batter, 1 to 1; lining, planks; slope of bottom = 0.0049; D. & B. 23, *R. H.* 423.

V	4.13	5.02	5.56	6.03	6.36	6.59	6.83	7.03	7.23	7.40	7.54	7.75
R	.327	.422	.494	.549	.597	.643	.683	.719	.752	.783	.814	.839
C	103	110	113	116	118	117	118	118	119	119	119	121

(12) and (13) **Trapezoidal;** nearly rectangular, bottom width, 5.91′; side batter, 1 to 10; lining, rubble masonry pointed with cement; side walls in better condition than bottom, which was broken in places, especially in the lower portions, having a slope of 0.037; furthermore, the bottom was covered with a light deposit of sticky mud. *R. H.* 446–447.

	D. & B. 32 Bottom slope = 0.101					D. & B. 33 Bottom slope = 0.037			
V	12.3	16.2	18.7	21.1	*V*	9.04	11.5	13.6	15.1
R	.324	.467	.580	.662	*R*	.424	.620	.745	.852
C	68	75	77	82	*C*	72	76	82	85

(14) and (15) **Trapezoidal;** bottom width, 5.91′ to 6.56′; side batter, 3 to 2; lining, dry masonry in rather poor condition; two series on the same channel; *R. H.* 446–447.

	D. & B. 34 Walls completely covered with vegetable growths Bottom slope = 0.0146					D. & B. 35 Walls carefully cleaned Bottom slope = 0.0142				
V	4.19	5.75	7.20	8.27	8.99	*V*	5.66	7.36	8.94	10.1	11.3
R	.856	1.087	1.383	1.586	1.694	*R*	.703	.930	1.227	1.394	1.491
C	38	46	51	54	57	*C*	57	64	68	72	77

(16) and (17) **Trapezoidal**; side batter about 2.3 to 1; **lining**, rough earth; *R. H.*, 454–446; two parts of same channel.

D. & B. 49
Bottom width, 6.44′
Bottom and slopes rough but quite regular and completely free of vegetable growth
Slope of bottom = 0.000367

V	.89	1.34	1.36	1.47
R	.96	1.32	1.57	1.78
C	57	70	69	66

D. & B. 50
Bottom width, 6.29′
Bottom and slopes rough with considerable vegetable growth
Slope of bottom = 0.00033

V	.82	1.26	1.30	1.41
R	1.05	1.42	1.65	1.85
C	45	62	56	57

(18) **Sudbury Aqueduct** of the Metropolitan W. W., Boston; horseshoe shape; 9.0′ wide × 7.7′ high (inside dimensions); hard brick lining, well laid. Experiments of Fteley & Stearns. TRANS. AM. SOC. C. E., vol. 12, pp. 114–117; the aqueduct being new. Bottom slope = .000189.

Values of *C* for various values of *R*

R	.5	.6	.7	.8	.9	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
C	117	119	122	124	125	127	130	132	134	136	138	139	140

Limits of experimental velocities, 0.5 to 2.33

The following formula will give results identical with the values immediately preceding:

$$V = 127\, R^{0.62}\, S^{0.5}.$$

In a portion of the conduit (600 feet long) chosen on account of its uniformity of section and grade, the surface was scraped clean; in this case the flow was represented by the formula:

$$V = 129\, R^{0.62}\, S^{0.5}.$$

In a portion of the conduit lined with a coating of neat cement mortar, the following formula very nearly represented the flow:

$$V = 137\, R^{0.62}\, S^{0.5}.$$

In tunnel through rock (4614 feet long) in which the rock sides had been left ragged for 4362 feet without lining, but with smooth concrete floor, the rock excavation being at no place less than 2 feet wider than the brick-lined section, the following formula very nearly represented the flow:

$$V = 96\, R^{0.62}\, S^{0.5}.$$

After 20 years in service with a collection of one year's slime the flow was represented by:

$$V = 129(RS)^{1/2}.$$

One month later, after being cleaned:

$$V = 143(RS)^{1/2}.$$

(19) **Croton Aqueduct,** New York Waterworks; horseshoe shape, 13.6′ wide × 13.53′ high for about 125,000 feet; of circular form, 12.25′ diameter for about 35,000 feet, and for short distances somewhat varied; hard brick lining well laid; *Aqueduct Commissioners' Report*, 1895, p. 97. Slope of bottom = 0.00013257. Values of C for various values of R.

R	1.0	1.5	2.0	2.5	3.0	3.5	4.0
C	119	126	130	132	133	134	134

The flow was represented by the formula:

$$V = 124\, R^{0.56}\, S^{0.5}.$$

After $9\frac{1}{2}$ years' use a measurement reported by J. R. Freeman at $V = 2.3$ gave $C = 105$ (in the Chezy formula).

(20) **Lake Katrine Aqueduct,** Glasgow, Scotland, Corporation Waterworks; horseshoe shape, straight, 5763 feet long; 10′ wide × 9′ high, lined with concrete for about 0.53 of its length, and unlined rock 12′ wide × 9′ high, with concrete bottom, for 0.47 of its length.

Experiments by A. F. Bruce, Proc. Institution C. E., vol. 123, p. 414.

Bottom slope = 0.0001818

V	1.871	2.069	2.108	2.216	2.128	2.155	2.175	2.206	2.233
R	1.227	1.471	1.471	1.490	1.498	1.500	1.547	1.599	1.607
C	125	127	129	135	129	130	130	129	131
n	.0123	.0125	.0123	.0119	.0124	.0123	.0124	.0125	.0125

V	2.226	2.245	2.255	2.259
R	1.611	1.621	1.627	1.740
C	130	131	131	127
n	.0125	.0124	.0124	.0129

(21) **Brick Sewers;** Metropolitan Sewerage Board, Boston; hard brick lining. Experiments by R. E. Horton, Trans. Am. Soc. C. E., vol. 46, p. 84 Shape. basket-handle; 6′ wide × 6.67′ high; lining, brick, cement washed.

Bottom slope = 0.0005

1896 after 10 months' use	V	1.99	2.46	2.82	3.13	3.44
	R	.688	.958	1.187	1.387	1.539
	C	107	112	115	118	124
	n	.0129	.0131	.0132	.0133	.0130

1897 after 2 years 2 months' use	V	2.97	3.16	1900 after 3 years 9 months' use	V	2.66	2.86	3.04
	R	1.54	1.65		R	1.342	1.508	1.645
	C	107	111		C	102	104	106
	n	.0149	.0147		n	.0151	.0152	.0152

Shape, circular, 9 feet in diameter; same lining. Slope of bottom = 0.000333.

1896	V	1.58	2.21	2.70	3.03	3.48	3.73	4.18
	R	.619	.928	1.208	1.408	1.830	1.999	2.309
	C	110	126	134	139	141	145	150
	n	.0122	.0117	.0116	.0115	.0117	.0116	.0115

1897	V	2.55	2.90	3.06	3.18	1900	V	2.38	2.82	3.16	3.30
	R	1.280	1.560	1.762	1.771		R	1.120	1.606	1.952	2.130
	C	123	127	126	131		C	119	121	124	124
	n	.0126	.0127	.0129	.0126		n	.0130	.0132	.0133	.0136

(22) **Rectangular;** width 4.5′, depth 3.13′, cement plastered, 8 years old, but free from growths; current meter; ENGINEERING NEWS, June 6, 1907.

$$V = 5.01, \quad R = 1.31, \quad C = 123.3$$

(23) **Round;** 42″ concrete pipe, new, 800′ long, very smooth, perfect joints; weir; DEPT. OF AGRIC. BULLETIN 852. Note that this gives complete transition from flow filling ¼ of pipe to full flow.

Depth in feet	*Area in sq. ft.*	*w. p. in feet*	*R in feet*	*Vel. in ft. per sec.*	*C*
.72	1.42	3.27	.434	2.37	114
1.21	2.97	4.39	.677	3.13	121
1.67	4.52	5.34	.846	3.74	129
2.17	6.25	6.34	.986	4.16	132
Full	9.62	10.99	.875	4.22	143

Same pipe, another section, 15 months later, 1,986′ long; color velocity.

1.26	3.11	4.50	.691	3.11	115
1.30	3.24	4.57	.710	3.53	132
1.65	4.47	5.30	.843	4.22	142
1.88	5.27	5.76	.916	4.62	148
2.08	5.94	6.20	.958	4.71	146

(24) **Horseshoe;** max. width 17.5′, very smooth concrete; Venturi meter; DEPT. OF AGRIC. BULLETIN 852.

4.54	71.80	23.90	3.0	3.64	145
5.40	86.60	25.70	3.37	3.99	150

(25) **A dredged ditch,** 55′ wide, maximum depth 13.5′. Sticky gumbo clay.

$$V = 1.93, \quad R = 7.47, \quad C = 65.5, \quad N = .0342$$

(26) **A rough rock channel,** excavated by explosives; Engineering News-Record, Dec. 6, 1923; average of four measurements.

Depth	*Area*	*P*	*R*	*Vel.*	*C*	*n*
5.64′	46.2	18.1	2.56	3.65	43.4	.0396

RIVERS

(27) **South Forked Deer River,** near Jackson, Tenn. Fairly straight, but very irregular in section; sandy clay loam, sides covered with trees, roots and vines; logs and branches in bottom of channel. ENGINEERING NEWS, March 13, 1919.

Width	*R*	*Vel.*	*C*	*n*
48′	4.44	2.01	39.5	.0500
	5.09	2.27	37.2	.0550
	5.56	2.68	34.4	.0620
	7.19	2.64	44.1	.0505

Another section of same river; very crooked, and more irregular and in worse condition; badly blocked by fallen trees.

63′	4.15	1.64	13.1	.1520
	4.99	1.75	12.7	.1620
	5.56	2.01	14.6	.1500
	5.68	1.90	15.3	.1460
	7.60	1.85	17.4	.1400

(28) **St. Francis River,** near Marked Tree, Ark. A rather irregular, sluggish stream, with an average fall of about half a foot per mile; banks partly covered with brush and trees.

Width	*R*	*Vel.*	*C*	*n*
230′	9.13	1.88	69.6	.0343

(29) **Miami River,** at Tadmor, Ohio. Stream bed composed of clay, sand, and gravel, no rocks or boulders; edges of channel partly obstructed by bushes and trees.

Width	*R*	*Vel.*	*C*	*n*
192	1.66	.40	57	.0250
204	10.8	4.66	72.5	.0316

(30) **Miami River,** at Dayton. Comparatively uniform channel in clay, sand, and gravel with a preponderance of gravel.

Width	*R*	*Vel.*	*C*	*n*
382	5.06	.89	66.2	.0317
605	12.3	6.08	95.4	.0236

(31) **Merrimack River,** above Lowell. Moderately winding; fairly regular section; gravel and silt. Measurements by Locks and Canals, Lowell, Mass.

±660′ (ave.)	15.2	3.52	86.3	.0290
	16.1	4.08	86.5	.0288
	16.4	4.25	89.0	.0279

(32) **Susquehanna River,** near McCall's Ferry, Pa. Very irregular and rocky channel, many small islands covered with trees and brush, which are overflowed at high stages. Engineering News, Aug. 4, 1904.

10.12	2.60	52.3	.0460
15.51	7.86	54.8	.0462
16.68	8.38	54.9	.0468
26.20	9.96	52.4	.0545

(33) **Mississippi River,** at Carrollton, La. Comparatively irregular channel, no breaks in levees.

3,330′	50–70	5–6	135–120	.0275–.0252

Note. The above examples are given to show the great variation in value of *C* that may occur in various rivers. No two rivers are the same, and coefficients must be based on a thorough study of the individual river. These examples, however, will give some idea of what to use for a very rough approximation. Except where noted the values given above are from Part IV of the Technical Reports of the Miami Conservancy District, *Calculation of Flow in Open Channels,* Ivan E. Houk. Both this and H. M. King's *Handbook of Hydraulics* (McGraw-Hill) give much information regarding flow in open channels.

CHAPTER XVII

DYNAMIC ACTION OF FLOWING WATER

348. Definitions and Notation. *Absolute velocity* is velocity relative to a point on the earth's surface. *Relative velocity* is velocity relative to a point on some body stationary or moving; if the body is stationary, relative velocity is the same as absolute velocity.

Direction of forces and motion will be stated in degrees from 0° to 360° measured from the X-axis. For a stationary solid the X-axis will be horizontal; and angles will read contra-clockwise, as in trigonometry. For motion in a straight line, the X-axis will be the line of motion of the solid; and the direction of its motion will be 0°. For circular (or curvilinear) motion, the X-axis will be a tangent to the path of motion of the solid at the points where the water enters or leaves the solid; and the direction of its motion will be 0°.

In certain types of water motors, the water enters and leaves the vane at practically a constant distance from the center of rotation. In other types the points of entrance and exit are at different distances from the center of rotation; in this latter case there will be two sets of axes of reference.

Forces and motion are assumed to be in, or reduced to, a common plane, or parallel planes. Forces will be expressed in pounds; velocities, in feet per second; areas, in square feet; heads, in feet; intensity of pressure, in pounds per square inch measured from atmospheric pressure; power, in foot pounds per second.

F = a constant force producing a change in velocity, direction, position, or pressure.

F_X and F_Y = X and Y components of F.

P = a constant force equal in magnitude to F, but opposite in direction = $F \cos 180° = F \times -1$.

P_X and P_Y = X and Y components of P.

V_0 = the component of the original velocity parallel to the line of action of F.

V_1 = the component of the final velocity parallel to the line of action of F.

$\frac{dv}{dt}$ = the constant acceleration or retardation in feet per second per second parallel to the line of action of F.

V_α = absolute mean velocity from guide vanes or nozzles; absolute entrance velocity.

V_ϵ = absolute mean velocity from a wheel vane or vessel; absolute exit velocity.

v_β = relative mean velocity at entrance to a vane or vessel.

v_δ = relative mean velocity at exit from a vane or vessel.

S = velocity of the point of application of a force.

S_1 = absolute velocity of a bucket at the point where the water leaves it.

S_0 = absolute velocity of a bucket at the point where the water enters it; $S_0 = \pi r_0 N/30$. The direction of S_0 is 0°, being tangent to the runner at this point.

α = angle of absolute velocity at entrance, direction of V_α.

β = angle of relative velocity at entrance, direction of v_β.

δ = angle of relative velocity at exit, direction of v_δ.

ϵ = angle of absolute velocity at exit, direction of V_ϵ.

a_α, a_β, and a_δ = cross-sectional areas of a stream or jet measured at right angles to the direction of flow; each area corresponding in the order named to the velocities V_α, v_β, and v_δ.

H = total head acting on wheel.

h_1 = head within wheel, if any. In other words, the difference in height between the point of entrance and the point of exit.

P_α = atmospheric pressure.

P_β = pressure at entrance to runner.

E = Power (rate of work done), in foot pounds per second.

W = Weight of a solid, or of volume of water, in pounds.

w = 62.4 pounds, weight of a cubic foot of water.

Q = flow in cubic feet per second.

g = acceleration due to gravity in feet per second per second, = 32.16.

r_0 = radius of runner, where water enters, in feet.

r_1 = radius of runner, where water leaves, in feet.

C_v, C_1 = coefficients of velocity from nozzles or buckets.

m = coefficient of loss in buckets.

K = hydraulic efficiency.

349. The Force required to produce Acceleration or Retardation. A constant force F which, acting upon a body weighing W, will cause an acceleration or retardation, dv, in an interval of time, dt, may be found as follows:

$$F = \frac{W}{g}\frac{dv}{dt} = \frac{W}{g}\frac{(V_1 - V_0)}{t}, \tag{1}$$

or

$$Fdt = \frac{W}{g}dv; \quad \text{and} \quad Ft = \frac{W}{g}(V_1 - V_0). \tag{2}$$

The impulse, therefore, which is the product of the force multiplied by the time during which it acts, equals the change in momentum in the same time. If the time be one second, Ft, the impulse equals F, the force itself.

A stream of water, or other fluid, impinging upon a solid body which is stationary, or moving at a different velocity or in a different direction than the stream, will by the impact or collision have its velocity or direction changed; and in the process impart to the body itself a force or pressure, which is equal in magnitude, but opposite in direction, to the force (frequently called the reaction) which is exerted by the body upon the stream.

In hydraulic problems dealing chiefly with continuous streams, not isolated bodies of water, the exact interval of time in which the change in velocity or direction occurs can rarely be determined. It is only necessary, however, to consider the weight of the mass of water which acts, or is acted upon, in one second.

If Q is the rate of discharge of a stream in cubic feet per second: then W in equation (2) corresponding to a time t of one second equals wQ pounds per second. Then equation (2) becomes

$$F = \frac{wQ}{g}(V_1 - V_0), \tag{3}$$

where F is the constant force which must be exerted on the water to change the velocity of Q cubic feet per second from V_0 to V_1 feet per second, and where V_0 and V_1 are the components of the original and final velocity in the line of action of the force F.

350. The Components of F for any Given Set of Axes. The horizontal or X components of the original and final velocity for absolute velocities are

$$V_0 = V_\alpha \cos \alpha, \qquad V_1 = V_\epsilon \cos \epsilon; \tag{4}$$

and for relative velocities,

$$V_0 = v_\beta \cos \beta, \qquad V_1 = v_\delta \cos \delta. \tag{5}$$

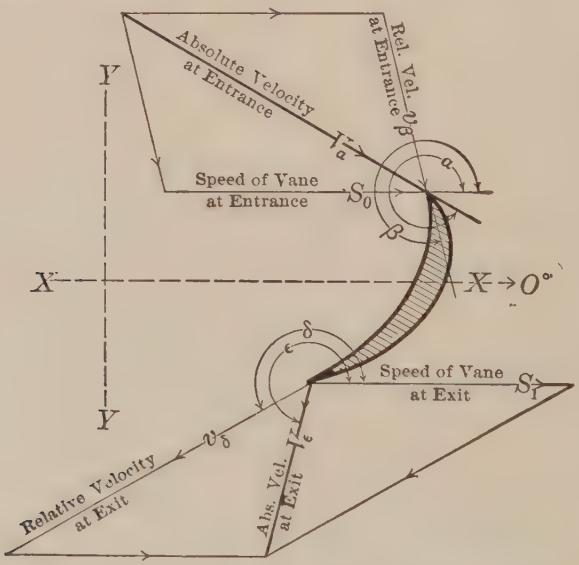


Fig. 98. — Velocity Diagram for a Moving Vane

The horizontal or X component of F, in terms of absolute velocity, by (3), is

$$F_X = \frac{wQ}{g}(V_\epsilon \cos \epsilon - V_\alpha \cos \alpha), \tag{6}$$

or, in terms of relative velocity,

$$F_X = \frac{wQ}{g}(v_\delta \cos \delta - v_\beta \cos \beta). \tag{7}$$

The vertical or Y components of the original and final velocities, for absolute velocities, are

$$V_0 = V_\alpha \sin \alpha, \qquad V_1 = V_\epsilon \sin \epsilon; \tag{8}$$

and for relative velocities,

$$V_0 = v_\beta \sin \beta, \qquad V_1 = v_\delta \sin \delta. \tag{9}$$

The vertical or Y component of F, in terms of absolute velocity, by (3), is

$$F_Y = \frac{wQ}{g}(V_\epsilon \sin \epsilon - V_\alpha \sin \alpha), \tag{10}$$

or, in terms of relative velocity,

$$F_Y = \frac{wQ}{g}(v_\delta \sin \delta - v_\beta \sin \beta). \tag{11}$$

351. Pressures caused by Moving Water. The force or pressure exerted on a body in consequence of changes in motion of moving water, in contact with a body, necessarily implies a resistance equal in magnitude, opposite in direction, and having a common point of application. The body must therefore have sufficient strength in its material to withstand the stresses set up by the force, and must supply in itself or transmit from other external forces resistance equal to the force exerted.

The force exerted upon moving water to change its velocity or direction sets up an equal and opposite force which is exerted on the body by the water; and its value is

$$P = F \cos 180° = F \times (-1) = \frac{wQ}{g}(V_0 - V_1). \tag{12}$$

352. The Components of P for any Given Set of Axes. The horizontal component of P, in terms of absolute velocity, is

$$P_X = \frac{wQ}{g}(V_\alpha \cos \alpha - V_\epsilon \cos \epsilon), \tag{13}$$

or, in terms of relative velocity,

$$P_X = \frac{wQ}{g}(v_\beta \cos \beta - v_\delta \cos \delta). \tag{14}$$

The vertical component of P, in terms of absolute velocity, is

$$P_Y = \frac{wQ}{g}(V_\alpha \sin \alpha - V_\epsilon \sin \epsilon), \tag{15}$$

or, in terms of relative velocity,

$$P_Y = \frac{wQ}{g}(v_\beta \sin \beta - v_\delta \sin \delta), \tag{16}$$

353. Discussion of Results Obtained. *The equations above derived* (1) *to* (16) *are general,* and applicable either to stationary or moving bodies.

Either absolute or relative velocities may be used in computing forces or work due to changes in flowing water while in contact with a body. *The difference between the components of original and final absolute velocities* with reference to a given axis *equals the difference between the components of original and final relative velocities*; since the component of the motion of the body with reference to the same axis has a constant and an equal effect on each difference. For convenience and uniformity final results will be expressed herein as far as possible in terms of absolute velocities.

Inspection of the equations (1) to (16) shows:

That the magnitude of the force F or P depends upon:

(*a*) The weight of water delivered every second by the stream;

(*b*) The change in velocity and direction at entrance and exit; viz., the algebraic difference between the components of original and final velocity parallel to the direction of F or P.

That the magnitude of F or P is independent of the radius of curvature of the path of the stream during contact with a body, except in so far as the smoothness and gradual curvature decrease friction losses, and in so far as curvature determines the direction of entrance and exit velocities; hence theoretically the path may be simply bent at an angle, not curved.

That the direction of the force F or P depends upon the algebraic sign (+ *or* −) *of the algebraic difference between the components of original and final velocities with reference to the given axes.* If an X component force has a plus sign, its direction is 0°; if a minus sign, its direction is 180°; if a Y component force has a plus sign, its direction is 90°; if a minus sign, its direction is 270°. The direction of F or P may be computed from the magnitudes of its X and Y components.

The point of application of the force F or P must be somewhere on the solid between the entrance and exit points of the stream. If there is a well-defined place through which the resistance is transmitted, that place would be the point of application; otherwise it will usually be indeterminate.

354. The Effect of a Force. If a body is at rest and opposes a force due to changes in velocity or direction of impinging water with sufficient resistance to overcome the tendency to move the body, the force will merely *set up stresses* tending to deform the body; if, however, the resistance is insufficient to prevent motion, the force will not only set up stresses, but will also impart to the body an onward *motion.*

If the body is already in motion, the force due to a change in velocity or direction will in addition to setting up stresses which tend to deform the body, impart to it a *change of motion.*

355. Work done by Moving Water. The work done by moving water is due to changes in velocity or direction, or both. If a force is exerted against a resistance and their common point of application is moving, work is done; that is, energy is expended. If St is the distance traveled by the point of application during the period of time (t) of the exertion of a force F or P, the work done is

$$FSt = -PSt \text{ (foot pounds).}$$

NOTE. t in most problems concerning the work done by flowing water is *one second;* in such cases, the distance $St = S$, and the work done $FSt = FS$, numerically.

356. Work done in Changing Velocity. If a stream in a contact with a constant resistance, which is moving at the rate of S feet per second, has its velocity changed from V_0 to V_1, the work done in one second against the stream is

$$(17) \qquad E = FS = \frac{wQ}{g}(V_1 - V_0)S;$$

or the work done in one second by the stream is

$$(18) \qquad -E = PS = \frac{wQ}{g}(V_0 - V_1)S.$$

In terms of kinetic energy, we may write

$$(19)\ Et = FSt = wQt\left(\frac{V_1^2}{2g} - \frac{V_0^2}{2g}\right), \text{ whence } F = \frac{wQ}{S}\left(\frac{V_1^2}{2g} - \frac{V_0^2}{2g}\right),$$

or

$$(20)\ Et = PSt = wQt\left(\frac{V_0^2}{2g} - \frac{V_1^2}{2g}\right), \text{ whence } P = \frac{wQ}{S}\left(\frac{V_0^2}{2g} - \frac{V_1^2}{2g}\right).$$

If V_0 is greater than V_1, the moving water must have parted with or transferred some of its energy, and if it is transmitted to a suitable machine, this energy may be transformed into useful work, as in the case of water motors. If V_1 is greater than V_0, energy must have been transferred to the water from some other source, as in the case of pumps.

357. Work done in Changing Elevation. If wQ pounds of water fall from a higher to a lower elevation against a constant moving resistance, or are raised from a lower to a higher elevation, work is performed, in the first instance by the force of gravity in the water, or in the second instance against the force of gravity. The work done in one second against the stream is

$$(21) \qquad E = FS = wQ(h_1 - h_0), \quad \text{whence} \quad F = \frac{wQ}{S}(h_1 - h_0);$$

or the work done in one second by the stream is

$$(22) \qquad -E = PS = wQ(h_0 - h_1), \quad \text{whence} \quad P = \frac{wQ}{S}(h_0 - h_1),$$

where h_0 is the original elevation above a datum plane (in feet), and h_1 is the final elevation above the same datum plane (in feet).

358. Work done in Changing Intensity of Pressure. If wQ pounds of water per second flowing in a closed vessel have its intensity of pressure increased or diminished against a constant moving resistance, the work done per second against the stream is

$$(23) \qquad E = FS = wQ\left(\frac{p_1}{w} - \frac{p_0}{w}\right), \quad \text{whence} \quad F = \frac{wQ}{S}\left(\frac{p_1}{w} - \frac{p_0}{w}\right);$$

or the work done in one second by the stream is

$$(24) \qquad -E = PS = wQ\left(\frac{p_0}{w} - \frac{p_1}{w}\right), \quad \text{whence} \quad P = \frac{wQ}{S}\left(\frac{p_0}{w} - \frac{p_1}{w}\right),$$

where p_0 is original intensity of pressure, and p_1 is final intensity of pressure

359. Equivalent Expressions for Work. The force exerted or the energy expended in consequence of changes in velocity, direction, elevation, or pressure of moving water may be expressed according to the conditions in which it exists in terms of velocity,

nead of water, or intensity of pressure, or in terms of two or more forms. All expressions may be freely interconverted to facilitate computation by the equivalent expressions in feet. Thus we have

$$\frac{V^2}{2g} = h = \frac{p}{w}.$$

If a stream divides upon entering a channel, or follows diverging paths through a channel upon its departure, for instance, in a complex waterwheel runner, unless the divergence is symmetrical, each subdivision of the stream must be computed independently. This makes the computations for work done by certain kinds of turbines very complicated.

360. A Jet of Water Impinging upon a Flat Plate. Let us suppose that a jet of water of cross-sectional area a_α, having a velocity $(V_\alpha, 0°)$, impinges upon a sufficiently large flat plate set normal to the direction of flow. We shall assume that the water flows symmetrically over the plate and leaves it in a direction parallel to the surface. Let us compute the X and Y forces exerted upon the plate by the water, and the work done, neglecting the friction.

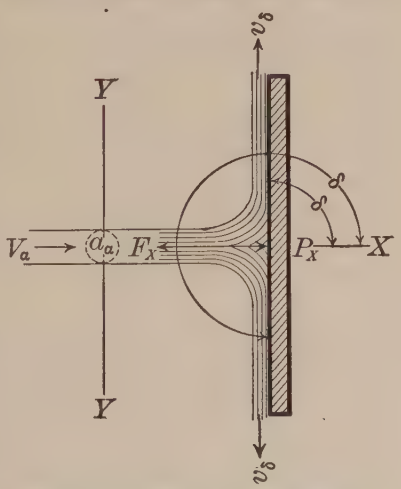


Fig. 99. — Flat Vane, Stationary

361. Stationary Vane. Let us suppose, in Fig. 99, that $a_\alpha = 0.5$, $V_\alpha = 40$, and $\alpha = 0°$. Since the stream is symmetrical, δ may be taken as 90° for the upper $Q/2$, and as 270° for the lower $Q/2$; hence $Q = a_\alpha V_\alpha$. Since the vane is stationary, $V_\epsilon = v_\delta = V_\alpha$. Hence, by (13), we have

$$P_X = \frac{wQ}{g}(V_\alpha \cos\alpha - V_\epsilon \cos\epsilon);$$

and since

$$V_\alpha \cos\alpha = V_\alpha \cos 0°; \quad V_\epsilon \cos_\epsilon = V_\delta \cos\delta = V_\alpha \cos 90°,$$

we find

$$P_X = \frac{wa_\alpha V_\alpha}{g}[(V_\alpha \times 1) - (V_\alpha \times 0)] = + \frac{wa_\alpha V_\alpha^2}{g}$$

$$= + \frac{62.4 \times 0.5 \times 40^2}{32.16} = 1552 \text{ pounds, direction } 0°.$$

Moreover, by (15),

$$P_Y = \frac{wQ}{g}(V_\alpha \sin \alpha - V_\epsilon \sin \epsilon),$$

and since

$$V_\alpha \sin \alpha = V_\alpha \sin 0°;\ V_\epsilon \sin \epsilon = V_\alpha(\sin 90° + \sin 270°) = V_\alpha \times 0,$$

we find

$$P_Y = \frac{wa_\alpha V_\alpha}{g}[(V_\alpha \times 0) - (V_\alpha \times 0)] = 0.$$

Since the vane is stationary, $E = 0$.

This analysis also applies to every plane through the axis of the jet which is at right angles to the plate

362. Single Flat Moving Vane. Let us suppose that a single flat vane moves against a resistance with a velocity (20, 0°) in a straight line; and that the other data are the same as in § 361. (See Fig. 100.) Then, by (13), we shall have

$$P_X = \frac{wQ}{g}(V_\alpha \cos \alpha - V_\epsilon \cos \epsilon).$$

Since $v_\beta = V_\alpha - S$, and since, neglecting losses on the vane, there should be no change in relative velocity in passing over the vane, we have also $v_\delta = V_\alpha - S$.

Since the vane is moving, only part of the stream strikes it; and we have $Q = a_\alpha(V_\alpha - S)$. Moreover we have

$$V_\alpha \cos \alpha = V_\alpha \cos 0°;\ V_\epsilon \cos \epsilon = (V_\alpha - S) \cos 90° + S,$$

so that

$$P_X = \frac{wa_\alpha(V_\alpha - S)}{g}[(V_\alpha \times +1) - (V_\alpha - S) \times 0 - S]$$

$$= + \frac{wa_\alpha(V_\alpha - S)^2}{g} = + \frac{62.4 \times 0.5 \times (40 - 20)^2}{32.16}$$

$$= 388 \text{ pounds, direction } 0°.$$

Since the resultant of the vertical components is zero, $P_Y = 0$. Finally, we have

$$E = P_X \times S = \frac{wa_\alpha(V_\alpha - S)^2}{g} S = 388 \times 20$$

$$= 7760 \text{ foot pounds per second.}$$

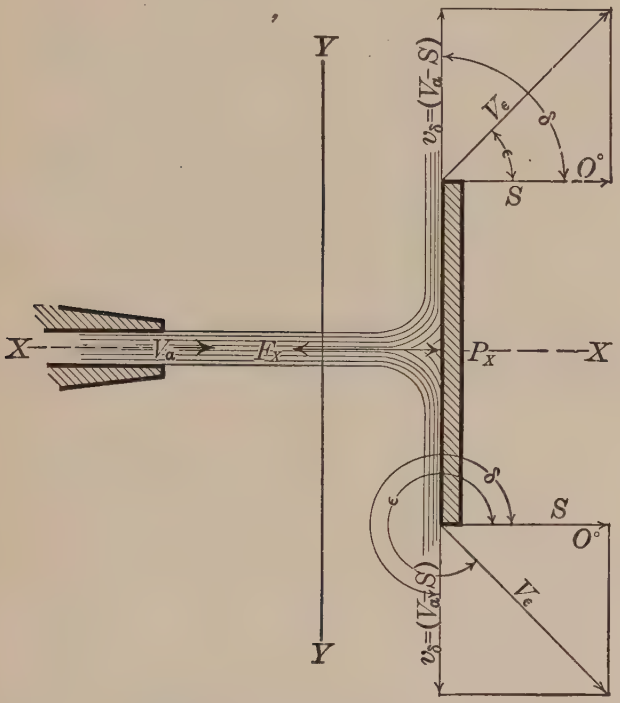


FIG. 100. — FLAT VANE, MOVING

363. Maximum Power for a Single Flat Vane. To find the ratio S/V_a which shall make E a maximum, place $dE/dS = 0$, and solve for S/V_a. Since, by § 362,

$$E = \frac{wa_\alpha}{g}(V_\alpha^2 S - 2\,V_\alpha S^2 + S^3),$$

we find

$$\frac{dE}{dS} = \frac{wa_\alpha}{g}(V_\alpha^2 - 4\,V_\alpha S + 3\,S^2) = 0.$$

Equating this to zero, and solving for S/V_α, we find

$$\frac{S}{V_\alpha} = \frac{1}{3}. \tag{25}$$

Substituting $S = V_\alpha/3$ in the equation for E, we find that the maximum power E_M is

$$E_M = \frac{4}{27}\frac{wa_\alpha V_\alpha^3}{g} = \frac{8}{27}wQ\frac{V_\alpha^2}{2g},$$

that is, E_M is 8/27 of the total kinetic energy available in the stream per second.

364. Series of Moving Vanes. Let us suppose that, with the hydraulic data in § 361, a series of flat vanes moves against a resistance with a velocity of (20, 0°). This would happen, for example, if the plates were set on the rim of a wheel. This is the case of an undershot wheel or an impulse wheel with flat vanes.

Here $Q = a_\alpha V_\alpha$, because with a series of vanes, succeeding vanes take whatever water the preceding ones have left behind (compare with a single vane § 362). Substituting this value of Q in the equation of § 362, we find

$$P_X = +\frac{wa_\alpha V_\alpha}{g}(V_\alpha - S) = \frac{62.4 \times 0.5 \times 40 \times 20}{32.16}$$

$$= 776 \text{ pounds, direction } 0°.$$

Since the resultant of the vertical components is zero, $P_Y = 0$. Hence we have

$$E = P_X S = \frac{wa_\alpha V_\alpha}{g}(V_\alpha - S)S$$

$$= 776 \times 20 = 15{,}520 \qquad \text{foot pounds per second.}$$

365. Maximum Power for a Series of Flat Vanes. To find the ratio S/V_α which shall make E a maximum, we compute dE/dS and equate it to zero, as in § 363. Doing so, we find

$$V_\alpha^2 - 2V_\alpha S = 0,$$

which gives

$$\frac{S}{V_\alpha} = \frac{1}{2}. \tag{27}$$

Substituting S in the equation for E, we find that the maximum power E_M is

$$E_M = \frac{wa_\alpha V_\alpha^3}{4g} = \frac{1}{2}wQ\frac{V_\alpha^2}{2g}, \tag{28}$$

that is, one-half of the total kinetic energy available in the stream per second.

366. A Jet of Water impinging on Curved or Bent Vanes. Theoretically the effect is identical for vanes bent with a sharp angle or with a smooth curve; actually, the latter offers less resistance to flow.

We shall suppose that a jet of water of area $a_\alpha = 0.5$ with a velocity $V_\alpha = (40, 0°)$, impinges on a curved vane having boundaries. (See Fig. 101.) Let us compute the X and Y forces exerted on the vane by the water.

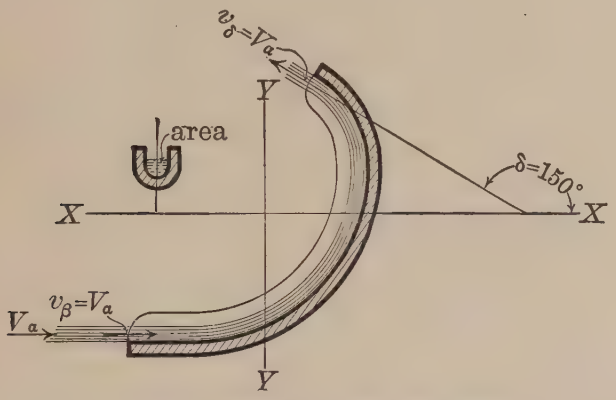


FIG. 101. — CURVED VANE, STATIONARY

367. Stationary Curved Vane. Taking the values $\delta = \epsilon = 150°$, $a_\alpha = a_\beta = a_\delta = 0.5$, $v_\delta = V_\epsilon = V_\alpha$, we shall have, by (13),

$$P_X = \frac{wQ}{g}(V_\alpha \cos\alpha - V_\epsilon \cos\epsilon)$$

$$= \frac{wa_\alpha V_\alpha^2}{g}(\cos 0° - \cos 150°)$$

$$= \frac{62.4 \times 0.5 \times 40^2}{32.16}[1 - (-0.866)] = +2896$$

$$= 2896 \quad \text{pounds, direction } 0°.$$

Moreover, by (15),

$$P_Y = \frac{wQ}{g}(V_\alpha \sin\alpha - V_\epsilon \sin\epsilon),$$

whence

$$P_Y = \frac{wa_\alpha V_\alpha^2}{g}(\sin 0° - \sin 150°)$$

$$= \frac{62.4 \times 0.5 \times 40^2}{32.16}(0 - (+0.5)) = -776$$

$$= 776 \quad \text{pounds, direction } 270°.$$

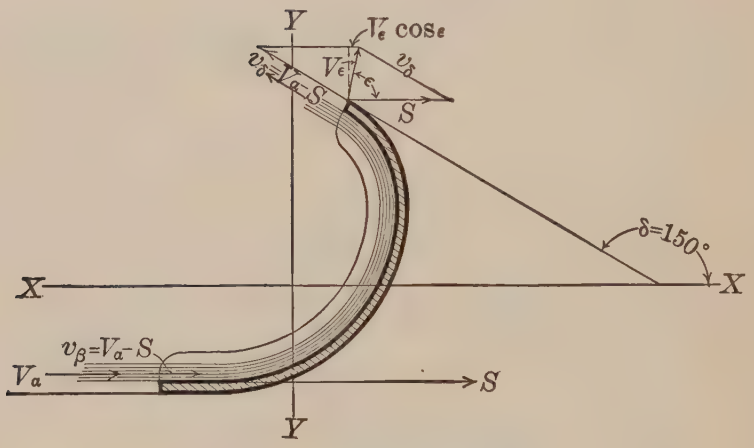


Fig. 102. — Curved Vane, Moving

368. Series of Moving Curved Vanes. Let us consider a series of curved vanes moving against a resistance with a velocity of (20, 0°), the other data being the same as § 366. (See Fig. 102.) Then we have, by (13),

$$P_X = \frac{wQ}{g}(V_\alpha \cos\alpha - V_\epsilon \cos\epsilon);$$

and since

$$Q = a_\alpha V_\alpha, \qquad V_\alpha \cos\alpha = 40 \cos 0°.$$

$$V_\epsilon \cos\epsilon = v_\delta \cos\delta + S = (V_\alpha - S)\cos 150° + S,$$

we may write

$$P_X = \frac{62.4 \times 0.5 \times 40(40 - 20)}{32.16}[1 - (-0.866)]$$

$$= +1448 = 1448 \quad \text{pounds, direction } 0°.$$

Moreover, by (15),

$$P_Y = \frac{wQ}{g}(V_\alpha \sin\alpha - V_\epsilon \sin\epsilon);$$

and since

$$V_\alpha \sin \alpha = 0,$$

$$V_\epsilon \sin \epsilon = (V_\alpha - S) \sin 150° = (40 - 20)(+ 0.5),$$

we have also

$$P_Y = \frac{62.4 \times 0.5 \times 40}{32.16} (20 \times - 0.5) = - 388$$

$$= 388 \quad \text{pounds, direction } 270°.$$

It follows that

$$E_X = P_X S = 1448 \times 20 = 28{,}960 \text{ foot pounds per second.}$$

Since movement of the vanes is only in the X direction, E_Y and E_Z are zero. Therefore total $E = E_X$.

Maximum efficiency for any series of vanes where the points of entrance and exit are moving in the same direction occurs theoretically (equation 27) when $S = V_a/2$. Therefore, at maximum efficiency the maximum power E_M is

$$(29) \qquad E_M = \frac{w a_\alpha V_\alpha}{g}\left(V_\alpha - \frac{V_\alpha}{2}\right) \times \frac{V_\alpha}{2} (\cos \alpha - \cos \epsilon).$$

Hence, when $\alpha = 0°$, and $\epsilon = 180°$, $(\cos \alpha - \cos \epsilon) = 1 - (- 1) = 2$, the maximum power E_M is

$$E_M = \frac{w a_\alpha V_\alpha}{g}\left(\frac{V_\alpha^2}{4}\right) \times 2 = \frac{wQ}{2g} V_\alpha^2,$$

which is equal to the total kinetic energy available in the jet per second.

369. The Runner of a Water Wheel. The runner of a water wheel is a series of vanes which are fixed to a shaft set in bearings and are moved against the resistance of the load on the water-wheel shaft. In this case S is the linear speed of a point on the rim of the wheel and the component P_X is the force tending to rotate the wheel, both of these being perpendicular to a radius drawn to the point under consideration. The P_Y component will ordinarily do no work, but will merely create stresses in the water wheel.

In some types of wheels the water enters the bucket at a different radius r_0 from that at which it leaves the bucket r_1 so that the rim speed at these two points S_0 and S_1 will also vary. In § 349 the fundamental equation for energy was derived on the basis of a linear reaction from the change in linear momentum.

(See equations (19) and (20).) If r_0 and r_1 vary, we may derive this equation from a consideration of angular momentum and angular impulse.

370. Angular Momentum. The *angular momentum* relative to a fixed point of a body having a motion of translation is found by multiplying its momentum by the perpendicular distance from the fixed point to the line of motion of the body's center of gravity at any instant.

371. Angular Impulse. The *angular impulse* is the product of a force (F or P) by the perpendicular distance (r) from a fixed point to the line of action of the force, and by the time during which it acts (t).

372. Variation in Angular Momentum. The variation in angular momentum in a given time is equal to the angular impulse producing that variation, that is,

$$Fr\,\partial t = \frac{W}{g}\,\partial v\,\partial r. \tag{30}$$

Applied to a stream of water of Q c.f. per second this becomes

$$Fr\,\partial t = \frac{wQ\partial t}{g}\,\partial v\,\partial r. \tag{31}$$

373. The Couple Acting. If wQ pounds of water per second undergo a change in angular momentum in passing through a wheel, the angular impulse becomes the couple acting on the wheel; and we may write

$$Pr = \frac{wQ}{g}(V_0r_0 - V_1r_1). \tag{32}$$

374. The Work Done. If the body is moving against a constant resistance with an angular velocity (ω radians per second), the work done in one second, E is

$$E = P\omega r = \frac{wQ}{g}(V_0\omega r_0 - V_1\omega r_1). \tag{33}$$

The equation (33) is the fundamental equation for power exerted (work done per second) on moving vanes in terms of the changes in velocity and direction of the water flowing on it. Perhaps a more satisfactory analysis giving a clearer idea of what actually takes place is to derive the equation of work by the aid of

Bernoulli's theorem as is done in the later consideration of reaction wheels.

375. Impulse or Free-Jet Wheels. The runner of a tangential impulse or free-jet wheel is a series of buckets around the rim of a wheel of such a diameter that in effect the buckets are a series of curved vanes moving in a straight line with a direction parallel with a jet from a nozzle. The water may be considered as entering and leaving the bucket at points where the radius is the same. This then becomes practically the same problem as the

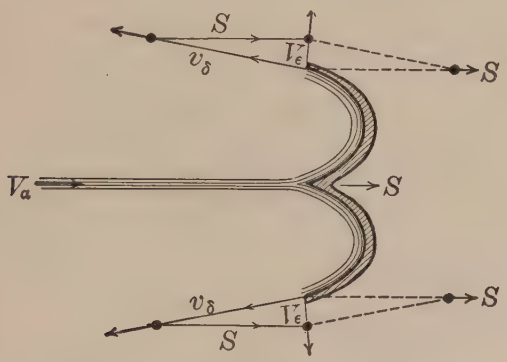


FIG. 103. — BUCKET OF IMPULSE WHEEL

series of curved vanes moving against a resistance. As in § 368, neglecting losses, we may obtain the theoretical power output as follows:

$$E = P_X S = \frac{wQ}{g}(V_\alpha \cos\alpha - V_\epsilon \cos\epsilon)S.$$

Since we have also

$$Q = a_\alpha V_\alpha, \quad \alpha = 0°, \quad V_\alpha \cos\alpha = V_\alpha,$$

and

$$V_\epsilon \cos\epsilon = v_\delta \cos\delta + S = (V_\alpha - S)\cos\delta + S,$$

we find

$$E = \frac{wQ}{g}(V_\alpha - S)(1 - \cos\delta)\,S. \tag{34}$$

It will be noticed that the larger the angle δ becomes the greater will be the power obtained from the wheel. Actually, this cannot be made much more than 160° to 170° to allow the water discharging from one bucket to clear the succeeding bucket.

376. Losses in an Impulse Wheel. In addition to the point just mentioned there are certain losses as the water passes through the nozzle and the buckets which prevent us from obtaining the full theoretical power of the jet. These losses, omitting those chargeable to the pipe line, are the nozzle losses and the bucket losses.

The nozzle losses will be (see § 163)

$$\frac{V_\alpha^2}{2g}\left(\frac{1}{C_v^2} - 1\right) = \frac{V_\alpha^2}{C_v^2\, 2g} - \frac{V_\alpha^2}{2g},$$

where C_v = the velocity coefficient of the nozzle, and the actual velocity V_α on the wheel will be

$$V_\alpha = C_v\sqrt{2gH},$$

where H is the total head on the nozzle.

If the bucket losses, determined actually by experiment, are assumed to be similar to orifice losses, the total head at the entrance and at the exit of the bucket may be found as follows, by Bernoulli's theorem,

$$\frac{p_\alpha}{w} + \frac{v_\beta^2}{2g} = \frac{p_\alpha}{w} + \frac{v_\delta^2}{2g} + \text{bucket losses},$$

$$\frac{v_\beta^2}{2g} = \frac{v_\delta^2}{2g} + \frac{v_\delta^2}{2g}\left(\frac{1}{C_1^2} - 1\right).$$

We have also

$$v_\delta = C_1 v_\beta = C_1(V_\alpha - S);$$

and for the work done per second, or power output, by (34),

$$E = \frac{wQ}{g}(V_\alpha - S)(1 - C_1\cos\delta)S. \tag{35}$$

The full theoretical power available due to the head H on the nozzle, that is, the input, is

$$wQH; \tag{36}$$

hence the efficiency, by (35) and (36), is

$$K = \frac{(V_\alpha - S)(1 - C_1\cos\delta)S}{gH}. \tag{37}$$

As in § 368, the maximum theoretical efficiency occurs when $S = V_a/2$, and the maximum work E_M then obtainable, is

$$E_M = \frac{wQ}{4g} V_a^2 (1 - C_1 \cos \delta). \tag{38}$$

377. Reaction Wheels. In the impulse or free-jet wheel considered above, the effective head acting on the wheel is all in the form of kinetic energy; *i.e.*, the head is all converted into velocity before the water enters the buckets, and the whole conversion into mechanical energy takes place in open channels

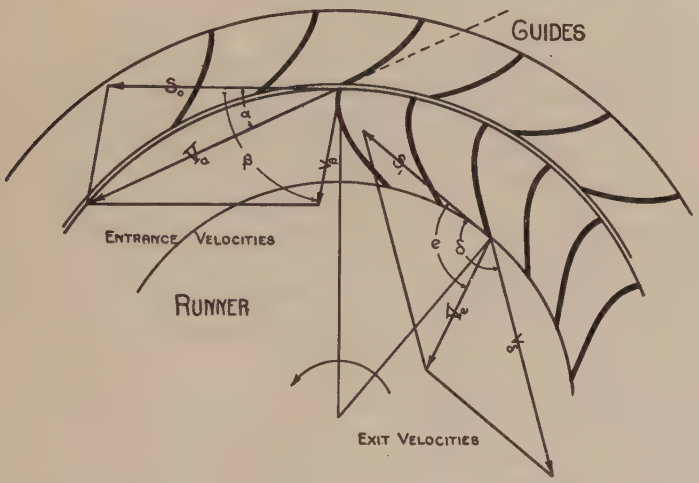


FIG. 104. — GUIDES AND BUCKETS OF A REACTION WHEEL, SHOWING VELOCITIES

under atmospheric pressure. The water is assumed to leave the wheel at the same distance from the center of the wheel as it enters.

In the case of the reaction wheel or turbine, energy delivered to the buckets is only in part kinetic energy in the form of velocity. The remainder is potential energy in the form of pressure, which requires a closed passage for the reaction to take place. Moreover, the water usually leaves the wheel at a different radius from that at which it enters, for example, at a point where the linear speed of the bucket S_0 is different from S_1. On the following pages we shall derive the general theory of a reaction wheel. The example used is an inward-flow type.

378. Assumptions made in Deriving Formulas. The equations which follow are based on the following conditions, which are most nearly fulfilled when the wheel is moving at an efficient speed and gate-opening.

(*a*) That the wheel is working against a steady resistance or load just sufficient to retain the assumed speed.

(*b*) That the water enters the wheel without violent changes in velocity.

(*c*) That all guides and buckets are sealed and are running full, so that

$$Q = a_\alpha V_\alpha = a_\beta v_\beta = a_\delta v_\delta.$$

(*d*) That the pressure at the exit from the bucket is atmospheric, a condition obtained if there is no draft tube.

(*e*) That the changes in velocity or direction in the interior of all channels are gradual, and that all surfaces in contact with flowing water are smooth.

On these assumptions, referring to Fig. 105, the velocity at the entrance to the bucket, V_α, is the resultant of v_β and S_0; and by trigonometry

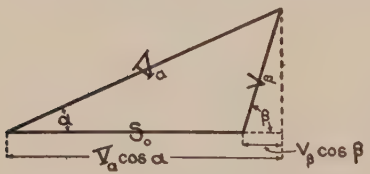


FIG. 105. — ENTRANCE VELOCITIES (from fig. 104)

(39) $$V_\alpha^2 = v_\beta^2 + S_0^2 + 2\, v_\beta S_0 \cos \beta,$$

and

(40) $$V_\alpha \cos \alpha = v_\beta \cos \beta + S_0;$$

and, at the exit to the bucket (Fig. 106),

(41) $$V_\epsilon^2 = v_\delta^2 + S_1^2 + 2\, v_\delta S_1 \cos \delta$$

and

(42) $$V_\epsilon \cos \epsilon = v_\delta \cos \delta + S_1.$$

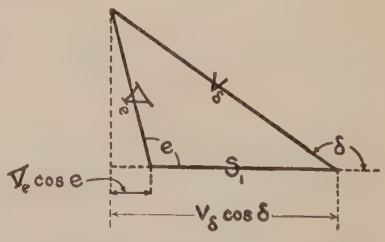


FIG. 106. — EXIT VELOCITIES (from fig. 104)

379. General Formulas for Reaction Turbine neglecting Losses. By the application of Bernoulli's theorem, general equations for the flow through and the work done in any wheel due to changes in velocity and direction may be derived as follows. Assuming that the water enters and leaves the wheel at the

same elevation (Fig. 104), and neglecting friction and other losses we obtain the following values.

The *power in the water entering the wheel*, in foot pounds per second, if measured at the exit of the guide orifices, is

$$wQ\left(\frac{p_\alpha}{w}+\frac{p_\beta}{w}+\frac{V_\alpha^2}{2g}\right). \tag{43}$$

The *power carried away*,, in foot pounds per second, by the water from the exit orifices at the bucket is

$$wQ\left(\frac{p_\alpha}{w}+\frac{V_\epsilon^2}{2g}\right). \tag{44}$$

Then E, the difference between the power in the water entering the wheel and the power carried away by the leaving water, is the *power delivered to the wheel;* its value is

$$E = wQ\left(\frac{p_\beta}{w}+\frac{V_\alpha^2}{2g}-\frac{V_\epsilon^2}{2g}\right). \tag{45}$$

These values are difficult to determine in any given case. Their equivalents may be determined as follows. Dividing both sides of (39) by $2g$, we find

$$\frac{V_\alpha^2}{2g}=\frac{v_\beta^2}{2g}+\frac{S_0^2}{2g}+\frac{v_\beta S_0\cos\beta}{2g}. \tag{46}$$

If there are no hydraulic losses in the buckets, the total energy at their entrance must equal the total energy at their exit. Hence, by Bernoulli's theorem, adding to the energy at the exit the loss in energy of rotation as the water goes through the buckets, we may write

$$wQ\left(\frac{p_\alpha}{w}+\frac{p_\beta}{w}+\frac{v_\beta^2}{2g}\right)=wQ\left(\frac{p_\alpha}{w}+\frac{v_\delta^2}{2g}+\frac{S_0^2}{2g}-\frac{S_1^2}{2g}\right), \tag{47}$$

and

$$\frac{p_\beta}{w}=\frac{v_\delta^2}{2g}-\frac{v_\beta^2}{2g}+\frac{S_0^2}{2g}-\frac{S_1^2}{2g}. \tag{48}$$

The absolute exit-velocity V_ϵ is the resultant of v_δ and S_1. Dividing both sides of (41) by $2g$, we have

$$\frac{V_\epsilon^2}{2g}=\frac{v_\delta^2}{2g}+\frac{S_1^2}{2g}+\frac{2v_\delta S_1\cos\delta}{2g}. \tag{49}$$

Inserting in (45), which gives the power delivered to the wheel, the values found in three equations (46), (48), and (49), we find

$$E = \frac{wQ}{2g}(v_\beta^2 + S_0^2 + 2\,v_\beta S_0 \cos\beta + v_\delta^2 - v_\beta^2 + S_0^2 - S_1^2 - v_\delta^2 - S_1^2 - 2\,v_\delta S_1 \cos\delta),$$

Simplifying this, we have

$$E = \frac{wQ}{g}[(v_\beta \cos\beta + S_0)S_0 - (v_\delta \cos\delta + S_1)S_1],$$

or, since we know, by (40), that

$$v_\beta \cos\beta + S_0 = V_\alpha \cos\alpha,$$

we may write

$$E = \frac{wQ}{g}[V_\alpha \cos\alpha\, S_0 - (v_\delta \cos\delta + S_1)S_1]. \tag{50}$$

This is a general expression for the power delivered to the wheel in terms of quantity of water, the absolute entrance velocity, the relative exit velocity, and the wheel speed. If we know the discharge and wheel dimensions, these are items usually most easily determined in any given case.

This equation may be related to the general equation for work done derived in § 374 by means of the principle of angular impulse and momentum as follows. By (42), we have

$$v_\delta \cos\delta + S_1 = V_\epsilon \cos\epsilon,$$

and

$$E = \frac{wQ}{g}(V_\alpha \cos\alpha\, S_0 - V_\epsilon \cos\epsilon\, S_1). \tag{51}$$

But since we know, by definition, that

$$S_0 = \omega r_0,\ S_1 = \omega r_1,\ V_\alpha \cos\alpha = V_0 \text{ and } V_\epsilon \cos\epsilon = V_1,$$

we find

$$E = \frac{wQ}{g}(V_0 \omega r_0 - V_1 \omega r_1).$$

It will be noted that the equations for the impulse wheel are merely special cases of the more general expressions just derived.

380. Reaction Coefficient for a Reaction Wheel. A certain proportion of the total head acting on a turbine is used in producing the velocity (V_α) with which the water enters the runner,

The ratio between this head and the total effective head acting on the wheel is called the *reaction coefficient* k. Its value varies from about 0.4 to 0.6 depending upon the type of wheel. If the reaction coefficient becomes 1.0, the pressure p_β becomes atmospheric. In that case, the wheel ceases to be a reaction turbine and becomes an impulse wheel. The value of the reaction coefficient k is

$$\frac{V_\alpha^2}{2\,gC_v^2H}.$$

In designing wheels the reaction coefficient is assumed to suit the requirements.

381. Inward-Flow Reaction Turbine including Losses. Let us suppose that we know the dimensions and the angles of the guides and buckets, the speed, the head H, the discharge Q, the reaction coefficient k, and the coefficients of velocity in the guides and buckets C_v and C_1 respectively. By (50), we have

$$E = \frac{wQ}{g}[V_\alpha \cos\alpha\, S_0 - (v_\delta \cos\delta + S_1)S_1].$$

We may give values to the terms as follows:

$$V_\alpha = C_v\sqrt{2\,g}\,(kH)^{1/2}. \tag{52}$$

The orifice losses in the guides will be

$$\frac{V_\alpha^2}{2\,g}\left(\frac{1}{C_v^2} - 1\right).$$

There are also losses in the clearance, which may be assumed as $mV_\alpha^2/(2\,g)$, m being coefficient which depends on the wheel design. Then we shall have, allowing for losses,

$$\frac{p_\beta}{w} = H - \frac{V_\alpha^2}{2\,g} - \frac{V_\alpha^2}{2\,g}\left(\frac{1}{C_v^2} - 1\right) - m\,\frac{V_\alpha^2}{2\,g}. \tag{53}$$

By Bernoulli's theorem and equation (47) allowing for losses in the bucket of

$$\frac{v_\delta^2}{2\,g}\left(\frac{1}{C_1^2} - 1\right)$$

the fall h_1 through the wheel, if any, and the change in energy of

rotation, the total energy at the entrance and at the exit of the buckets will be equal; hence we may write

$$(54)\quad wQ\left(\frac{p_\beta}{w}+\frac{v_\beta^2}{2g}+h_1\right)=wQ\left[\frac{v_\delta^2}{2g}+\frac{v_\delta^2}{2g}\left(\frac{1}{C_1^2}-1\right)+\frac{S_0^2}{2g}-\frac{S_1^2}{2g}\right],$$

and

$$(55)\qquad v_\delta=C_1\left[v_\beta^2+S_1^2-S_0^2+2g\left(\frac{p_\beta}{w}+h_1\right)\right]^{1/2}.$$

The values of V_α and v_δ from (52) and (55) corrected to include losses may be substituted in equation (50) for the power delivered to the wheel. From this, the efficiency may be found, and its value turns out to be

$$K=\frac{\frac{wQ}{g}\left[V_\alpha S_0\cos\alpha-(v_\delta\cos\delta+S_1)S_1\right]}{wQH},$$

or

$$K=\frac{V_\alpha S_0\cos\alpha-(v_\delta\cos\delta+S_1)S_1}{gH}.$$

382. American-Type Runner. The example given above is for an inward flow reaction wheel. In the modern American-type runner, the water enters the runner in a horizontal direction very much as in the example given, so that the parallelogram of velocities at the entrance is in a horizontal plane. In going through the runner, however, the water is turned and the discharge is in a nearly vertical direction, the wheel being of an inward and downward or *mixed-flow* type. There is therefore some fall h_1 through the wheel, and the parallelogram of velocities at the exit is in a nearly vertical plane.

In a modern wheel with a draft tube, the pressure at the exit of the buckets may be less than atmospheric, in which case p_β in the preceding discussion will be in effect the difference between the pressures at the entrance and at the exit of the buckets. The draft tube allows a higher absolute exit velocity V_ϵ, as described in the next chapter.

383. Water-Hammer. If water flowing in a pipe is stopped, the resulting surge causes a pressure rise, the magnitude of which depends upon the length of the pipe, the velocity of the water, and the extent of time over which the stoppage of flow occurs. If this time is short, the pressure rise is considerable, usually

resulting in the thudding pound known as water-hammer, if not in more serious damage.

If we have a mass of water m, moving at a velocity v, the kinetic energy to be absorbed in arresting its movement is

$$E = \tfrac{1}{2} mv^2,$$

whether the time of absorbtion is one second or one week. If the mass be brought to rest with a uniform retardation in the time t, the distances through which the arresting force will have to be exerted is

$$s = \tfrac{1}{2} vt.$$

The force or pressure F, which arrests the motion, must do work of the amount

$$E = \tfrac{1}{2} vtF.$$

The work of retardation must obviously be equal to the kinetic energy of the mass to be arrested in movement, that is,

$$E = \tfrac{1}{2} vtF = \tfrac{1}{2} mv^2,$$

whence we find

$$F = \frac{mv}{t}.$$

This value is the average force required during the time t. If the pressure is assumed to rise uniformly, the maximum will be twice the average, or $2F$; then the maximum force F_M will be

$$F_M = \frac{2\,mv}{t}.$$

Consider a column whose cross section is one square inch, and whose length is L feet. Taking the weight as 0.0361 lbs. per cubic inch, its mass is

$$m = \frac{0.0361 \times 12\,L}{g}.$$

Then

$$F_M = 2\,\frac{0.0361 \times 12\,L \times v}{gt},$$

in pounds per square inch. Changing pounds per square inch to feet head, we have

$$h = \frac{2 \times 0.0361 \times 12 \times 2.31\,Lv}{gt} = \frac{2\,Lv}{gt},$$

where h is the additional head over and above the normal head resulting from the shutdown.

This expression does not hold when the time becomes very short, since the compressibility of the water and the expansion of the pipe under the resulting pressure tend to reduce the amount of the pressure rise.

NOTE. The major part of the preceding discussion is taken from a pamphlet issued by the Worthington Pump and Machinery Corporation.

Professor Joukovsky gave a formula for water-hammer applicable to cases of practically instantaneous shutdown.[1] If we write

$$h = \frac{av}{g},$$

where a denotes the velocity of translation of the wave of pressure through the pipe, the value of a is given by the formula

$$a = \frac{12}{\left[\frac{62.4}{g}\left(\frac{1}{k} + \frac{d}{Ee}\right)\right]^{1/2}},$$

where k = voluminal modulus of water in pounds per square inch
= 294,000.

d = diameter of pipe in inches.

e = thickness of pipe wall in inches.

E = modulus of elasticity of material forming the pipe walls.

The problem of water-hammer and surge in pipe lines is probably the most complicated one met with in hydraulics, and can only be touched upon very lightly in a book of this nature.

It is now thought that the amount of rise in pressure resulting from a complete or partial stoppage of flow is not independent of the initial normal head, as was formerly believed. Various formulas are offered to cover all cases, but they are extremely complicated and cumbersome.

The two formulas given above are sufficient for simple problems. For the more complicated ones which arise in the study of water-wheel regulation, recourse must be had to the formulas

[1] For a résumé of Joukowsky's experiments, see *Proc. Amer. Waterworks Assn.* for 1904.

of Alliévi, Johnson, and Gibson.[1] Gibson's formulas largely are based upon Joukovsky's theory, and are amplifications of his formula.

PROBLEMS

1. A pipe bent in the form of a semicircle having a radius of 10 feet discharges 5 cubic feet of water per second with a velocity of 25 feet per second. The pipe is suspended by two strings so that the two ends are in the same horizontal plane and water is supplied in a vertical direction and without shock. What is the tension in the strings due only to the change in direction of the water?

2. Water is flowing at a velocity of 10 feet per second through a pipe 2 feet in diameter, which is bent through a central angle of 120 degrees. Compute the force ex rted parallel to either tangent and the force exerted perpendicular to either tangent, by the change in direction of the water.

3. A stream with a transverse section of 6 square inches delivers 2 cubic feet per second against a single flat vane, in a direction normal to its surface. Find the force acting on the vane, if the water after striking glides over it parallel to the surface:

(*a*) If the vane is stationary.

(*b*) If the vane is moving at a velocity that will give the maximum work.

(*c*) Compute the work done on the vane in (*b*).

4. A nozzle $1\frac{1}{2}$ inches in diameter, of which the coefficient of discharge is 0.98, delivers water under an effective head of 169 feet, against a series of flat vanes fixed on the circumference of a wheel at a point 10 feet from the center of rotation. (*a*) Determine the velocity of the vanes that will give maximum efficiency. (*b*) What will be the angular velocity of the wheel?

5. A street sprinkler is running on the tracks of a street railway at the rate of 15 miles per hour. From the front 500 streams issue, each $\frac{1}{8}$ inch effective diameter. What is the retarding force in pounds when the head upon the openings has an average of 7 feet? Neglect friction. Solve (*a*) when streams flow parallel with track; (*b*) when openings make a complete semicircle.

6. The jet from a nozzle strikes a series of curved vanes in a direction tangent to the rim of the wheel on which they are set, and at a point 1.5 feet from the axle. The lips of the vanes where the water enters are parallel to the jet and are bent through 165° from this line. The velocity of the jet is 172 feet per second and the wheel is revolving at the rate of 548 revolutions per minute. $Q = 3.75$ cubic feet per second.

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[1] The following discussions of water-hammer should be examined:

M. M. Warren, *Penstock and surge-tank problems*, TRANSACTIONS A. S. C. E., vol. 79, 1915.

H. C. Vensano, *Pulsations in pipe lines*, TRANSACTIONS A. S. C. E., vol. 82, 1918.

Norman R. Gibson, *Pressure in penstocks*, TRANSACTIONS A. S. C. E., vol. 83, 1919–20.

A. H. Gibson, *Hydraulics and its Applications*, Van Nostrand, 1923, pp. 222–244.

Compute (*a*) the constant force exerted on the vanes, and (*b*) the horse power developed.

7. Water discharging from a nozzle impinges on a wheel with an absolute velocity of 200 feet per second, and leaves the wheel with an absolute velocity of 10 feet per second. $Q = 10$ cubic feet per second. What horse power has been developed by the wheel?

8. A penstock of riveted steel 8 feet in diameter and 1000 feet long leads from an open reservoir to a turbine. Assume, that when the velocity is 7 feet per second and the pressure at the turbine is 20 pounds per square inch, the gate at the lower end is instantly closed.

(*a*) Compute the pressure due to water-hammer, and the necessary thickness of the pipe.

(*b*) If the pipe were 10,000 feet long, make a similar computation.

(*c*) If the time of closure in each case is 3 seconds.

CHAPTER XVIII

WATER WHEELS AND PUMPS

384. Water Wheels. The power of falling water may be utilized by means of a water wheel. Waterwheels are of many kinds and types, but they are all fundamentally the same in that water is supplied to them either under pressure or at high velocity and is discharged at a lower pressure or velocity. Modern water wheels are of two general types, reaction turbines and impulse wheels. The first operates largely through the reactive pressure of the water, while the impulse wheel is driven by a free jet striking one or more of the buckets simultaneously.

385. The Development of the Water Wheel. The earliest form of water mill was the *Norse Mill*. It occasionally is still found in use today in remote hilly districts in Europe. The wheel consisted of a vertical wooden shaft to the top of which a mill stone was fixed, the whole being carried by a step-bearing. Four or more flat wooden blades were fastened to the lower end of the shaft, and a jet of water was directed against them. These blades were later made curved or spoon-shaped.

The next development was the *undershot wheel* (Fig. 107), which was merely the wheel of the Norse Mill turned over on its side with the jet of water striking its blades, or *floats*, horizontally and underneath. A development of this type, the saw mill *flutter wheel*, was much used in the United States during the last two centuries. It was of very small diameter, giving the high wheel speed necessary to drive a saw.

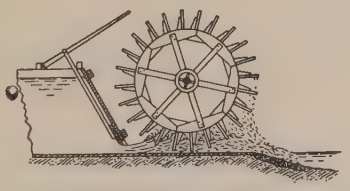


FIG. 107. — UNDERSHOT WHEEL

Both of these types required that the water should be supplied to the wheel as a high-velocity jet impinging upon a flat blade. The maximum theoretical efficiency of such a combination is only fifty per cent, and these wheels realized far less.

A great improvement changed the flat floats into buckets, enclosed them on the ends, and supplied the water to the wheel

at its highest point at a very low velocity (Fig. 108). The *overshot wheel* operates almost entirely by gravity and its theoretical efficiency is nearly one hundred per cent. Modern overshot wheels have shown test efficiencies of over ninety per cent. The overshot wheel is a very excellent wheel, and it is still in use in many places. It is still perhaps the best and cheapest wheel for very small developments where the head is nearly constant and the winter is not severe enough to cause trouble from ice. Some overshot wheels of very large size have been built; the Burden wheel at Troy, N. Y., now in ruins, was sixty feet in diameter with buckets twenty feet long.

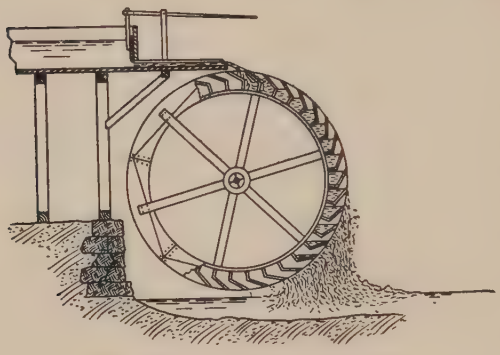


FIG. 108. — OVERSHOT WHEEL

The *breast wheel* is similar to the overshot wheel, except that the water is admitted to the wheel at a point varying between half to two-thirds of the height of the wheel. It was used in the large New England factories before the introduction of the turbine.

The *tub wheel* (Fig. 109) was an improvement upon the wheel of the Norse Mill. It was made in many forms, but generally the blades were slightly curved, inclined at about 30–45° with the horizontal, and surrounded by a ring of wooden staves or "tub." The water was delivered to the wheel from above through a spout, tangentially and downwards. It is from this wheel that the modern turbine was developed.

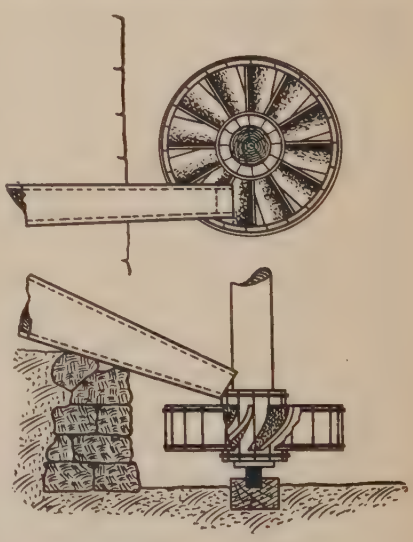


FIG. 109. — TUB WHEEL

Until about 1820, water power was developed by the use of one of these types of wheels. The beginnings of the factory system called for more improved forms of water wheels. The first modern turbine was built by Fourneyron in France. It was what was called an *outward-flow turbine.* The water was supplied to the wheel from the inside and discharged outward. The *Boyden wheel,* long used in New England, was of this type. Meanwhile another form of turbine was gradually developing in the little shops and mills of the northern United States, where ice conditions were severe. This was the *inward-flow wheel,* the forerunner of all modern reaction wheels. It first appeared as the *Howd-Francis wheel* in 1847.

During the years from about 1830 until 1876 a great many forms and types of small turbines were developed by inventors and millwrights throughout the country. The idea of the turbine had been evolved, but it was not yet perfected. The result of this half century of turbine development was the *Hercules wheel,* which appeared in 1876. All modern reaction wheels are a direct development of this type. The modern inward-flow reaction turbine is an American invention, and it was not until about 1880 that this type of water wheel was taken up in Europe.

Most of the development since 1876, aside from mechanical refinement, has been in the direction of increasing the power and speed for a given diameter of wheel. There has been a very great increase in the amount of maximum allowable head, so that reaction turbines are now used on heads as high as eight hundred feet.

386. The Modern Hydraulic Turbine. A water wheel consists of two main parts, the *runner* (Fig. 110), or rotating part, and the *wheel-case* (Fig. 111), to which one of the main bearings is attached. The runner varies considerably in form with different wheels. It consists of a *crown,* or *hub,* a number of *buckets,* or *blades,* and a *rim* surrounding the bottom of the buckets. The rim has been omitted in some very recent wheels. The casing surrounds the runner and contains around its periphery the *gate* or gates, which regulate and direct the flow to the runner. The water enters the casing on the outside, passes through the runner, and then discharges out of the bottom of the casing. Figures 110

and 111 show the parts for what is usually known as a Francis type wheel.

The gate is either a cylinder gate or a wicket gate, but the former is now used but slightly. The *cylinder gate* is a ring, like a section of pipe surrounding the runner, and moving vertically up and down. A *wicket gate* consists of a number of *guides*, placed vertically and pivoting in a horizontal plane, between which the entering water passes in a number of streams. The wicket gate is more complicated mechanically, but it is better hydraulically.

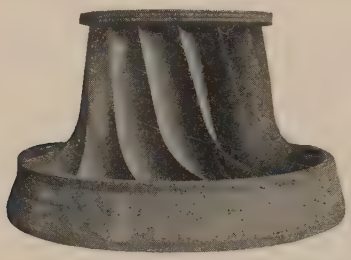


FIG. 110.—WATER WHEEL RUNNER
(Courtesy of the James Leffel Co.)

387. The Propeller-Type Turbine. A type of water wheel to which much attention has been directed recently is the *propeller-type turbine*, or *Nagler wheel* (Fig. 112). The casing of this wheel is the same as that of the ordinary reaction turbine, but the runner is shaped like a marine propeller. The angle of the blades is flat, giving a very high wheel-speed, but this is secured only by a sacrifice of efficiency, particularly at part gate. This decrease in efficiency, however, is not great, and it is often more than balanced by the increased speed. The efficiency of this type of wheel holds up well over a wide range of relative velocity, making it particularly suitable for low-head hydro-electric plants where the head is apt to vary considerably. This type of runner, moreover, is particularly rugged.

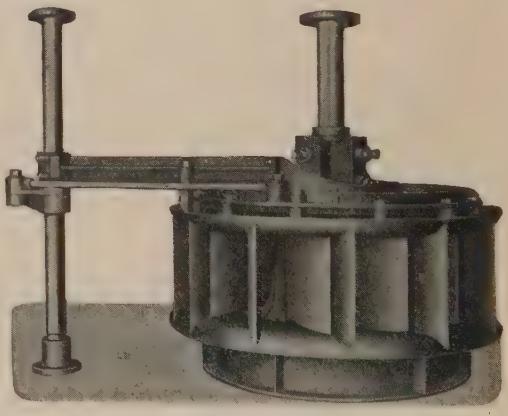


FIG. 111. — WATER WHEEL CASING
(Courtesy of the James Leffel Co.)

388. Efficiency. The modern water wheel under the best conditions has a very high efficiency, attaining a maximum of

perhaps 93%. So high an efficiency, however, is obtained only within very narrow limits, and a wheel which averages 85% over its usual operating range is considered to have a very good efficiency.

FIG. 112. — PROPELLER TYPE RUNNER
(Courtesy of the Allis-Chalmers Mfg. Co.)

Generally speaking, the efficiency of a water wheel decreases as the gate opening is decreased, but most wicket-gate wheels have their maximum efficiency at from seven-tenths to nine-tenths gate, and the efficiency falls off either way from that point.

To operate at its best efficiency, not only must a water wheel be run at a certain gate-opening, but its speed must have a definite

relation to the spouting velocity equivalent to the head acting on the wheel. This ratio between the peripheral speed of the wheel and the spouting velocity of the water is known as the *relative velocity*. This will vary with different wheels from perhaps 0.60 almost up to as high as 2.00, but this last value is reached only with the propeller type of runner.

389. Relation between Diameter, Head, Speed, Power, and Discharge. The rim velocity of a water wheel varies directly with the spouting velocity of the water, or, in other words, with the square root of the head acting. The horsepower of a given wheel varies with the three-halves power of the head. Water wheels are made in many different types, but each of these types is usually made in several sizes, and all the wheels of a type are or should be homologous. For homologous wheels under a given head, the speed varies inversely with the diameter of the wheel, and the horsepower and discharge directly with the diameter squared. Knowing these facts and the performance and characteristics of a number of types of water wheels of varying specific speeds, one can easily determine the type and necessary size of wheel to meet any condition.

EXAMPLE. A 30-inch wheel at 220 R.P.M., discharges 210 cubic feet per second under 20-foot head. What will a homologous 48-inch wheel discharge, and what will be its speed, when operating under the same head?

$$\frac{48^2}{30^2} \times 210 = \frac{2304}{900} \times 210 = 538 \qquad \text{cu. ft. per sec.}$$

$$\frac{30}{48} \times 220 = 137.5 \qquad \text{R.P.M.}$$

If the 30-inch wheel develops 395 H.P. under 20-foot head, what will the 48-inch wheel develop under 28-foot head?

$$395 \times \frac{48^2}{30^2} \times \frac{28^{3/2}}{20^{3/2}} = 395 \times \frac{2304}{900} \times \frac{148.2}{89.4} = 1677 \qquad \text{H.P.}$$

390. The Testing of Water Wheels. At the present time there is a great deal of testing of water wheels. At least one example of every type made has been carefully tested, so that all its characteristics are known. This is usually done in a special testing flume; but many tests are also made after the wheel has been installed.

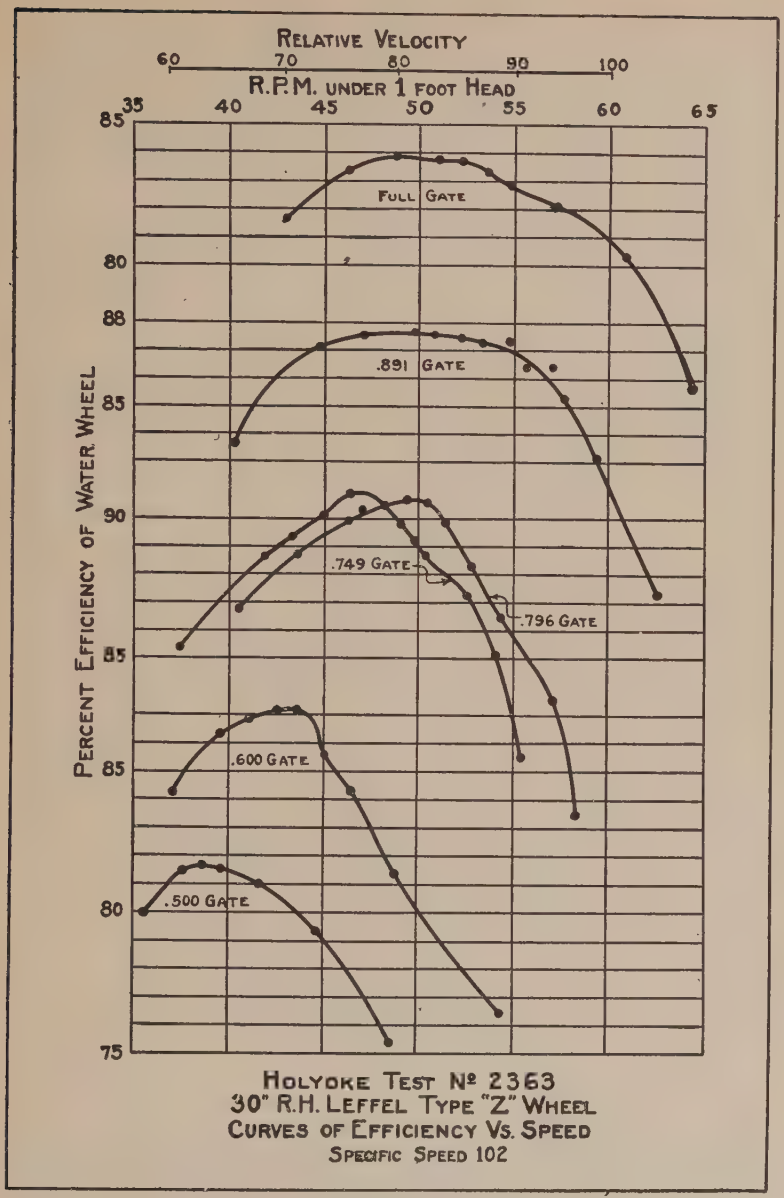

Relative Velocity
60 70 80 90 100
R.P.M. under 1 foot Head
35 40 45 50 55 60 65
85
80
88
85
90
85
85
80
75
Full Gate
.891 Gate
.749 Gate
.796 Gate
.600 Gate
.500 Gate
Percent Efficiency of Water Wheel
Holyoke Test № 2363
30" R.H. Leffel Type "Z" Wheel
Curves of Efficiency Vs. Speed
Specific Speed 102

Fig. 113

The water used by the wheel is passed over a weir and the power of the wheel is absorbed by a dynamometer or a Prony brake. The head on the wheel is read and the speed of the wheel recorded. The gate is set at a definite opening and a series of runs is made at varying speeds, and accordingly at varying relative velocities, since the head is kept nearly constant. The gate-opening is then changed and another series of runs is made. This is continued until the whole range of gate-openings has been covered. The results are all reduced to a common head and are tabulated. They also are usually plotted as a series of curves as shown in Fig. 113. Hydro-electric units require a constant wheel-speed regardless of head and gate-opening. A constant-speed curve for various gate-openings can be constructed by picking values off the test curves. The speed of the wheel and the head acting determine the value of the relative velocity for this condition of

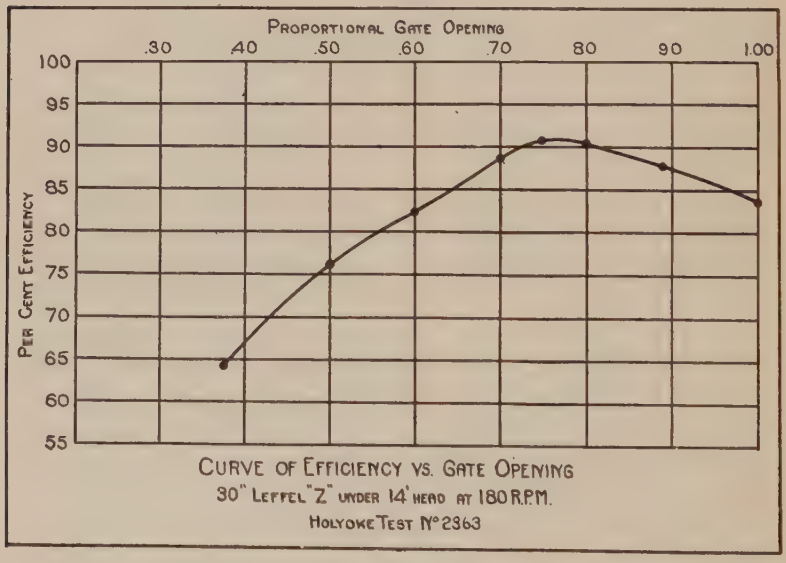


Fig. 114

operation (Fig. 114). If the wheel operates at a different head the relative velocity changes, and another similar curve can be drawn. The performance of a wheel under different heads and at different gate-openings can be shown by a family of curves. Figure 115 shows such a series of curves.

In this case they are discharge curves; but a similar series can be constructed showing efficiency or power. When a water wheel has been installed, it can be used as a water meter after a family of discharge curves has been constructed. The curves should be based preferably on actual measurements in place of both water

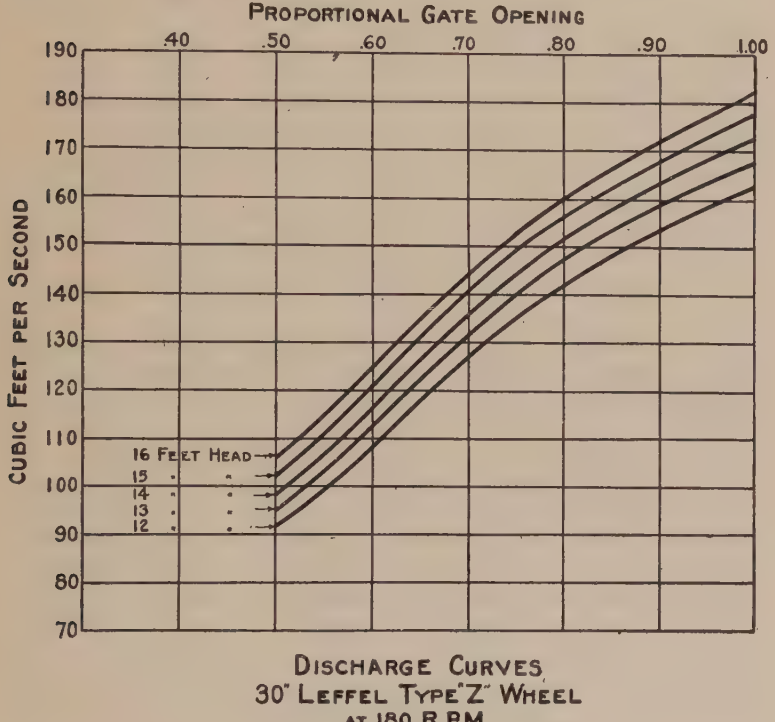


FIG. 115

and head, but good results may often be obtained by increasing or reducing the results of a test on a homologous wheel.

391. Maximum and Rated Diameters of Water Wheels. Modern wheels of medium and high specific speeds (see § 393) have a maximum diameter much larger than that at which they are rated. The rated diameter is usually measured at a diameter opposite the mid-height of the gate. Propeller wheels and high-head wheels of low specific speed are rated at their maximum diameter.

392. R. P. M. at One-Foot Head. There is a decided tendency at present to tabulate and plot wheel performances, not against relative velocity, but against the revolutions per minute that the wheel would make when running under one foot head. There is, of course a definite relation between the relative velocity and the R. P. M. under one-foot head; and, when the wheel diameter is known, they are interchangeable.

393. Specific Speed. The *specific speed* of a water wheel is a figure which expresses the relation between power and the speed as a numerical value. It is defined as *that speed at which a homologous wheel would revolve under one foot head when the size of this wheel is such as to develop one horsepower under one foot head,* or numerically,

$$N_s = \frac{RPM\sqrt{HP}}{h^{5/4}}.$$

The specific speed varies with the gate-opening and with the relative velocity, but it is customary to rate a wheel upon its specific speed at the point at which the maximum power is developed.

The higher the specific speed, the greater the power for a given size wheel. With hydro-electric units, high rotative speeds are desirable. To secure this when the head on the wheel is low a wheel of high specific speed must be used. On the other hand, under very high heads, wheels of low specific speed must be used in order to keep the speed low enough for direct connection to electric generators. The specific speed of modern reaction water wheels varies from 20 or less for high-head Francis runners to 200 or more for those of the propeller type.

394. The Draft Tube. The *draft tube* is a device whereby the water wheel may be set above tailwater without losing any of the available head. In its simplest form it is a cylindrical tube extending from the discharge opening of the wheel case down to the tailwater. Since the draft tube operates under negative pressure, the joint with the wheel case must be air-tight, and the lower end must be submerged below the tailwater surface. The total draft head includes not only the elevation of the top of the draft tube above tailwater, but also the velocity head of the water entering the top or throat of the draft tube. This total draft head must always be less than 34 feet.

For example, the discharge opening of the wheel case of a water wheel is placed 12 feet above tailwater, and under 25-foot head the discharge is 232 cubic feet per second through a draft tube with a throat 4.32 feet in diameter.

Elevation of top of draft tube above tailwater . . 12 ft.

Velocity head

$$\frac{232}{\pi\left(\frac{4.32}{2}\right)^2} = 15.83 \text{ ft./sec.}$$

$$\frac{15.83^2}{64.32} = 3.9 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \underline{3.9}$$

Total draft head 15.9 ft.

If the power development is at a high altitude, due allowance must be made for the fact. At an elevation 3,000 feet above sea level, the limiting value of the total draft head becomes about 30.6 feet. It is always desirable to keep the total draft head well within the allowable limit.

If a draft tube is of constant size throughout its length, the final discharge velocity will be the same as at the throat and will be wasted in turbulence. On the other hand, if the draft tube is made with diverging sides, the velocity of discharge will be converted into negative pressure-head on the under side of the runner, and the total head acting on the wheel will be increased as with the diverging tube described in § 165. At the present time, all draft tubes are made diverging, so that they act as velocity regainers as well as true draft tubes.

There are many special forms of modern draft tubes or diffusers, but the best draft tube is a straight-sided flaring tube with a divergence of 15 to 18 degrees and of length about three times the throat diameter. Wheels of high specific speed allow the use of a draft tube with even greater flare; a 24-degree tube has given good results with a wheel of a specific speed of about 200.

The final velocity head at the discharge end of the draft tube is lost, and it is advantageous, particularly with large units, to keep this at a minimum. For good results, especially with wheels of high specific speed, the draft tube must be symmetric about a vertical line for at least a major part of its length. If a straight flaring draft tube is extended far enough to secure the desired velocity of discharge (in low-head work usually from 4 to 6 feet per second), it will often reach an inconvenient length. To over-

come this difficulty, two forms of draft tube have been developed within the last few years which secure a greater amount of effective divergence in a given length. These are the *Hydraucone* [1] and the *Moody Spreading Draft Tube*.[2] While not quite as effective as a straight flaring tube of the requisite length, they allow the over-all length of the draft tube to be shortened very materially.

395. Pitting. A water wheel has a long life, if well made and correctly installed. The mechanism is simple and there is little to wear or break. Bearings, of course, wear loose, and the gate mechanism occasionally jams or breaks, but, generally speaking, a water wheel requires very few repairs. Trouble has been had with the runner "pitting" in certain installations. The metal becomes spongy at certain points on the buckets, and eventually a hole is eroded clear through. A recent report [3] on pitting of water wheels shows that it is due to the action of free oxygen at points on the wheel where the water breaks free from the bucket surface because of excessive vacuum at the under side of the runner. This is shown to be due to too high a total draft head. The critical total draft head above which pitting occurs is shown to be about 27 feet for a 30-inch runner and decreasing down to about 20 for a 150-inch runner.

396. Modern Hydro-Electric Stations. The discussion of power-station design is not within the scope of this book, but a very brief résumé of modern water-power practice is in order. Practically all water wheels are placed vertically and are directly connected to a generator. Until recently, a pair of wheels on a horizontal shaft was customary, but the best practice does not now countenance this except in the case of high-head units.

There is no need to mention the dam for creating the head, the canal, or penstock for conveying the water to the station and the tailrace for taking it off, but a few words must be said about the power station. The water wheel is placed either in an open or in a closed setting. The former is usually the best and simplest, but it can only be used for small units and low heads. The wheel is placed at the bottom of a box-like structure or flume, into which

[1] Allis-Chalmers Mfg. Co., Milwaukee, Wis.
[2] I. P. Morris Dept., Wm. Cramp & Sons, Philadelphia.
[3] N. E. L. A. Report in *Power*, July 1, 1924.

the water is directed. The upper surface of the water in the flume is exposed to the air. The closed setting is similar except that a water-tight cover is used and the water in the flume is under pressure. This form is very often modified into a scroll, particularly with large units. The draft tube and the wheel setting are generally molded in concrete and form the foundations and substructure of the power house (Fig. 116).

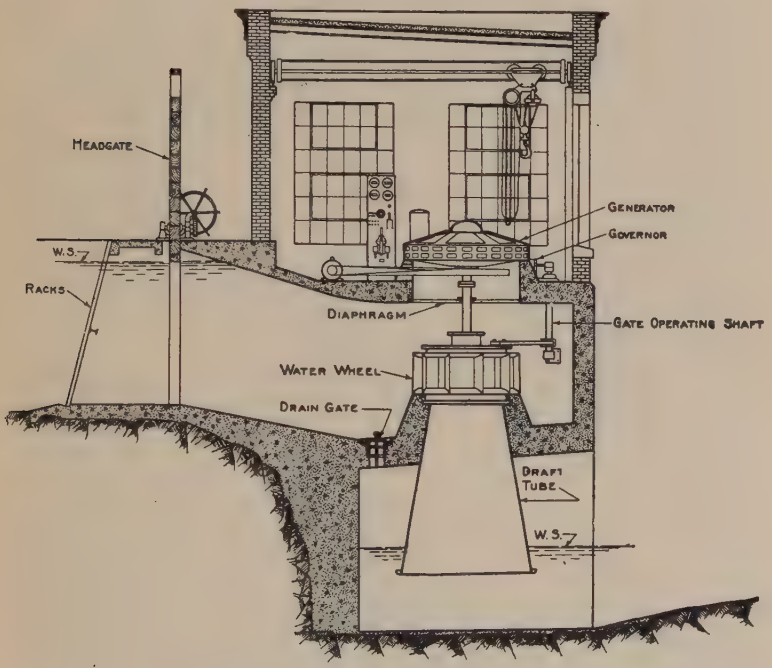


FIG. 116. — SMALL, LOW HEAD HYDRO-ELECTRIC STATION.

For electrical operation, the speed of the water wheel must be kept constant. This is done by a water-wheel governor, which consists primarily of a movable piston connected to the gate mechanism of the wheel. Oil is stored under pressure in a small tank and is admitted to one side or to the other side of the piston by a pilot valve operated by a rise or fall in the flyballs connected to the water-wheel shaft. The flow of water to the wheel is varied accordingly and the speed of the wheel is brought back

to normal, when the flyballs resume their normal position and the pilot valve closes.

The main basic principles of all hydraulic design apply particularly to power-station design. The water should be moved

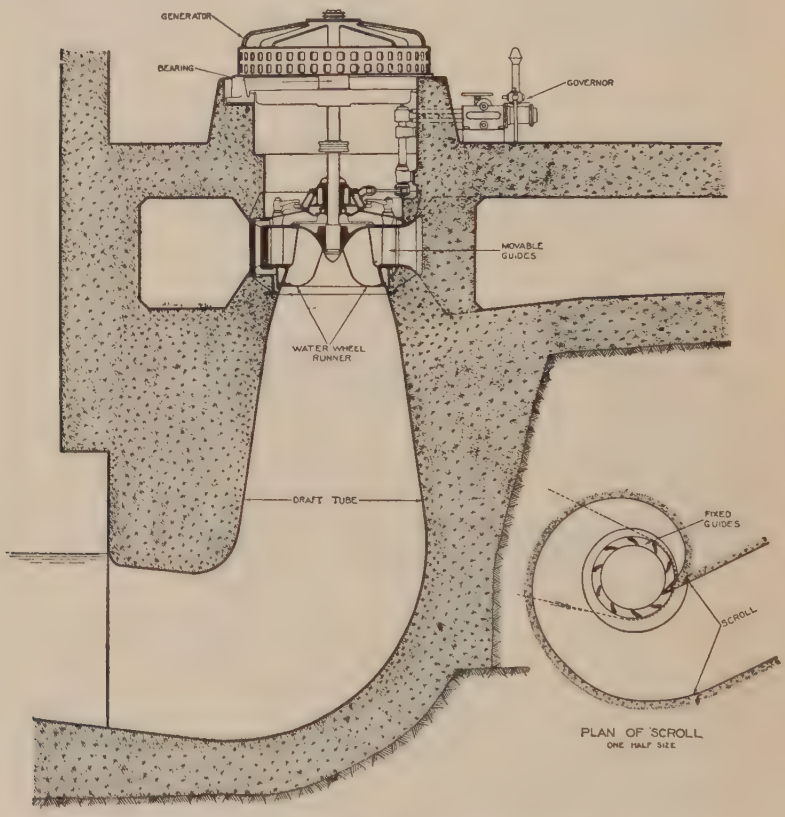


FIG. 117. — MEDIUM HEAD HYDRO-ELECTRIC UNIT

(Courtesy of the S. Morgan Smith Co.)

at the minimum allowable velocity through channels of maximum smoothness around smooth and easy curves, and all abrupt changes should be avoided.

Two recently built hydro-electric units are shown in Figures 117 and 118. The student should note the tendency to avoid all sharp corners and abrupt changes and to use smooth and easy curves and transitions.

397. Impulse Turbines. Hitherto most of the remarks have been confined to reaction turbines. *Impulse turbines* or *jet turbines* are also used, but only under comparatively high heads. The impulse wheel is a development of the undershot wheel, but

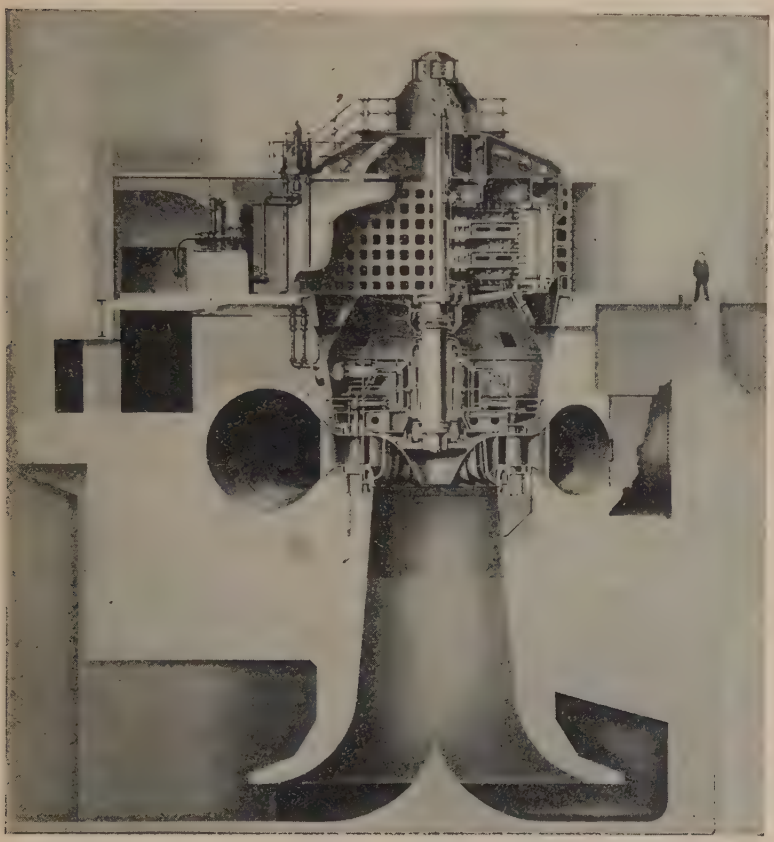


FIG. 118. — 70,000 H. P. UNIT AT NIAGARA FALLS

(Courtesy of the Allis-Chalmers Mfg. Co.)

the flat floats have been changed into curved buckets, so that efficiencies of impulse wheels are almost equal to those obtained with reaction turbines. One of the best known of the impulse wheels is the *Pelton*, which is much used on the Pacific Coast and in other places where very high heads are obtained. The present day increase in allowable heads for reaction turbines has largely

decreased the field of the impulse wheel for heads under a thousand feet.

The impulse wheel consists simply of a wheel with a number of spoon-shaped buckets of special form attached around the

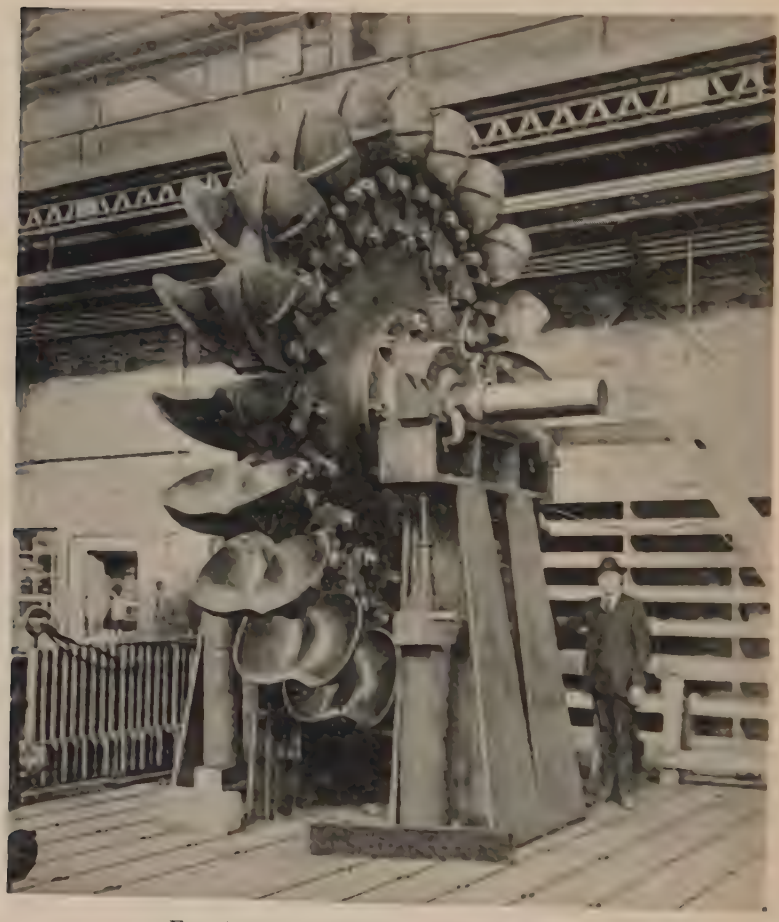


FIG. 119. — RUNNER OF IMPULSE WHEEL
(Courtesy of the Allis-Chalmers Mfg. Co.)

rim (Fig. 119). One or more jets, impinging upon the buckets, drive the wheel. The relative velocity of impulse wheels varies from about 0.40 to 0.50.

A special nozzle with a movable central needle is generally used (Fig. 120). The area of the jet can be regulated, while the velocity of the jet remains practically the same. Thus the power of the wheel may be varied without any change in the wheel speed.

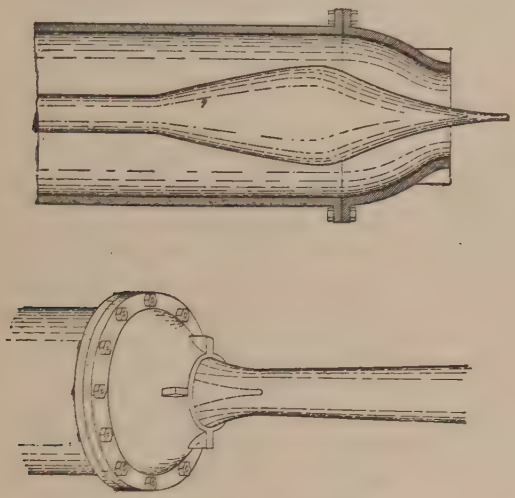


FIG. 120. — DOBLE NOZZLE

398. Pumps. Pumps are of two general types: positive displacement pumps and centrifugal pumps. All reciprocating pumps are of the first class. They consist of a piston moving up and down in a cylinder with the admission and discharge controlled by two sets of valves (Fig. 121). Rotary-gear pumps are also of the positive displacement type.

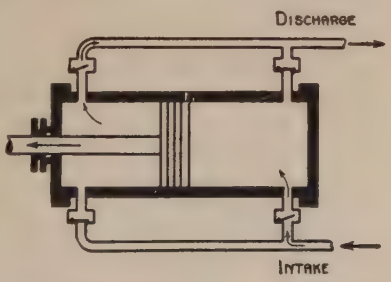


FIG. 121. — DOUBLE-ACTING PUMP

Centrifugal pumps have been known for a great many years, but they did not come into general use until the steam turbine and the electric motor became available to furnish the high speeds necessary for direct connection. Essentially, a *centrifugal pump* consists of a multi-bladed impeller revolving inside a casing, with the water admitted at the center of the case and discharged

through an opening at the periphery. The *turbine centrifugal pump* has a series of guides similar to water-wheel guide-vanes, around the impeller, which diffuse the water discharged from the impeller and convert the kinetic energy imparted to the water by the impeller into pressure. In the *volute centrifugal pump* (Fig. 122) the guides are omitted and the casing is made in a spiral form in order to accomplish the same end. The volute pump is used for the lower heads, while the turbine centrifugal pump is generally used for the higher.

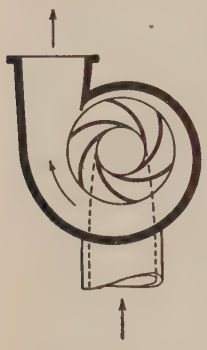


FIG. 122. — VOLUTE CENTRIFUGAL PUMP

Centrifugal pumps are made in all sizes up to as much as 16 feet in diameter, but such large ones as this are used only for very low heads. They are also made multi-stage with a number of impellers on a single shaft, the discharge from one being fed to the intake of the next. A single-stage pump is used only for heads up to 100–200 feet, but multi-stage pumps have been made to work against heads of 2,000 feet or more.

The centrifugal pump has a number of advantages over the piston pump. It is less complicated, having only one moving part. It is much smaller than a corresponding piston pump, except when the head is very high; and it is correspondingly cheaper. The efficiency of the two types is almost on a par; centrifugal pumps have attained efficiencies of 85 per cent.

Another advantage of the centrifugal type is that in case of a stoppage at some point on the discharge side of the pump the pressure will rise only to a certain point, when the pump will merely churn. On the other hand, with a positive displacement pump, the pressure would continue to rise until the power ceased to be sufficient, or more likely, until something burst. Moreover, the discharge from a centrifugal pump is uniform, and not a series of pulsations as in the piston type of pump.

For small quantities under very high heads, a positive displacement pump is smaller and cheaper than a centrifugal pump. Another advantage of the piston pump is that it can be started without priming; but, generally speaking, practically every need can now be met better by the use of a centrifugal pump than by the use of the older piston type.

PROBLEM

The following test was made at the Holyoke Testing Flume on a 30-inch water wheel. It is required to determine the size of an homologous wheel which will utilize 600 c.f.p.s. under a 25-foot head at full gate. What maximum power will it develop?

This wheel is to be directly connected to an alternating current generator. The available synchronous speeds in which the generator can be obtained are: 128.6, 138.5, 150, 163.6, 180, and 200 R.P.M. At what speed will the wheel be operated? Assuming an electrical efficiency of 92%, what power will the unit deliver at full gate? at $\frac{1}{2}$ gate? What is the overfall efficiency at each point?

If the unit runs at an average gate opening of 0.70 for 300 ten-hour days per year, and the total charges (interest, taxes, depreciation, and operation) are 15% on a total installation cost of $85,000, what is the cost of power production per kilowatt hour?

NOTE. This test is shown in Fig. 113 plotted as curves of efficiency against speed for the various gate openings. Determine the wheel size to the nearest inch only. Choose a generator speed such as will give a relative velocity at 25 feet head of as near 0.8 as possible. Remember that the generator speed and relative velocity of the wheel must remain constant for all gate openings.

HOLYOKE WATER POWER CO.

Report of tests of a 30″ Right Hand Leffel Type Z Turbine Wheel.
Test No. 2363 made October 7–8, 1914.

Swing Gate. Conical Draft-tube.

PROPORTIONAL GATE OPENING	HEAD IN FT.	R.P.M.	RELATIVE VELOCITY FT. PER SEC.	DISCHARGE c. f. p. s.	H.P.	EFFICIENCY
1.000	16.16	172.75	0.701	180.49	269.65	81.58
"	16.08	185.25	0.754	183.36	278.45	83.34
"	16.04	195.25	0.796	185.23	282.19	83.82
"	16.03	204.67	0.834	186.82	283.97	83.68
"	15.98	209.50	0.855	187.69	284.62	83.74
"	15.97	214.00	0.874	188.85	284.55	83.26
"	15.97	219.00	0.894	190.16	284.87	82.78
"	15.93	228.75	0.935	191.92	284.32	82.07
"	15.84	242.50	0.994	194.55	280.39	80.29
"	15.84	256.20	1.051	196.32	266.60	75.66
0.891	16.40	163.33	0.658	168.36	261.55	83.59
"	16.27	180.25	0.729	172.00	276.14	87.08
"	16.24	189.75	0.769	173.97	279.73	87.37
"	16.21	200.00	0.811	176.08	283.28	87.58
"	16.21	210.00	0.851	177.92	285.30	87.29
"	16.15	214.25	0.870	178.64	284.88	87.14
"	16.14	220.25	0.895	179.78	286.49	87.13
"	16.13	223.75	0.909	180.35	284.58	86.33
"	16.09	229.33	0.933	181.06	285.04	86.34
"	16.08	238.00	0.969	181.78	275.18	83.08
"	16.06	251.00	1.022	183.36	261.19	78.27
"	16.15	264.00	1.072	182.93	228.93	68.38
0.796	16.52	164.67	0.661	158.32	257.04	86.72
"	16.49	177.00	0.711	160.50	266.05	88.71
"	16.45	187.50	0.755	162.56	270.99	89.43
"	16.41	199.75	0.805	164.48	277.15	90.61
"	16.39	204.25	0.823	165.04	277.49	90.53
"	16.41	207.75	0.837	165.31	276.24	89.86
"	16.43	213.75	0.861	165.45	271.86	88.25
"	16.47	221.00	0.889	166.42	268.30	86.38
"	16.46	231.33	0.931	167.25	260.78	83.59
"	16.48	237.00	0.953	166.28	246.62	79.42
"	16.47	249.25	1.002	167.39	230.55	73.80
"	16.74	153.00	0.610	150.21	243.24	85.36
"	16.70	171.00	0.683	153.16	257.03	88.68
"	16.67	183.00	0.732	155.46	264.49	90.06
"	16.64	188.80	0.755	156.00	267.41	90.91
"	16.63	191.75	0.767	156.41	266.05	90.26
"	16.60	196.00	0.785	156.68	266.28	90.35
"	16.59	199.00	0.797	156.82	264.60	89.75

HOLYOKE WATER POWER CO. — *Continued*

PROPORTIONAL GATE OPENING	HEAD IN FT.	R.P.M.	RELATIVE VELOCITY FT. PER SEC.	DISCHARGE c. f. p. s.	H.P.	EFFICIENCY
0.749	16.61	202.25	0.810	156.68	263.08	89.21
"	16.62	206.25	0.826	157.09	262.32	88.66
"	16.63	214.25	0.858	158.18	260.11	87.26
"	16.66	220.20	0.881	158.45	254.60	85.11
"	16.65	225.75	0.903	157.23	241.44	81.39
"	16.65	236.60	0.946	158.18	232.53	77.91
0.700	16.90	148.00	0.588	142.90	231.01	84.41
"	16.86	166.00	0.660	145.41	234.72	88.09
"	16.85	177.50	0.706	147.40	251.41	89.33
"	16.83	183.75	0.731	147.93	254.95	90.37
"	16.85	187.25	0.745	148.46	254.40	89.74
"	16.83	194.00	0.772	148.46	252.35	89.12
"	16.81	201.50	0.802	149.27	250.45	88.08
"	16.80	210.67	0.839	149.13	243.58	85.80
"	16.81	231.75	0.923	150.47	227.76	79.46
0.600	17.18	153.33	0.604	126.91	208.31	84.31
"	17.15	164.00	0.646	128.17	215.22	86.40
"	17.11	170.67	0.673	128.80	217.07	86.92
"	17.12	175.75	0.693	129.06	218.45	87.25
"	17.13	180.50	0.712	129.44	219.13	87.21
"	17.17	186.00	0.733	129.18	215.06	85.56
"	17.17	192.50	0.758	128.93	211.45	84.29
"	17.18	202.00	0.795	129.06	204.36	81.34
"	17.18	223.00	0.878	130.08	193.38	76.36
0.500	17.48	149.00	0.582	108.83	172.28	79.92
"	17.51	157.25	0.613	109.67	177.29	81.46
"	17.46	161.67	0.631	109.79	177.58	81.75
"	17.51	166.00	0.647	109.79	177.54	81.50
"	17.51	174.67	0.681	109.91	176.71	81.03
"	17.50	187.00	0.730	109.79	172.97	79.45
"	17.50	203.00	0.792	109.91	164.30	75.38
"	17.50	223.33	0.871	111.24	154.93	70.23

NOTES. The turbine and dynamometer carried during test on ball-bearing. The proportional gates based on the measurement of gate.

With the flume empty, a force of 0.5 lb., applied at a distance of 3.0 ft. from the center of the shaft, sufficed to start the wheel.

HOLYOKE WATER POWER COMPANY
By A. F. Sickman, Hydraulic Engineer
W. C. Gaylord, Engineer in charge of Experiments

APPENDIX

TABLE XXVIII

H = head in feet of water = $1.1330\ H_g = 2.3077\ p$.
H_g = head in inches of mercury = $.88264\ H = 2.0369\ p$.
p = pressure in pounds per square inch = $.43333\ H = .49095\ H_g$.
V = velocity in feet per second due to a head of H feet = $8.02(H)^{\frac{1}{2}} = 8.5365(H_g)^{\frac{1}{2}} = 12.183\ (p)^{\frac{1}{2}}$.
Q = discharge in cubic feet per second of a stream having an actual area of one square inch and a mean velocity of V feet per second = $\frac{V}{144}$.
G = discharge in gallons per minute of a stream having an actual **area of one square inch** and a mean velocity of V feet per second = $\frac{V \times 1728 \times 60}{144 \times 231}$.

$(2\,g)^{\frac{1}{2}} = 8.02$. Weight of one cubic foot of water, 62.4 pounds. One gallon = 231 cubic inches. $g = 32.16$.

H	H_g	p	V	Q	G
.001600	.001412	.0006933	.3208	.002225	**1**
.01555	.01372	.006737	**1.000**	.006944	3.117
.06219	.05489	.02695	**2.000**	.01389	6.234
.1399	.1235	.06063	**3.000**	.02083	9.351
.2488	.2196	.1078	**4.000**	.02778	12.47
.3887	.3431	.1684	**5.000**	.03472	15.58
.5597	.4940	.2425	**6.000**	.04167	18.70
.7618	.6724	.3301	**7.000**	.04861	21.82
.9950	.8782	.4311	**8.000**	.05556	24.94
1.000	.8826	.4333	**8.02**	.05569	**25.00**
1.133	**1.000**	.4909	8.537	.05928	26.61
1.259	1.111	.5457	**9.000**	.06250	28.05
1.555	1.372	.6737	**10.00**	.06944	31.17
1.881	1.660	.8151	**11.00**	.07639	34.29
2.000	1.765	.8667	11.34	.07875	35.35
2.239	1.976	.9701	**12.00**	.08333	37.40
2.266	**2.000**	.9819	12.07	.08384	37.63
2.308	2.037	**1.000**	12.18	.08461	37.97
2.627	2.319	1.138	**13.00**	.09028	40.52
3.000	2.648	1.300	13.89	.09646	43.30
3.047	2.690	1.320	**14.00**	.09722	43.64
3.399	**3.000**	1.473	14.79	.1027	46.09
3.497	3.087	1.516	**15.00**	.1042	46.75
3.980	3.513	1.725	**16.00**	.1111	49.87
4.000	3.531	1.733	16.04	.1114	**50.00**
4.493	3.966	1.947	**17.00**	.1181	52.99
4.532	**4.000**	1.964	17.07	.1186	53.21
4.615	4.074	**2.000**	17.23	.1196	53.69
5.000	4.413	2.167	17.93	.1245	55.90
5.037	4.446	2.183	**18.00**	.1250	56.10
5.612	4.954	2.432	**19.00**	.1319	59.22
5.665	**5.000**	2.455	19.09	.1326	59.50
6.000	5.296	2.600	19.65	.1364	61.23
6.219	5.489	2.695	**20.00**	.1389	62.34
6.798	**6.000**	2.946	20.91	.1452	65.17
6.856	6.051	2.971	**21.00**	.1458	65.45
6.923	6.111	**3.000**	21.10	.1465	65.75
7.000	6.178	3.033	21.22	.1474	66.15
7.525	6.641	3.270	**22.00**	.1528	68.57
7.931	**7.000**	3.437	22.59	.1568	70.40
8.000	7.061	3.467	22.69	.1575	70.70
8.224	7.259	3.564	**23.00**	.1597	71.69
8.955	7.904	3.880	**24.00**	.1667	74.81
9.000	7.944	3.900	24.06	.1671	**75.00**
9.064	**8.000**	3.928	24.15	.1677	75.26
9.231	8.148	**4.000**	24.37	.1692	75.95

TABLE XXVIII — *Continued*

H	H_g	p	V	Q	G
9.717	8.576	4.211	**25.00**	.1736	77.92
10.00	8.826	4.333	25.36	.1761	79.05
10.20	**9.000**	4.419	25.61	.1779	79.82
11.00	9.709	4.767	26.60	.1847	82.93
11.33	**10.00**	4.909	27.00	.1875	84.14
11.54	10.18	**5.000**	27.24	.1892	84.90
12.00	10.59	5.200	27.78	.1929	86.60
12.46	**11.00**	5.400	28.31	.1966	88.25
13.00	11.47	5.633	28.92	.2008	90.15
13.60	**12.00**	5.891	29.57	.2054	92.17
13.85	12.22	**6.000**	29.84	.2072	92.99
13.99	12.35	6.063	**30.00**	.2083	93.51
14.00	12.36	6.067	30.01	.2084	93.54
14.73	**13.00**	6.382	30.78	.2138	95.93
15.00	13.24	6.500	31.06	.2157	96.83
15.86	**14.00**	6.873	31.94	.2218	99.55
16.00	14.12	6.933	32.08	.2228	**100.0**
16.15	14.26	**7.000**	32.24	.2239	100.5
17.00	**15.00**	7.367	33.07	.2296	103.1
18.00	15.89	7.800	34.03	.2363	106.1
18.13	**16.00**	7.855	34.15	.2371	106.4
18.46	16.30	**8.000**	34.46	.2393	107.4
19.00	16.77	8.233	34.96	.2428	109.0
19.26	**17.00**	8.346	35.20	.2444	109.7
20.00	17.65	8.667	35.87	.2491	111.8
20.39	**18.00**	8.837	36.22	.2515	112.9
20.77	18.33	**9.000**	36.55	.2538	113.9
21.00	18.54	9.100	36.75	.2552	114.6
21.53	**19.00**	9.328	37.21	.2584	116.0
22.00	19.42	9.533	37.62	.2612	117.3
22.66	**20.00**	9.819	38.18	.2651	119.0
23.00	20.30	9.967	38.46	.2671	119.9
23.08	20.37	**10.00**	38.53	.2676	120.1
23.79	**21.00**	10.31	39.12	.2717	121.9
24.00	21.18	10.40	39.29	.2729	122.5
24.88	21.96	10.78	**40.00**	.2778	124.7
24.93	**22.00**	10.80	40.04	.2780	124.8
25.00	22.07	10.83	40.10	.2785	**125.0**
25.38	22.41	**11.00**	40.41	.2806	126.0
26.00	22.95	11.27	40.89	.2840	127.5
26.06	**23.00**	11.29	40.94	.2843	127.6
27.00	23.83	11.70	41.67	.2894	129.9
27.19	**24.00**	11.78	41.82	.2904	130.4
27.69	24.44	**12.00**	42.20	.2930	131.5
28.00	24.71	12.13	42.44	.2947	132.3
28.33	**25.00**	12.27	42.68	.2964	133.0
29.00	25.60	12.57	43.19	.2999	134.6
29.46	**26.00**	12.76	43.53	.3023	135.7
30.00	26.48	**13.00**	43.93	.3051	136.9
30.59	**27.00**	13.26	44.36	.3080	138.3
31.00	27.36	13.43	44.65	.3101	139.2
31.72	**28.00**	13.75	45.17	.3137	140.8
32.00	28.24	13.87	45.37	.3151	141.4
32.31	28.52	**14.00**	45.59	.3166	142.1
32.86	**29.00**	14.24	45.97	.3192	143.3
33.00	29.13	14.30	46.07	.3200	143.6
33.99	**30.00**	14.73	46.76	.3247	145.7
34.00	30.01	14.73	46.77	.3248	145.8
34.62	30.55	**15.00**	47.19	.3277	147.1
35.00	30.89	15.17	47.45	.3295	147.9
36.00	31.78	15.60	48.12	.3341	**150.0**
36.92	32.59	**16.00**	48.73	.3384	151.9
37.00	32.66	16.03	48.78	.3388	152.1
38.00	33.54	16.47	49.44	.3434	154.1
38.87	34.31	16.84	**50.00**	.3472	155.8
39.00	34.42	16.90	50.09	.3478	156.1

TABLE XXVIII — *Continued*

H	H_g	p	V	Q	G
39.23	34.63	**17.00**	50.23	.3488	156.6
40.00	35.31	17.33	50.72	.3522	158.1
41.00	36.19	17.77	51.35	.3566	160.1
41.54	36.66	**18.00**	51.69	.3590	161.1
42.00	37.07	18.20	51.98	.3610	162.0
43.00	37.95	18.63	52.59	.3652	163.9
43.85	38.70	**19.00**	53.11	.3688	165.5
44.00	38.84	19.07	53.20	.3694	165.8
45.00	39.72	19.50	53.80	.3736	167.7
45.32	**40.00**	19.64	53.99	.3749	168.3
46.00	40.60	19.93	54.39	.3777	169.6
46.15	40.74	**20.00**	54.48	.3784	169.8
47.00	41.48	20.37	54.98	.3818	171.4
48.00	42.37	20.80	55.56	.3858	173.2
48.46	42.77	**21.00**	55.83	.3877	174.0
49.00	43.25	21.23	56.14	.3898	**175.0**
50.00	44.13	21.67	56.71	.3938	176.8
50.77	44.81	**22.00**	57.14	.3968	178.1
51.00	45.01	22.10	57.27	.3977	178.5
52.00	45.90	22.53	57.83	.4016	180.3
53.00	46.78	22.97	58.39	.4055	182.0
53.08	46.85	**23.00**	58.43	.4058	182.1
54.00	47.66	23.40	58.93	.4092	183.7
55.00	48.55	23.83	59.48	.4130	185.4
55.38	48.89	**24.00**	59.69	.4145	186.1
55.97	49.40	24.25	**60.00**	.4167	187.0
56.00	49.43	24.27	60.02	.4168	187.1
56.65	**50.00**	24.55	60.36	.4192	188.1
57.00	50.31	24.70	60.55	.4205	188.8
57.69	50.92	**25.00**	60.92	.4231	189.9
58.00	51.19	25.13	61.08	.4242	190.4
59.00	52.08	25.57	61.60	.4278	192.0
60.00	52.96	**26.00**	62.12	.4314	193.6
61.00	53.84	26.43	62.64	.4350	195.2
62.00	54.72	26.87	63.15	.4385	196.9
62.31	55.00	**27.00**	63.31	.4396	197.3
63.00	55.61	27.30	63.66	.4421	198.4
64.00	56.49	27.73	64.16	.4455	**200.0**
64.62	57.03	**28.00**	64.47	.4477	200.9
65.00	57.37	28.17	64.66	.4490	201.6
66.00	58.25	28.60	65.16	.4525	203.1
66.92	59.07	**29.00**	65.61	.4556	204.5
67.00	59.14	29.03	65.65	.4559	204.6
67.98	**60.00**	29.46	66.12	.4592	206.1
68.00	60.02	29.47	66.14	.4593	206.2
69.00	60.90	29.90	66.62	.4626	207.7
69.23	61.11	**30.00**	66.73	.4634	208.0
70.00	61.78	30.33	67.10	.4659	209.2
71.00	62.67	30.77	67.58	.4693	210.7
71.54	63.14	**31.00**	67.83	.4711	211.4
72.00	63.55	31.20	68.05	.4726	212.1
73.00	64.43	31.63	68.52	.4759	213.6
73.85	65.18	**32.00**	68.92	.4786	214.8
74.00	65.32	32.07	68.99	.4791	215.1
75.00	66.20	32.50	69.46	.4823	216.5
76.00	67.08	32.93	69.92	.4855	218.0
76.15	67.22	**33.00**	69.99	.4861	218.2
76.18	67.24	33.01	**70.00**	.4861	218.2
77.00	67.96	33.37	70.38	.4887	219.4
78.00	68.85	33.80	70.84	.4919	220.8
78.46	69.25	**34.00**	71.04	.4933	221.4
79.00	69.73	34.23	71.29	.4950	222.2
79.31	**70.00**	34.37	71.42	.4960	222.6
80.00	70.61	34.67	71.73	.4981	223.6
80.77	71.29	**35.00**	72.08	.5005	224.7
81.00	71.49	35.10	72.18	.5012	**225.0**

TABLE XXVIII — *Continued*

H	H_g	p	V	Q	G
82.00	72.38	35.53	72.63	.5043	226.4
83.00	73.26	35.97	73.07	.5074	227.7
83.08	73.33	**36.00**	73.10	.5076	227.8
84.00	74.14	36.40	73.51	.5104	229.1
85.00	75.02	36.83	73.94	.5135	230.5
85.38	75.37	**37.00**	74.11	.5147	231.0
86.00	75.91	37.27	74.38	.5165	231.9
87.00	76.79	37.70	74.81	.5195	233.2
87.69	77.40	**38.00**	75.10	.5215	234.1
88.00	77.67	38.13	75.23	.5224	234.5
89.00	78.56	38.57	75.66	.5254	235.9
90.00	79.44	**39.00**	76.08	.5284	237.2
90.64	**80.00**	39.28	76.35	.5302	238.0
91.00	80.32	39.43	76.51	.5314	238.5
92.00	81.20	39.87	76.93	.5342	239.8
92.31	81.48	**40.00**	77.05	.5352	240.2
93.00	82.09	40.30	77.34	.5371	241.1
94.00	82.97	40.73	77.76	.5400	242.4
94.62	83.51	**41.00**	78.01	.5417	243.1
95.00	83.85	41.17	78.17	.5429	243.7
96.00	84.73	41.60	78.58	.5457	245.0
96.92	85.55	**42.00**	78.96	.5483	246.1
97.00	85.62	42.03	78.99	.5485	246.2
98.00	86.50	42.47	79.39	.5513	247.5
99.00	87.38	42.90	79.80	.5542	248.8
99.23	87.59	**43.00**	79.89	.5548	249.0
99.50	87.82	43.11	**80.00**	.5556	249.4
100.0	88.26	43.33	80.20	.5569	**250.0**
101.5	89.62	**44.00**	80.81	.5613	251.9
102.0	**90.00**	44.19	80.98	.5624	252.4
103.8	91.66	**45.00**	81.73	.5676	254.7
106.2	93.70	**46.00**	82.63	.5738	257.5
108.5	95.73	**47.00**	83.52	.5800	260.3
110.0	97.09	47.67	84.12	.5842	262.2
110.8	97.77	**48.00**	84.41	.5862	263.1
113.1	99.81	**49.00**	85.28	.5922	265.8
113.3	**100.0**	49.09	85.37	.5928	266.1
115.4	101.8	**50.00**	86.15	.5983	268.5
120.0	105.9	52.00	87.85	.6101	273.9
124.6	**110.0**	54.00	89.53	.6218	279.1
125.9	111.1	54.57	**90.00**	.6250	280.5
130.0	114.7	56.33	91.44	.6350	285.0
136.0	**120.0**	58.91	93.51	.6494	291.5
138.5	122.2	**60.00**	94.37	.6554	294.1
140.0	123.6	60.67	94.89	.6590	295.8
147.3	**130.0**	63.82	97.33	.6759	303.4
150.0	132.4	65.00	98.22	.6821	306.2
155.5	137.2	67.37	**100.0**	.6944	311.7
158.6	**140.0**	68.73	101.0	.7014	314.8
160.0	141.2	69.33	101.5	.7045	316.2
161.5	142.6	**70.00**	101.9	.7080	317.7
170.0	**150.0**	73.67	104.6	.7261	326.0
180.0	158.9	78.00	107.6	.7472	335.4
181.3	**160.0**	78.55	108.0	.7499	336.6
184.6	163.0	**80.00**	109.0	.7567	339.6
190.0	167.7	82.33	110.6	.7677	344.6
192.6	**170.0**	83.46	111.3	.7730	346.9
200.0	176.5	86.67	113.4	.7875	353.6
203.9	**180.0**	88.37	114.5	.7954	357.0
207.7	183.3	**90.00**	115.6	.8027	360.2
215.3	**190.0**	93.28	117.7	.8171	366.8
226.6	**200.0**	98.19	120.7	.8384	376.3
230.8	203.7	**100.0**	121.8	.8461	379.7

TABLE XXIX

$\frac{V^2}{2g}$ HEADS DUE TO VELOCITIES FROM 0 TO 10 FEET PER SECOND*

V	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001
.1	.0002	.0002	.0002	.0003	.0003	.0003	.0004	.0004	.0005	.0006
.2	.0006	.0007	.0008	.0008	.0009	.0010	.0011	.0011	.0012	.0013
.3	.0014	.0015	.0016	.0017	.0018	.0019	.0020	.0021	.0022	.0024
.4	.0025	.0026	.0027	.0029	.0030	.0031	.0033	.0034	.0036	.0037
0.5	.0039	.0040	.0042	.0044	.0045	.0047	.0049	.0051	.0052	.0054
.6	.0056	.0058	.0060	.0062	.0064	.0066	.0068	.0070	.0072	.0074
.7	.0076	.0078	.0081	.0083	.0085	.0087	.0090	.0092	.0095	.0097
.8	.0099	.0102	.0105	.0107	.0110	.0112	.0115	.0118	.0120	.0123
.9	.0126	.0129	.0132	.0134	.0137	.0140	.0143	.0146	.0149	.0152
1.0	.0155	.0159	.0162	.0165	.0168	.0171	.0175	.0178	.0181	.0185
.1	.0188	.0192	.0195	.0199	.0202	0206	.0209	.0213	.0216	.0220
.2	.0224	.0228	.0231	.0235	.0239	.0243	.0247	.0251	.0255	.0259
.3	.0263	.0267	.0271	.0275	.0279	.0283	.0288	.0292	.0296	.0300
.4	.0305	.0309	.0313	.0318	.0322	.0327	.0331	.0336	.0341	.0345
1.5	.0350	.0354	.0359	.0364	.0369	.0374	.0378	.0383	.0388	.0393
.6	.0398	.0403	.0408	.0413	.0418	.0423	.0428	.0434	.0439	.0444
.7	.0449	.0455	.0460	.0465	.0471	.0476	.0482	.0487	.0493	.0 '98
.8	.0504	.0509	.0515	.0521	.0526	.0532	.0538	.0544	.0549	.0555
.9	.0561	.0567	.0573	.0579	.0585	.0591	.0597	.0603	.0609	.0616
2.0	.0622	.0628	.0634	.0641	.0647	.0653	.0660	.0666	.0673	.0679
.1	.0686	.0692	.0699	.0705	.0712	.0719	.0725	.0732	.0739	.0746
.2	.0752	.0759	.0766	.0773	.0780	.0787	.0794	.0801	.0808	.0815
.3	.0822	.0830	.0837	.0844	.0851	.0859	.0866	.0873	.0881	.0888
.4	.0895	.0903	.0910	.0918	.0926	.0933	.0941	.0948	.0956	.0964
2.5	.0972	.0979	.0987	.0995	.1003	.1011	.1019	.1027	.1035	.1043
.6	.1051	.1059	.1067	.1075	.1084	.1092	.1100	.1108	.1117	.1125
.7	.1133	.1142	.1150	.1159	.1167	.1176	.1184	.1193	.1201	.1210
.8	.1219	.1228	.1236	.1245	.1254	.1263	.1272	.1281	.1289	.1298
.9	.1307	.1316	.1326	.1335	.1344	.1353	.1362	.1371	.1381	.1390
3.0	.1399	.1409	.1418	.1427	.1437	.1446	.1456	.1465	.1475	.1484
.1	.1494	.1504	.1513	.1523	.1533	.1543	.1552	.1562	.1572	.1582
.2	.1592	.1602	.1612	.1622	.1632	.1642	.1652	.1662	.1673	.1683
.3	.1693	.1703	.1714	.1724	.1734	.1745	.1755	.1766	.1776	.1787
.4	.1797	.1808	.1818	.1829	.1840	.1850	.1861	.1872	.1883	.1894
3.5	.1904	.1915	.1926	.1937	.1948	.1959	.1970	.1981	.1992	.2004
.6	.2015	.2026	.2037	.2049	.2060	.2071	.2083	.2094	.2105	.2117
.7	.2128	.2140	.2151	.2163	.2175	.2186	.2198	.2210	.2221	.2233
.8	.2245	.2257	.2269	.2280	.2292	.2304	.2316	.2328	.2340	.2352
.9	.2365	.2377	.2389	.2401	.2413	.2426	.2438	.2450	.2463	.2475
4.0	.2487	.2500	.2512	.2525	.2537	.2550	.2563	.2575	.2588	.2601
.1	.2613	.2626	.2639	.2652	.2665	.2677	.2690	.2703	.2716	.2729
.2	.2742	.2755	.2769	.2782	.2795	.2808	.2821	.2835	.2848	.2861
.3	.2875	.2888	.2901	.2915	.2928	.2942	.2955	.2969	.2982	.2996
.4	.3010	.3023	.3037	.3051	.3065	.3079	.3092	.3106	.3120	.3134
4.5	.3148	.3162	.3176	.3190	.3204	.3218	.3233	.3247	.3261	.3275
.6	.3290	.3304	.3318	.3333	.3347	.3362	.3376	.3390	.3405	.3420
.7	.3434	.3449	.3463	.3478	.3493	.3508	.3522	.3537	.3552	.3567
.8	.3582	.3597	.3612	.3627	.3642	.3657	.3672	.3687	.3702	.3717
.9	.3733	.3748	.3763	.3779	.3794	.3809	.3825	.3840	.3856	.3871

* *Lowell Hydraulic Experiments*, p. 253, for heads from 0 to 4.99.

$\frac{V^2}{2g}$

TABLE XXIX — *Continued*

V	0	1	2	3	4	5	6	7	8	9
5.0	.3887	.3902	.3917	.3934	.3949	.3965	.3981	.3996	.4012	.4028
.1	.4044	.4060	.4076	.4092	.4108	.4123	.4140	.4156	.4172	.4188
.2	.4204	.4220	.4236	.4253	.4269	.4285	.4302	.4318	.4334	.4351
.3	.4367	.4384	.4400	.4417	.4433	.4450	.4467	.4483	.4500	.4517
.4	.4534	.4551	.4567	.4584	.4601	.4618	.4635	.4652	.4669	.4686
5.5	.4703	.4720	.4737	.4755	.4772	.4789	.4806	.4824	.4841	.4858
.6	.4876	.4893	.4910	.4928	.4946	.4963	.4981	.4998	.5016	.5034
.7	.5051	.5069	.5087	.5105	.5123	.5141	.5158	.5176	.5194	.5212
.8	.5231	.5248	.5266	.5284	.5303	.5321	.5339	.5357	.5376	.5394
.9	.5412	.5431	.5449	.5467	.5486	.5504	.5523	.5541	.5560	.5579
6.0	.5598	.5616	.5634	.5653	.5672	.5691	.5710	.5728	.5747	.5766
.1	.5785	.5804	.5823	.5842	.5861	.5880	.5900	.5919	.5938	.5957
.2	.5977	.5996	.6015	.6034	.6054	.6073	.6093	.6112	.6132	.6151
.3	.6171	.6191	.6210	.6230	.6249	.6269	.6289	.6309	.6329	.6348
.4	.6368	.6388	.6408	.6428	.6448	.6468	.6489	.6508	.6529	.6549
6.5	.6569	.6589	.6609	.6630	.6650	.6670	.6691	.6711	.6732	.6752
.6	.6773	.6793	.6814	.6834	.6855	.6876	.6896	.6917	.6938	.6959
.7	.6979	.7000	.7021	.7042	.7063	.7084	.7105	.7126	.7147	.7168
.8	.7189	.7210	.7232	.7253	.7274	.7295	.7317	.7338	.7360	.7381
.9	.7402	.7424	.7445	.7467	.7488	.7510	.7532	.7553	.7575	.7597
7.0	.7618	.7640	.7662	.7684	.7706	.7727	.7749	.7771	.7794	.7815
.1	.7838	.7860	.7882	.7904	.7926	.7948	.7971	.7993	.8015	.8037
.2	.8060	.8082	.8105	.8127	.8150	.8172	.8195	.8217	.8240	.8263
.3	.8285	.8308	.8331	.8354	.8376	.8399	.8422	.8445	.8468	.8491
.4	.8514	.8537	.8560	.8583	.8606	.8629	.8653	.8676	.8699	.8722
7.5	.8746	.8769	.8792	.8816	.8839	.8863	.8886	.8910	.8933	.8957
.6	.8980	.9004	.9028	.9051	.9075	.9099	.9123	.9147	.9171	.9194
.7	.9218	.9242	.9266	.9290	.9314	.9338	.9362	.9387	.9411	.9435
.8	.9459	.9484	.9508	.9532	.9557	.9581	.9605	.9630	.9654	.9679
.9	.9703	.9728	.9753	.9777	.9802	.9827	.9851	.9876	.9901	.9926
8.0	.9951	.9975	1.000	1.002	1.005	1.007	1.010	1.012	1.015	1.018
.1	1.020	1.023	1.025	1.028	1.030	1.033	1.035	1.038	1.040	1.043
.2	1.045	1.048	1.051	1.053	1.056	1.058	1.061	1.063	1.066	1.069
.3	1.071	1.074	1.076	1.079	1.082	1.084	1.087	1.089	1.092	1.094
.4	1.097	1.099	1.102	1.105	1.108	1.110	1.113	1.115	1.118	1.120
8.5	1.123	1.126	1.129	1.131	1.134	1.136	1.139	1.142	1.145	1.147
.6	1.150	1.152	1.155	1.158	1.161	1.163	1.166	1.169	1.172	1.174
.7	1.177	1.180	1.183	1.185	1.188	1.190	1.193	1.196	1.199	1.201
.8	1.204	1.207	1.210	1.212	1.215	1.218	1.221	1.223	1.226	1.229
.9	1.232	1.234	1.237	1.240	1.243	1.245	1.248	1.251	1.254	1.257
9.0	1.260	1.262	1.265	1.268	1.270	1.273	1.276	1.279	1.281	1.285
.1	1.288	1.290	1.293	1.296	1.299	1.302	1.304	1.307	1.310	1.313
.2	1.316	1.319	1.322	1.324	1.327	1.330	1.333	1.336	1.339	1.342
.3	1.345	1.348	1.351	1.354	1.356	1.359	1.362	1.365	1.368	1.371
.4	1.374	1.377	1.379	1.383	1.386	1.389	1.392	1.394	1.397	1.400
9.5	1.403	1.406	1.409	1.412	1.415	1.418	1.421	1.424	1.427	1.430
.6	1.433	1.436	1.439	1.442	1.445	1.448	1.451	1.454	1.457	1.460
.7	1.463	1.466	1.469	1.472	1.475	1.478	1.481	1.484	1.487	1.490
.8	1.493	1.496	1.499	1.503	1.506	1.508	1.511	1.514	1.517	1.520
.9	1.524	1.526	1.530	1.533	1.536	1.539	1.542	1.545	1.549	1.551
10.0	1.555									

Possible error of 1 in last figure.

$g = 32.16$

TABLE XXX

Smith's Weir Coefficients

c_s = coefficient for weirs with the contraction suppressed at both ends and complete crest contraction

Effective Head in Feet, H	Length of Weir in Feet, L								
	.66	2	3	4	5	7	10	15	19
.1	.675				.659	.658	.658	.657	.657
.15	.662	.652	.649	.647	.645	.645	.644	.644	.643
.2	.656	.645	.642	.641	.638	.637	.637	.636	.635
.25	.653	.641	.638	.636	.634	.633	.632	.631	.630
.3	.651	.639	.636	.633	.631	.629	.628	.627	.626
.4	.650	.636	.633	.630	.628	.625	.623	.622	.621
.5	.650	.637	.633	.630	.627	.624	.621	.620	.619
.6	.651	.638	.634	.630	.627	.623	.620	.619	.618
.7	.653	.640	.635	.631	.628	.624	.620	.619	.618
.8	.656	.643	.637	.633	.629	.625	.621	.620	.618
.9		.645	.639	.635	.631	.627	.622	.620	.619
1.0		.648	.641	.637	.633	.628	.624	.621	.619
1.1			.644	.639	.635	.630	.625	.622	.620
1.2			.646	.641	.636	.632	.626	.623	.620
1.3			.648	.643	.638	.633	.628	.624	.621
1.4				.644	.640	.634	.629	.625	.622
1.5				.646	.641	.636	.630	.625	.622
1.6				.647	.642	.637	.631	.626	.623
1.7						.638	.632	.626	.623

c_c = coefficient for weirs with complete contraction at two ends and complete crest contraction

Effective Head in Feet, H	Length of Weir in Feet, L										
	.66	1	2	2.6	3	4	5	7	10	15	19
.1	.632	.639	.646	.650	.652	.653	.653	.654	.655	.655	.656
.15	.619	.625	.634	.637	.638	.639	.640	.640	.641	.642	.642
.2	.611	.618	.626	.629	.630	.631	.631	.632	.633	.634	.634
.25	.605	.612	.621	.623	.624	.625	.626	.627	.628	.628	.629
.3	.601	.608	.616	.618	.619	.621	.621	.623	.624	.624	.625
.4	.595	.601	.609	.612	.613	.614	.615	.617	.618	.619	.620
.5	.590	.596	.605	.607	.608	.610	.611	.613	.615	.616	.617
.6	.587	.593	.601	.604	.605	.607	.608	.611	.613	.614	.615
.7	.585	.590	.598	.601	.603	.604	.606	.609	.612	.613	.614
.8			.595	.598	.600	.602	.604	.607	.611	.612	.613
.9			.592	.596	.598	.600	.603	.606	.609	.611	.612
1.0			.590	.593	.595	.598	.601	.604	.608	.610	.611
1.1			.587	.591	.593	.596	.599	.603	.606	.609	.610
1.2			.585	.589	.591	.594	.597	.601	.605	.608	.610
1.3			.582	.586	.589	.592	.596	.599	.604	.607	.609
1.4			.580	.584	.587	.590	.594	.598	.602	.606	.609
1.5				.582	.585	.589	.592	.596	.601	.605	.608
1.6				.580	.582	.587	.591	.595	.600	.604	.607
1.7								.594	.599	.603	.607

TABLE XXXI

Weir Table

(Possible errors of 1 in the last figure)

$3.31\ (n^{\frac{3}{2}}) + 0.007$

Tenths of the Tabular Difference

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.01	.0103	.0108	.0114	.0119	.0125	.0131	.0137	.0143	.0150	.0157	1	1	2	2	3	4	4	5	5
2	.0164	.0171	.0178	.0185	.0193	.0201	.0209	.0217	.0225	.0233	1	2	2	3	4	5	5	6	7
3	.0242	.0251	.0259	.0268	.0278	.0287	.0296	.0306	.0315	.0325	1	2	3	4	5	6	6	7	8
4	.0335	.0345	.0355	.0365	.0376	.0386	.0397	.0407	.0418	.0429	1	2	3	4	5	6	7	8	9
.05	.0440	.0452	.0463	.0474	.0485	.0497	.0508	.0520	.0532	.0544	1	2	3	5	6	7	8	9	10
6	.0557	.0569	.0581	.0593	.0606	.0619	.0632	.0644	.0656	.0669	1	3	4	5	6	8	9	10	11
7	.0683	.0696	.0709	.0723	.0736	.0750	.0764	.0778	.0791	.0805	1	3	4	5	7	8	10	11	12
8	.0819	.0833	.0847	.0861	.0876	.0890	.0905	.0919	.0934	.0949	1	3	4	6	7	9	10	12	13
9	.0964	.0979	.0993	.1008	.1024	.1039	.1054	.1070	.1086	.1101	2	3	5	6	8	9	11	12	14
.10	.1117	.1133	.1148	.1164	.1180	.1196	.1212	.1229	.1245	.1261	2	3	5	6	8	10	11	13	14
1	.1277	.1294	.1311	.1327	.1344	.1361	.1378	.1395	.1412	.1429	2	3	5	7	8	10	12	13	15
2	.1446	.1464	.1480	.1498	.1515	.1533	.1551	.1568	.1586	.1604	2	4	5	7	9	11	12	14	16
3	.1621	.1640	.1657	.1675	.1693	.1712	.1730	.1749	.1767	.1786	2	4	5	7	9	11	13	15	16
4	.1804	.1822	.1841	.1860	.1879	.1897	.1917	.1936	.1955	.1973	2	4	6	8	9	11	13	15	17
.15	.1993	.2012	.2031	.2051	.2070	.2090	.2110	.2130	.2148	.2168	2	4	6	8	10	12	14	16	18
6	.2188	.2208	.2228	.2248	.2268	.2289	.2308	.2329	.2349	.2370	2	4	6	8	10	12	14	16	18
7	.2390	.2411	.2431	.2452	.2472	.2493	.2514	.2535	.2556	.2577	2	4	6	8	10	12	15	17	19
8	.2598	.2619	.2639	.2661	.2682	.2704	.2725	.2747	.2768	.2790	2	4	6	9	11	13	15	17	19
9	.2811	.2833	.2855	.2877	.2898	.2920	.2942	.2965	.2986	.3008	2	4	7	9	11	13	15	18	20
.20	.3030	.3053	.3075	.3098	.3120	.3142	.3165	.3187	.3210	.3233	2	4	7	9	11	13	16	18	20
1	.3255	.3277	.3300	.3323	.3347	.3370	.3393				2	5	7	9	11	14	16	18	21
1							.339	.342	.344	.346	0	0	1	1	1	1	2	2	2
2	.349	.351	.353	.356	.358	.360	.363	.365	.368	.370	0	0	1	1	1	1	2	2	2
3	.372	.375	.377	.380	.382	.384	.387	.389	.392	.394	0	0	1	1	1	1	2	2	2
4	.396	.399	.401	.404	.406	.409	.411	.414	.416	.419	0	0	1	1	1	1	2	2	2
.25	.421	.423	.425	.428	.430	.433	.435	.438	.440	.443	0	1	1	1	1	2	2	2	2
6	.446	.448	.451	.453	.456	.458	.461	.464	.466	.469	0	1	1	1	1	2	2	2	2
7	.471	.474	.477	.479	.482	.484	.487	.490	.492	.495	0	1	1	1	1	2	2	2	2
8	.498	.500	.503	.505	.508	.510	.513	.516	.519	.521	0	1	1	1	1	2	2	2	2
9	.524	.527	.529	.532	.535	.537	.540	.543	.546	.548	0	1	1	1	1	2	2	2	2
.30	.551	.554	.557	.559	.562	.565	.568	.570	.573	.576	0	1	1	1	1	2	2	2	2
1	.578	.581	.584	.586	.589	.592	.594	.597	.600	.603	0	1	1	1	1	2	2	2	3
2	.606	.609	.611	.614	.617	.620	.623	.626	.628	.631	0	1	1	1	1	2	2	2	3
3	.635	.637	.640	.643	.646	.649	.652	.654	.657	.660	0	1	1	1	1	2	2	2	3
4	.663	.666	.669	.672	.675	.678	.681	.684	.687	.690	0	1	1	1	1	2	2	2	3
.35	.693	.696	.698	.701	.704	.707	.710	.713	.716	.719	0	1	1	1	1	2	2	2	3
6	.722	.725	.728	.731	.734	.737	.740	.743	.746	.750	0	1	1	1	2	2	2	2	3
7	.752	.755	.758	.761	.764	.767	.770	.773	.776	.779	0	1	1	1	2	2	2	2	3
8	.782	.785	.788	.791	.795	.798	.801	.804	.807	.810	0	1	1	1	2	2	2	2	3
9	.813	.816	.819	.823	.826	.829	.832	.835	.838	.841	0	1	1	1	2	2	2	2	3
.40	.844	.847	.851	.854	.857	.860	.863	.867	.870	.873	0	1	1	1	2	2	2	3	3
1	.876	.879	.883	.886	.889	.892	.895	.899	.902	.905	0	1	1	1	2	2	2	3	3
2	.908	.912	.915	.917	.920	.924	.927	.930	.933	.937	0	1	1	1	2	2	2	3	3
3	.940	.943	.946	.950	.953	.956	.960	.963	.966	.970	0.	1	1	1	2	2	2	3	3
4	.973	.976	.980	.983	.986	.990	.993	.996	1.000	1.003	0	1	1	1	2	2	2	3	3
.45	1.006	1.010	1.013	1.016	1.020	1.023	1.026	1.030	1.033	1.036	0	1	1	1	2	2	2	3	3
6	1.040	1.043	1.047	1.050	1.053	1.057	1.060	1.063	1.067	1.071	0	1	1	1	2	2	2	3	3
7	1.074	1.077	1.081	1.083	1.087	1.090	1.094	1.097	1.101	1.104	0	1	1	1	2	2	2	3	3
8	1.108	1.111	1.114	1.118	1.121	1.125	1.128	1.132	1.135	1.139	0	1	1	1	2	2	2	3	3
9	1.142	1.146	1.149	1.153	1.156	1.160	1.163	1.167	1.170	1.174	0	1	1	1	2	2	2	3	3
.50	1.177																		

TABLE XXXI — *Continued*

WEIR TABLE

(Possible errors of 1 in the last figure)

$3.33\,(n^{\frac{3}{2}})$

TENTHS OF THE TABULAR DIFFERENCE

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8
.50	1.177	1.181	1.184	1.188	1.191	1.195	1.198	1.202	1.206	1.209	0	1	1	1	2	2	2	3
1	1.213	1.216	1.220	1.223	1.227	1.231	1.234	1.238	1.241	1.245	0	1	1	1	2	2	3	3
2	1.249	1.252	1.256	1.259	1.263	1.267	1.270	1.274	1.278	1.281	0	1	1	1	2	2	3	3
3	1.285	1.288	1.292	1.296	1.299	1.303	1.307	1.310	1.314	1.318	0	1	1	1	2	2	3	3
4	1.321	1.325	1.329	1.332	1.336	1.340	1.343	1.347	1.351	1.355	0	1	1	1	2	2	3	3
.55	1.358	1.362	1.366	1.369	1.373	1.377	1.381	1.384	1.388	1.392	0	1	1	1	2	2	3	3
6	1.396	1.399	1.403	1.407	1.411	1.414	1.418	1.422	1.426	1.429	0	1	1	2	2	2	3	3
7	1.433	1.437	1.441	1.444	1.448	1.452	1.456	1.460	1.463	1.467	0	1	1	2	2	2	3	3
8	1.471	1.475	1.479	1.482	1.486	1.490	1.494	1.498	1.501	1.505	0	1	1	2	2	2	3	3
9	1.509	1.513	1.517	1.520	1.524	1.528	1.532	1.536	1.540	1.544	0	1	1	2	2	2	3	3
.60	1.548	1.551	1.555	1.559	1.563	1.567	1.571	1.575	1.579	1.583	0	1	1	2	2	2	3	3
1	1.586	1.590	1.594	1.598	1.602	1.606	1.610	1.614	1.618	1.622	0	1	1	2	2	2	3	3
2	1.626	1.630	1.634	1.637	1.641	1.645	1.649	1.653	1.657	1.661	0	1	1	2	2	2	3	3
3	1.665	1.669	1.673	1.677	1.681	1.685	1.689	1.693	1.697	1.701	0	1	1	2	2	2	3	3
4	1.705	1.709	1.713	1.717	1.721	1.725	1.729	1.733	1.737	1.741	0	1	1	2	2	2	3	3
.65	1.745	1.749	1.753	1.757	1.761	1.765	1.769	1.773	1.778	1.782	0	1	1	2	2	2	3	3
6	1.786	1.790	1.794	1.798	1.802	1.806	1.810	1.814	1.818	1.822	0	1	1	2	2	2	3	3
7	1.826	1.830	1.834	1.838	1.842	1.847	1.851	1.855	1.859	1.863	0	1	1	2	2	2	3	3
8	1.867	1.871	1.875	1.880	1.884	1.888	1.892	1.896	1.900	1.904	0	1	1	2	2	2	3	3
9	1.909	1.913	1.917	1.921	1.925	1.929	1.933	1.938	1.942	1.946	0	1	1	2	2	2	3	3
.70	1.950	1.954	1.959	1.963	1.967	1.971	1.975	1.980	1.984	1.988	0	1	1	2	2	3	3	3
1	1.992	1.996	2.001	2.005	2.009	2.013	2.018	2.022	2.026	2.030	0	1	1	2	2	3	3	3
2	2.034	2.039	2.043	2.047	2.051	2.056	2.060	2.064	2.069	2.073	0	1	1	2	2	3	3	3
3	2.077	2.081	2.086	2.090	2.094	2.098	2.103	2.107	2.111	2.166	0	1	1	2	2	3	3	3
4	2.120	2.124	2.129	2.133	2.137	2.141	2.146	2.150	2.154	2.159	0	1	1	2	2	3	3	3
.75	2.163	2.167	2.171	2.176	2.180	2.184	2.189	2.193	2.197	2.202	0	1	1	2	2	3	3	3
6	2.206	2.211	2.215	2.219	2.224	2.228	2.232	2.237	2.241	2.246	0	1	1	2	2	3	3	3
7	2.250	2.254	2.259	2.263	2.267	2.272	2.276	2.281	2.285	2.290	0	1	1	2	2	3	3	4
8	2.294	2.298	2.303	2.307	2.312	2.316	2.320	2.325	2.329	2.334	0	1	1	2	2	3	3	4
9	2.338	2.343	2.347	2.352	2.356	2.360	2.365	2.369	2.374	2.378	0	1	1	2	2	3	3	4
.80	2.383	2.387	2.392	2.396	2.401	2.405	2.410	2.414	2.419	2.423	0	1	1	2	2	3	3	4
1	2.428	2.432	2.437	2.441	2.446	2.450	2.455	2.459	2.464	2.468	0	1	1	2	2	3	3	4
2	2.473	2.477	2.482	2.486	2.491	2.495	2.500	2.504	2.509	2.513	0	1	1	2	2	3	3	4
3	2.518	2.522	2.527	2.532	2.536	2.541	2.545	2.550	2.554	2.559	0	1	1	2	2	3	3	4
4	2.564	2.568	2.573	2.577	2.582	2.587	2.591	2.596	2.600	2.605	0	1	1	2	2	3	3	4
.85	2.610	2.614	2.619	2.623	2.628	2.633	2.637	2.642	2.647	2.651	0	1	1	2	2	3	3	4
6	2.656	2.660	2.665	2.670	2.674	2.679	2.684	2.688	2.693	2.698	0	1	1	2	2	3	3	4
7	2.702	2.707	2.712	2.716	2.721	2.726	2.730	2.735	2.740	2.744	0	1	1	2	2	3	3	4
8	2.749	2.754	2.758	2.763	2.768	2.773	2.777	2.782	2.787	2.791	0	1	1	2	2	3	3	4
9	2.796	2.801	2.806	2.810	2.815	2.820	2.824	2.829	2.834	2.838	0	1	1	2	2	3	3	4
.90	2.843	2.848	2.853	2.857	2.862	2.867	2.872	2.876	2.881	2.886	0	1	1	2	2	3	3	4
1	2.891	2.895	2.900	2.905	2.910	2.914	2.919	2.924	2.929	2.934	0	1	1	2	2	3	3	4
2	2.938	2.943	2.948	2.953	2.958	2.962	2.967	2.972	2.977	2.982	0	1	1	2	2	3	3	4
3	2.987	2.991	2.996	3.001	3.006	3.011	3.016	3.020	3.025	3.030	0	1	1	2	2	3	3	4
4	3.035	3.040	3.045	3.049	3.054	3.059	3.064	3.069	3.074	3.079	0	1	1	2	2	3	3	4
.95	3.083	3.088	3.093	3.098	3.103	3.108	3.113	3.118	3.123	3.127	0	1	1	2	2	3	3	4
6	3.132	3.137	3.142	3.147	3.152	3.157	3.162	3.166	3.171	3.176	0	1	1	2	2	3	3	4
7	3.181	3.186	3.191	3.196	3.201	3.206	3.211	3.216	3.221	3.226	0	1	1	2	2	3	3	4
8	3.231	3.235	3.240	3.245	3.250	3.255	3.260	3.265	3.270	3.275	0	1	1	2	2	3	3	4
9	3.280	3.285	3.290	3.295	3.300	3.305	3.310	3.315	3.320	3.325	0	1	1	2	2	3	3	4
1.00	3.33																	

TABLE XXXI — *Continued*

WEIR TABLE

$3.33\,(n^{\frac{3}{2}})$

(Possible errors of 1 in the last figure)

TENTHS OF THE TABULAR DIFFERENCE

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	3.33	3.38	3.43	3.48	3.53	3.58	3.63	3.69	3.74	3.79	1	1	2	2	3	3	4	4	5
.1	3.84	3.89	3.95	4.00	4.05	4.11	4.16	4.22	4.27	4.32	1	1	2	2	3	3	4	4	5
.2	4.38	4.43	4.49	4.54	4.60	4.66	4.71	4.77	4.82	4.88	1	1	2	2	3	3	4	4	5
.3	4.94	4.99	5.05	5.11	5.16	5.22	5.28	5.34	5.40	5.46	1	1	2	2	3	3	4	5	5
.4	5.52	5.57	5.63	5.69	5.75	5.81	5.87	5.93	5.99	6.06	1	1	2	2	3	4	4	5	5
1.5	6.12	6.18	6.24	6.30	6.36	6.43	6.49	6.55	6.61	6.68	1	1	2	2	3	4	4	5	6
.6	6.74	6.80	6.87	6.92	6.99	7.06	7.12	7.19	7.25	7.32	1	1	2	3	3	4	4	5	6
.7	7.38	7.45	7.51	7.58	7.64	7.71	7.78	7.84	7.91	7.98	1	1	2	3	3	4	5	5	6
.8	8.04	8.11	8.18	8.25	8.31	8.38	8.45	8.51	8.58	8.65	1	1	2	3	3	4	5	5	6
.9	8.72	8.79	8.86	8.93	9.00	9.07	9.14	9.21	9.28	9.35	1	1	2	3	3	4	5	6	6
2.0	9.42	9.49	9.56	9.63	9.70	9.77	9.85	9.92	9.99	10.06	1	1	2	3	4	4	5	6	6
.1	10.13	10.21	10.28	10.35	10.43	10.50	10.57	10.65	10.72	10.79	1	1	2	3	4	4	5	6	7
.2	10.87	10.94	11.02	11.09	11.17	11.24	11.32	11.39	11.47	11.54	1	1	2	3	4	4	5	6	7
.3	11.62	11.69	11.77	11.84	11.92	11.99	12.07	12.15	12.23	12.30	1	2	2	3	4	5	5	6	7
.4	12.38	12.46	12.54	12.61	12.69	12.77	12.85	12.93	13.01	13.08	1	2	2	3	4	5	5	6	7
2.5	13.16	13.24	13.32	13.40	13.48	13.56	13.64	13.72	13.80	13.88	1	2	2	3	4	5	6	6	7
.6	13.96	14.04	14.12	14.20	14.28	14.37	14.45	14.53	14.61	14.69	1	2	2	3	4	5	6	7	7
.7	14.78	14.86	14.94	15.02	15.10	15.18	15.27	15.35	15.43	15.52	1	2	2	3	4	5	6	7	7
.8	15.60	15.68	15.77	15.85	15.94	16.02	16.11	16.19	16.28	16.36	1	2	3	3	4	5	6	7	8
.9	16.45	16.53	16.62	16.70	16.79	16.87	16.96	17.04	17.13	17.22	1	2	3	3	4	5	6	7	8
3.0	17.30	17.39	17.48	17.56	17.65	17.74	17.83	17.91	18.00	18.09	1	2	3	3	4	5	6	7	8
.1	18.18	18.27	18.35	18.44	18.53	18.62	18.70	18.79	18.88	18.97	1	2	3	4	4	5	6	7	8
.2	19.06	19.15	19.24	19.33	19.42	19.51	19.60	19.69	19.78	19.87	1	2	3	4	5	5	6	7	8
.3	19.96	20.05	20.14	20.24	20.33	20.42	20.51	20.60	20.69	20.79	1	2	3	4	5	5	6	7	8
.4	20.88	20.97	21.06	21.15	21.25	21.34	21.43	21.53	21.62	21.71	1	2	3	4	5	6	6	7	8
3.5	21.80	21.90	21.99	22.08	22.18	22.27	22.37	22.46	22.56	22.65	1	2	3	4	5	6	7	8	8
.6	22.75	22.84	22.94	23.03	23.13	23.22	23.32	23.41	23.51	23.60	1	2	3	4	5	6	7	8	9
.7	23.70	23.80	23.89	23.99	24.09	24.18	24.28	24.38	24.47	24.57	1	2	3	4	5	6	7	8	9
.8	24.67	24.77	24.86	24.96	25.06	25.15	25.25	25.35	25.45	25.55	1	2	3	4	5	6	7	8	9
.9	25.65	25.75	25.84	25.94	26.04	26.14	26.24	26.34	26.44	26.54	1	2	3	4	5	6	7	8	9
4.0	26.64	26.74	26.84	26.94	27.04	27.14	27.24	27.34	27.44	27.55	1	2	3	4	5	6	7	8	9
.1	27.65	27.75	27.85	27.95	28.05	28.15	28.26	28.35	28.46	28.56	1	2	3	4	5	6	7	8	9
.2	28.66	28.76	28.87	28.97	29.07	29.18	29.28	29.38	29.49	29.59	1	2	3	4	5	6	7	8	9
.3	29.69	29.80	29.90	30.00	30.11	30.21	30.32	30.42	30.53	30.63	1	2	3	4	5	6	7	8	9
.4	30.74	30.84	30.95	31.05	31.16	31.26	31.37	31.47	31.58	31.68	1	2	3	4	5	6	7	8	9
4.5	31.79	31.89	32.00	32.11	32.21	32.32	32.42	32.53	32.64	32.75	1	2	3	4	5	6	7	9	10
.6	32.85	32.96	33.07	33.18	33.28	33.39					1	2	3	4	5	6	8	9	10
.6						33.4	33.5	33.6	33.7	33.8	0	0	0	0	1	1	1	1	1
.7	33.9	34.0	34.1	34.3	34.4	34.5	34.6	34.7	34.8	34.9	0	0	0	0	1	1	1	1	1
.8	35.0	35.1	35.2	35.4	35.5	35.6	35.7	35.8	35.9	36.0	0	0	0	0	1	1	1	1	1
4.9	36.1	36.2	36.3	36.5	36.6	36.7	36.8	36.9	37.0	37.1	0	0	0	0	1	1	1	1	1
5	37.2	38.4	39.5	40.6	41.8	43.0	44.1	45.3	46.5	47.7	1	2	4	5	6	7	8	9	11
6	49.0	50.2	51.4	52.6	53.9	55.2	56.5	57.7	59.0	60.3	1	3	4	5	6	8	9	10	11
7	61.7	63.0	64.3	65.7	67.0	68.4	69.8	71.2	72.5	73.9	1	3	4	5	7	8	10	11	12
8	75.4	76.8	78.2	79.6	81.1	82.5	84.0	85.4	86.9	88.4	1	3	4	6	7	9	10	12	13
9	89.9	91.4	92.9	94.4	96.0	97.5	99.0	100.6	102.2	103.7	2	3	5	6	8	9	11	12	14
10	105.3	106.9	108.5	110.1	111.7	113.3	114.9	116.6	118.2	119.8	2	3	5	6	8	10	11	13	15
1	121.5	123.1	124.8	126.5	128.2	129.9	131.6	133.3	135.0	136.7	2	3	5	7	8	10	12	14	15
2	138.4	140.2	141.9	143.7	145.4	147.2	149.0	150.7	152.5	154.3	2	4	5	7	9	11	12	14	16
3	156.1	157.9	159.7	161.5	163.3	165.2	167.0	168.9	170.7	172.6	2	4	6	7	9	11	13	15	17
4	174.4	176.3	178.2	180.1	182.0	183.8	185.8	187.7	189.6	191.5	2	4	6	8	10	11	13	15	17
15	193.4	195.4	197.3	199.3	201.2	203.2	205.2	207.2	209.1	211.1	2	4	6	8	10	12	14	16	18
6	213.1	215.1	217.1	219.1	221.1	223.2	225.2	227.3	229.3	231.4	2	4	6	8	10	12	14	16	18
7	233.4	235.5	237.5	239.6	241.7	243.8	245.9	248.0	250.1	252.2	2	4	6	8	10	13	15	17	19
8	254.3	256.4	258.5	260.7	262.8	265.0	267.1	269.3	271.4	273.6	2	4	6	9	11	13	15	17	19
9	275.8	278.0	280.2	282.4	284.5	286.7	288.9	291.2	293.4	295.6	2	4	7	9	11	13	15	18	20
20	297.8																		

TABLE XXXII

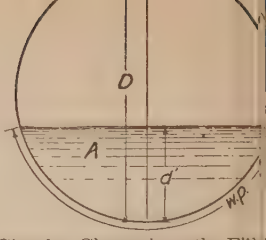
$$d' = \frac{D}{2}\left(1 - \cos\frac{\phi}{2}\right) \qquad \cos\frac{\phi}{2} = 1 - \frac{2d'}{D}$$

ϕ = angle subtended at center, in radians

$$w.p. = \frac{D}{2}\,\phi$$

$$A = \frac{D^2}{8}\,(\phi - \sin\phi)$$

$$R = \frac{A}{w.p.} = \frac{D}{4}\left(1 - \frac{\sin\phi}{\phi}\right)$$



Circular Channel partly Filled

$\frac{(w.p.)}{D}$ Wet perimeter

d'/D	0	1	2	3	4	5	6	7	8	9
0.0	0.000*	0.200*	0.284*	0.348*	0.403	0.451	0.495 +	0.536	0.574	0.609
.1	0.644	0.676	0.707	0.738	0.767	0.795 +	0.823	0.850 −	0.876	0.902
.2	0.927	0.952	0.976	1.000	1.024	1.047	1.070	1.093	1.115 +	1.137
.3	1.159	1.181	1.203	1.224	1.245 +	1.266	1.287	1.308	1.328	1.349
.4	1.369	1.390	1.410	1.430	1.450 +	1.471	1.491	1.511	1.531	1.551
0.5	1.571	1.591	1.611	1.631	1.651	1.671	1.691	1.711	1.731	1.752
.6	1.772	1.793	1.813	1.834	1.855 −	1.875 +	1.897	1.918	1.939	1.961
.7	1.982	2.004	2.026	2.049	2.071	2.094	2.118	2.141	2.165 +	2.190
.8	2.214	2.240	2.265 +	2.292	2.319	2.346	2.375 −	2.404	2.434	2.465
.9	2.498	2.532	2.568	2.606	2.647	2.691	2.739*	2.793*	2.858*	2.941*
1.0	3.142									

A/D^2 Wet area of cross section

d'/D	0	1	2	3	4	5	6	7	8	9
0.0	0.0000*	.0013*	.0037*	.0069*	.0105 +	.0147	.0192	.0242	.0294	.0350
.1	.0409	.0470	.0534	.0600	.0668	.0739	.0811	.0885 +	.0961	.1039
.2	.1118	.1199	.1281	.1365	.1449	.1535 +	.1623	.1711	.1800	.1890
.3	.1982	.2074	.2167	.2260	.2355 −	.2450 −	.2545 +	.2642	.2739	.2836
.4	.2934	.3032	.3130	.3229	.3328	.3428	.3527	.3627	.3727	.3827
0.5	.3927	.4027	.4127	.4227	.4327	.4426	.4526	.4625 −	.4724	.4822
.6	.4920	.5018	.5115 +	.5212	.5308	.5404	.5499	.5594	.5687	.5780
.7	.5872	.5964	.6054	.6143	.6231	.6319	.6405 −	.6489	.6573	.6655
.8	.6736	.6815 −	.6893	.6969	.7043	.7115 +	.7186	.7254	.7320	.7384
.9	.7445 +	.7504	.7560	.7612	.7662	.7707	.7749*	.7785 + *	.7816*	.7841*
1.0	.7854									

$\frac{R}{D} = \frac{A/D^2}{(w.p.)/D}$ Mean hydraulic radius

d'/D	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0066	.0132	.0197	.0262	.0326	.0389	.0451	.0513	.0575
.1	.0635	.0695	.0755	.0813	.0871	.0929	.0986	.1042	.1097	.1152
.2	.1206	.1259	.1312	.1364	.1416	.1466	.1516	.1566	.1614	.1662
.3	.1709	.1756	.1802	.1847	.1891	.1935 −	.1978	.2020	.2062	.2102
.4	.2142	.2181	.2220	.2258	.2295 −	.2331	.2366	.2401	.2435 −	.2468
0.5	.2500	.2531	.2562	.2592	.2621	.2649	.2676	.2703	.2728	.2753
.6	.2776	.2799	.2821	.2842	.2862	.2881	.2900	.2917	.2933	.2948
.7	.2962	.2975	.2987	.2998	.3008	.3017	.3024	.3031	.3036	.3039
.8	.3042	.3043	.3043	.3041	.3038	.3033	.3026	.3018	.3007	.2995
.9	.2980	.2963	.2944	.2921	.2895 −	.2865 −	.2829*	.2787*	.2735 + *	.2666
1.0	.2500									

* Interpolation at points marked (*) may be inaccurate.

TABLE XXXIII

Circumferences of Circles by Hundredths

These two pages give the circumferences of circles for any diameter n, from $n = 1$ to $n = 10$.
To obtain the circumference of a circle whose diameter is outside the range from 1 to 10, note that moving the decimal point column n is equivalent to moving it the same number of places in the body of the table.
The table serves also as a table of the multiples of π.

(n = diameter)

$n\pi$											Tenths of the Tabular Difference								
n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.0	3.142	3.173	3.204	3.236	3.267	3.299	3.330	3.362	3.393	3.424	3	6	9	13	16	19	22	25	28
.1	3.456	3.487	3.519	3.550	3.581	3.613	3.644	3.676	3.707	3.738	3	6	9	13	16	19	22	25	28
.2	3.770	3.801	3.833	3.864	3.896	3.927	3.958	3.990	4.021	4.053	3	6	9	13	16	19	22	25	28
.3	4.084	4.115	4.147	4.178	4.210	4.241	4.273	4.304	4.335	4.367	3	6	9	13	16	19	22	25	28
.4	4.398	4.430	4.461	4.492	4.524	4.555	4.587	4.618	4.650	4.681	3	6	9	13	16	19	22	25	28
.5	4.712	4.744	4.775	4.807	4.838	4.869	4.901	4.932	4.964	4.995	3	6	9	13	16	19	22	25	28
.6	5.027	5.058	5.089	5.121	5.152	5.184	5.215	5.246	5.278	5.309	3	6	9	13	16	19	22	25	28
.7	5.341	5.372	5.404	5.435	5.466	5.498	5.529	5.561	5.592	5.623	3	6	9	13	16	19	22	25	28
.8	5.655	5.686	5.718	5.749	5.781	5.812	5.843	5.875	5.906	5.938	3	6	9	13	16	19	22	25	28
.9	5.969	6.000	6.032	6.063	6.095	6.126	6.158	6.189	6.220	6.252	3	6	9	13	16	19	22	25	28
2.0	6.283	6.315	6.346	6.377	6.409	6.440	6.472	6.503	6.535	6.566	3	6	9	13	16	19	22	25	28
.1	6.597	6.629	6.660	6.692	6.723	6.754	6.786	6.817	6.849	6.880	3	6	9	13	16	19	22	25	28
.2	6.912	6.943	6.974	7.006	7.037	7.069	7.100	7.131	7.163	7.194	3	6	9	13	16	19	22	25	28
.3	7.226	7.257	7.288	7.320	7.351	7.383	7.414	7.446	7.477	7.508	3	6	9	13	16	19	22	25	28
.4	7.540	7.571	7.603	7.634	7.665	7.697	7.728	7.760	7.791	7.823	3	6	9	13	16	19	22	25	28
2.5	7.854	7.885	7.917	7.948	7.980	8.011	8.042	8.074	8.105	8.137	3	6	9	13	16	19	22	25	28
.6	8.168	8.200	8.231	8.262	8.294	8.325	8.357	8.388	8.419	8.451	3	6	9	13	16	19	22	25	28
.7	8.482	8.514	8.545	8.577	8.608	8.639	8.671	8.702	8.734	8.765	3	6	9	13	16	19	22	25	28
.8	8.796	8.828	8.859	8.891	8.922	8.954	8.985	9.016	9.048	9.079	3	6	9	13	16	19	22	25	28
.9	9.111	9.142	9.173	9.205	9.236	9.268	9.299	9.331	9.362	9.393	3	6	9	13	16	19	22	25	28
3.0	9.425	9.456	9.488	9.519	9.550	9.582	9.613	9.645	9.676	9.708	3	6	9	13	16	19	22	25	28
.1	9.739	9.770	9.802	9.833	9.865	9.896	9.927	9.959	9.990	10.022	3	6	9	13	16	19	22	25	28
.2	10.05	10.08	10.12	10.15	10.18	10.21	10.24	10.27	10.30	10.34	0	1	1	1	2	2	2	3	3
.3	10.37	10.40	10.43	10.46	10.49	10.52	10.56	10.59	10.62	10.65	0	1	1	1	2	2	2	3	3
.4	10.68	10.71	10.74	10.78	10.81	10.84	10.87	10.90	10.93	10.96	0	1	1	1	2	2	2	3	3
3.5	11.00	11.03	11.06	11.09	11.12	11.15	11.18	11.22	11.25	11.28	0	1	1	1	2	2	2	3	3
.6	11.31	11.34	11.37	11.40	11.44	11.47	11.50	11.53	11.56	11.59	0	1	1	1	2	2	2	3	3
.7	11.62	11.66	11.69	11.72	11.75	11.78	11.81	11.84	11.88	11.91	0	1	1	1	2	2	2	3	3
.8	11.94	11.97	12.00	12.03	12.06	12.10	12.13	12.16	12.19	12.22	0	1	1	1	2	2	2	3	3
.9	12.25	12.28	12.32	12.35	12.38	12.41	12.44	12.47	12.50	12.53	0	1	1	1	2	2	2	3	3
4.0	12.57	12.60	12.63	12.66	12.69	12.72	12.75	12.79	12.82	12.85	0	1	1	1	2	2	2	3	3
.1	12.88	12.91	12.94	12.97	13.01	13.04	13.07	13.10	13.13	13.16	0	1	1	1	2	2	2	3	3
.2	13.19	13.23	13.26	13.29	13.32	13.35	13.38	13.41	13.45	13.48	0	1	1	1	2	2	2	3	3
.3	13.51	13.54	13.57	13.60	13.63	13.67	13.70	13.73	13.76	13.79	0	1	1	1	2	2	2	3	3
.4	13.82	13.85	13.89	13.92	13.95	13.98	14.01	14.04	14.07	14.11	0	1	1	1	2	2	2	3	3
4.5	14.14	14.17	14.20	14.23	14.26	14.29	14.33	14.36	14.39	14.42	0	1	1	1	2	2	2	3	3
.6	14.45	14.48	14.51	14.55	14.58	14.61	14.64	14.67	14.70	14.73	0	1	1	1	2	2	2	3	3
.7	14.77	14.80	14.83	14.86	14.89	14.92	14.95	14.99	15.02	15.05	0	1	1	1	2	2	2	3	3
.8	15.08	15.11	15.14	15.17	15.21	15.24	15.27	15.30	15.33	15.36	0	1	1	1	2	2	2	3	3
.9	15.39	15.43	15.46	15.49	15.52	15.55	15.58	15.61	15.65	15.68	0	1	1	1	2	2	2	3	3

CIRCUMFERENCES OF CIRCLES BY HUNDREDTHS

(n = diameter)

$n\pi$											TENTHS OF THE TABULAR DIFFERENCE							
n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8
5.0	15.71	15.74	15.77	15.80	15.83	15.87	15.90	15.93	15.96	15.99	0	1	1	1	2	2	2	3
.1	16.02	16.05	16.08	16.12	16.15	16.18	16.21	16.24	16.27	16.30	0	1	1	1	2	2	2	3
.2	16.34	16.37	16.40	16.43	16.46	16.49	16.52	16.56	16.59	16.62	0	1	1	1	2	2	2	3
.3	16.65	16.68	16.71	16.74	16.78	16.81	16.84	16.87	16.90	16.93	0	1	1	1	2	2	2	3
.4	16.96	17.00	17.03	17.06	17.09	17.12	17.15	17.18	17.22	17.25	0	1	1	1	2	2	2	3
5.5	17.28	17.31	17.34	17.37	17.40	17.44	17.47	17.50	17.53	17.56	0	1	1	1	2	2	2	3
.6	17.59	17.62	17.66	17.69	17.72	17.75	17.78	17.81	17.84	17.88	0	1	1	1	2	2	2	3
.7	17.91	17.94	17.97	18.00	18.03	18.06	18.10	18.13	18.16	18.19	0	1	1	1	2	2	2	3
.8	18.22	18.25	18.28	18.32	18.35	18.38	18.41	18.44	18.47	18.50	0	1	1	1	2	2	2	3
.9	18.54	18.57	18.60	18.63	18.66	18.69	18.72	18.76	18.79	18.82	0	1	1	1	2	2	2	3
6.0	18.85−	18.88	18.91	18.94	18.98	19.01	19.04	19.07	19.10	19.13	0	1	1	1	2	2	2	3
.1	19.16	19.20	19.23	19.26	19.29	19.32	19.35+	19.38	19.42	19.45−	0	1	1	1	2	2	2	3
.2	19.48	19.51	19.54	19.57	19.60	19.63	19.67	19.70	19.73	19.76	0	1	1	1	2	2	2	3
.3	19.79	19.82	19.85+	19.89	19.92	19.95−	19.98	20.01	20.04	20.07	0	1	1	1	2	2	2	3
.4	20.11	20.14	20.17	20.20	20.23	20.26	20.29	20.33	20.36	20.39	0	1	1	1	2	2	2	3
6.5	20.42	20.45+	20.48	20.51	20.55−	20.58	20.61	20.64	20.67	20.70	0	1	1	1	2	2	2	3
.6	20.73	20.77	20.80	20.83	20.86	20.89	20.92	20.95+	20.99	21.02	0	1	1	1	2	2	2	3
.7	21.05−	21.08	21.11	21.14	21.17	21.21	21.24	21.27	21.30	21.33	0	1	1	1	2	2	2	3
.8	21.36	21.39	21.43	21.46	21.49	21.52	21.55+	21.58	21.61	21.65−	0	1	1	1	2	2	2	3
.9	21.68	21.71	21.74	21.77	21.80	21.83	21.87	21.90	21.93	21.96	0	1	1	1	2	2	2	3
7.0	21.99	22.02	22.05+	22.09	22.12	22.15−	22.18	22.21	22.24	22.27	0	1	1	1	2	2	2	3
.1	22.31	22.34	22.37	22.40	22.43	22.46	22.49	22.53	22.56	22.59	0	1	1	1	2	2	2	3
.2	22.62	22.65+	22.68	22.71	22.75−	22.78	22.81	22.84	22.87	22.90	0	1	1	1	2	2	2	3
.3	22.93	22.97	23.00	23.03	23.06	23.09	23.12	23.15+	23.18	23.22	0	1	1	1	2	2	2	3
.4	23.25−	23.28	23.31	23.34	23.37	23.40	23.44	23.47	23.50−	23.53	0	1	1	1	2	2	2	3
7.5	23.56	23.59	23.62	23.66	23.69	23.72	23.75+	23.78	23.81	23.84	0	1	1	1	2	2	2	3
.6	23.88	23.91	23.94	23.97	24.00	24.03	24.06	24.10	24.13	24.16	0	1	1	1	2	2	2	3
.7	24.19	24.22	24.25+	24.28	24.32	24.35−	24.38	24.41	24.44	24.47	0	1	1	1	2	2	2	3
.8	24.50+	24.54	24.57	24.60	24.63	24.66	24.69	24.72	24.76	24.79	0	1	1	1	2	2	2	3
.9	24.82	24.85−	24.88	24.91	24.94	24.98	25.01	25.04	25.07	25.10	0	1	1	1	2	2	2	3
8.0	25.13	25.16	25.20	25.23	25.26	25.29	25.32	25.35+	25.38	25.42	0	1	1	1	2	2	2	3
.1	25.45−	25.48	25.51	25.54	25.57	25.60	25.64	25.67	25.70	25.73	0	1	1	1	2	2	2	3
.2	25.76	25.79	25.82	25.86	25.89	25.92	25.95−	25.98	26.01	26.04	0	1	1	1	2	2	2	3
.3	26.08	26.11	26.14	26.17	26.20	26.23	26.26	26.30	26.33	26.36	0	1	1	1	2	2	2	3
.4	26.39	26.42	26.45+	26.48	26.52	26.55−	26.58	26.61	26.64	26.67	0	1	1	1	2	2	2	3
8.5	26.70	26.73	26.77	26.80	26.83	26.86	26.89	26.92	26.95+	26.99	0	1	1	1	2	2	2	3
.6	27.02	27.05−	27.08	27.11	27.14	27.17	27.21	27.24	27.27	27.30	0	1	1	1	2	2	2	3
.7	27.33	27.36	27.39	27.43	27.46	27.49	27.52	27.55+	27.58	27.61	0	1	1	1	2	2	2	3
.8	27.65−	27.68	27.71	27.74	27.77	27.80	27.83	27.87	27.90	27.93	0	1	1	1	2	2	2	3
.9	27.96	27.99	28.02	28.05+	28.09	28.12	28.15−	28.18	28.21	28.24	0	1	1	1	2	2	2	3
9.0	28.27	28.31	28.34	28.37	28.40	28.43	28.46	28.49	28.53	28.56	0	1	1	1	2	2	2	3
.1	28.59	28.62	28.65+	28.68	28.71	28.75−	28.78	28.81	28.84	28.87	0	1	1	1	2	2	2	3
.2	28.90	28.93	28.97	29.00	29.03	29.06	29.09	29.12	29.15+	29.19	0	1	1	1	2	2	2	3
.3	29.22	29.25−	29.28	29.31	29.34	29.37	29.41	29.44	29.47	29.50−	0	1	1	1	2	2	2	3
.4	29.53	29.56	29.59	29.63	29.66	29.69	29.72	29.75+	29.78	29.81	0	1	1	1	2	2	2	3
9.5	29.85−	29.88	29.91	29.94	29.97	30.00	30.03	30.07	30.10	30.13	0	1	1	1	2	2	2	3
.6	30.16	30.19	30.22	30.25+	30.28	30.32	30.35−	30.38	30.41	30.44	0	1	1	1	2	2	2	3
.7	30.47	30.50+	30.54	30.57	30.60	30.63	30.66	30.69	30.72	30.76	0	1	1	1	2	2	2	3
.8	30.79	30.82	30.85+	30.88	30.91	30.94	30.98	31.01	31.04	31.07	0	1	1	1	2	2	2	3
.9	31.10	31.13	31.16	31.20	31.23	31.26	31.29	31.32	31.35+	31.38	0	1	1	1	2	2	2	3
10.0	31.42																	

TABLE XXXIV

Area of Circles by Hundredths

(n = diameter)

These two pages give the area of circles for any diameter n, from $n = 1$ to $n = 10$. To obtain the area of a circle whose diameter is outside the range from 1 to 10, note that moving the decimal point one place in column n is equivalent to moving it two places in the body of the table. (Compare heading to table of squares.)

$\frac{\pi}{4}n^2$

n	0	1	2	3	4	5	6	7	8	9	Tenths of the Tabular Difference								
											1	2	3	4	5	6	7	8	9
1.0	0.785	0.801	0.817	0.833	0.849	0.866	0.882	0.899	0.916	0.933	2	3	5	7	8	10	12	13	15
1.1	0.950	0.968	0.985	1.003	1.021	1.039	1.057	1.075	1.094	1.112	2	4	5	7	9	11	13	14	16
1.2	1.131	1.150	1.169	1.188	1.208	1.227	1.247	1.267	1.287	1.307	2	4	6	8	10	12	14	16	18
1.3	1.327	1.348	1.368	1.389	1.410	1.431	1.453	1.474	1.496	1.517	2	4	6	8	11	13	15	17	19
1.4	1.539	1.561	1.584	1.606	1.629	1.651	1.674	1.697	1.720	1.744	2	5	7	9	11	14	16	18	21
1.5	1.767	1.791	1.815	1.839	1.863	1.887	1.911	1.936	1.961	1.986	2	5	7	10	12	15	17	19	22
1.6	2.011	2.036	2.061	2.087	2.112	2.138	2.164	2.190	2.217	2.243	3	5	8	10	13	16	18	21	23
1.7	2.270	2.297	2.324	2.351	2.378	2.405	2.433	2.461	2.488	2.516	3	5	8	11	14	16	19	22	25
1.8	2.545	2.573	2.602	2.630	2.659	2.688	2.717	2.746	2.776	2.806	3	6	9	12	15	17	20	23	26
1.9	2.835	2.865	2.895	2.926	2.956	2.986	3.017	3.048	3.079	3.110	3	6	9	12	15	18	21	25	28
2.0	3.142	3.173	3.205	3.237	3.269	3.301	3.333	3.365	3.398	3.431	3	6	10	13	16	19	23	26	29
2.1	3.464	3.497	3.530	3.563	3.597	3.631	3.664	3.698	3.733	3.767	3	7	10	14	17	20	24	27	30
2.2	3.801	3.836	3.871	3.906	3.941	3.976	4.011	4.047	4.083	4.119	4	7	11	14	18	21	25	28	32
2.3	4.155	4.191	4.227	4.264	4.301	4.337	4.374	4.412	4.449	4.486	4	7	11	15	18	22	26	30	33
2.4	4.524	4.562	4.600	4.638	4.676	4.714	4.753	4.792	4.831	4.870	4	8	12	15	19	23	27	31	35
2.5	4.909	4.948	4.988	5.027	5.067	5.107	5.147	5.187	5.228	5.269	4	8	12	16	20	24	28	32	36
2.6	5.309	5.350	5.391	5.433	5.474	5.515	5.557	5.599	5.641	5.683	4	8	12	17	21	25	29	33	37
2.7	5.726	5.768	5.811	5.853	5.896	5.940	5.983	6.026	6.070	6.114	4	9	13	17	22	26	30	35	39
2.8	6.158	6.202	6.246	6.290	6.335	6.379	6.424	6.469	6.514	6.560	4	9	13	18	22	27	31	36	40
2.9	6.605	6.651	6.697	6.743	6.789	6.835	6.881	6.928	6.975	7.022	5	9	14	19	23	28	32	37	42
3.0	7.069	7.116	7.163	7.211	7.258	7.306	7.354	7.402	7.451	7.499	5	10	14	19	24	29	34	38	43
3.1	7.548	7.596	7.645	7.694	7.744	7.793	7.843	7.892	7.942	7.992	5	10	15	20	25	30	35	40	45
3.2	8.042	8.093	8.143	8.194	8.245	8.296	8.347	8.398	8.450	8.501	5	10	15	20	26	31	36	41	46
3.3	8.553	8.605	8.657	8.709	8.762	8.814	8.867	8.920	8.973	9.026	5	11	16	21	26	32	37	42	47
3.4	9.079	9.133	9.186	9.240	9.294	9.348	9.402	9.457	9.511	9.566	5	11	16	22	27	33	38	43	49
3.5	9.621	9.676	9.731	9.787	9.842	9.898	9.954	10.010			6	11	17	22	28	33	39	44	50
								10.01	10.07	10.12	1	1	2	2	3	3	4	5	5
3.6	10.18	10.24	10.29	10.35	10.41	10.46	10.52	10.58	10.64	10.69	1	1	2	2	3	3	4	5	5
3.7	10.75	10.81	10.87	10.93	10.99	11.04	11.10	11.16	11.22	11.28	1	1	2	2	3	4	4	5	5
3.8	11.34	11.40	11.46	11.52	11.58	11.64	11.70	11.76	11.82	11.88	1	1	2	2	3	4	4	5	5
3.9	11.95	12.01	12.07	12.13	12.19	12.25	12.32	12.38	12.44	12.50	1	1	2	2	3	4	4	5	6
4.0	12.57	12.63	12.69	12.76	12.82	12.88	12.95	13.01	13.07	13.14	1	1	2	3	3	4	4	5	6
4.1	13.20	13.27	13.33	13.40	13.46	13.53	13.59	13.66	13.72	13.79	1	1	2	3	3	4	5	5	6
4.2	13.85	13.92	13.99	14.05	14.12	14.19	14.25	14.32	14.39	14.45	1	1	2	3	3	4	5	5	6
4.3	14.52	14.59	14.66	14.73	14.79	14.86	14.93	15.00	15.07	15.14	1	1	2	3	3	4	5	5	6
4.4	15.21	15.27	15.34	15.41	15.48	15.55	15.62	15.69	15.76	15.83	1	1	2	3	3	4	5	6	6
4.5	15.90	15.98	16.05	16.12	16.19	16.26	16.33	16.40	16.47	16.55	1	1	2	3	4	4	5	6	6
4.6	16.62	16.69	16.76	16.84	16.91	16.98	17.06	17.13	17.20	17.28	1	1	2	3	4	4	5	6	7
4.7	17.35	17.42	17.50	17.57	17.65	17.72	17.80	17.87	17.95	18.02	1	1	2	3	4	4	5	6	7
4.8	18.10	18.17	18.25	18.32	18.40	18.47	18.55	18.63	18.70	18.78	1	2	2	3	4	5	5	6	7
4.9	18.86	18.93	19.01	19.09	19.17	19.24	19.32	19.40	19.48	19.56	1	2	2	3	4	5	5	6	7

TABLE XXXIV — *Continued*

AREAS OF CIRCLES BY HUNDREDTHS

$\frac{\pi}{4}n^2$ (n = diameter)

TENTHS OF THE TABULAR DIFFERENCE

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5.0	19.63	19.71	19.79	19.87	19.95	20.03	20.11	20.19	20.27	20.35	1	2	2	3	4	5	6	6	7
.1	20.43	20.51	20.59	20.67	20.75	20.83	20.91	20.99	21.07	21.16	1	2	2	3	4	5	6	6	7
.2	21.24	21.32	21.40	21.48	21.57	21.65	21.73	21.81	21.90	21.98	1	2	2	3	4	5	6	7	7
.3	22.06	22.15	22.23	22.31	22.40	22.48	22.56	22.65	22.73	22.82	1	2	3	3	4	5	6	7	8
.4	22.90	22.99	23.07	23.16	23.24	23.33	23.41	23.50	23.59	23.67	1	2	3	3	4	5	6	7	8
5.5	23.76	23.84	23.93	24.02	24.11	24.19	24.28	24.37	24.45	24.54	1	2	3	3	4	5	6	7	8
.6	24.63	24.72	24.81	24.89	24.98	25.07	25.16	25.25	25.34	25.43	1	2	3	4	4	5	6	7	8
.7	25.52	25.61	25.70	25.79	25.88	25.97	26.06	26.15	26.24	26.33	1	2	3	4	5	5	6	7	8
.8	26.42	26.51	26.60	26.69	26.79	26.88	26.97	27.06	27.15	27.25	1	2	3	4	5	6	6	7	8
.9	27.34	27.43	27.53	27.62	27.71	27.81	27.90	27.99	28.09	28.18	1	2	3	4	5	6	7	7	8
6.0	28.27	28.37	28.46	28.56	28.65	28.75	28.84	28.94	29.03	29.13	1	2	3	4	5	6	7	8	9
.1	29.22	29.32	29.42	29.51	29.61	29.71	29.80	29.90	30.00	30.09	1	2	3	4	5	6	7	8	9
.2	30.19	30.29	30.39	30.48	30.58	30.68	30.78	30.88	30.97	31.07	1	2	3	4	5	6	7	8	9
.3	31.17	31.27	31.37	31.47	31.57	31.67	31.77	31.87	31.97	32.07	1	2	3	4	5	6	7	8	9
.4	32.17	32.27	32.37	32.47	32.57	32.67	32.78	32.88	32.98	33.08	1	2	3	4	5	6	7	8	9
6.5	33.18	33.29	33.39	33.49	33.59	33.70	33.80	33.90	34.00	34.11	1	2	3	4	5	6	7	8	9
.6	34.21	34.32	34.42	34.52	34.63	34.73	34.84	34.94	35.05	35.15	1	2	3	4	5	6	7	8	9
.7	35.26	35.36	35.47	35.57	35.68	35.78	35.89	36.00	36.10	36.21	1	2	3	4	5	6	7	8	10
.8	36.32	36.42	36.53	36.64	36.75	36.85	36.96	37.07	37.18	37.28	1	2	3	4	5	6	8	9	10
.9	37.39	37.50	37.61	37.72	37.83	37.94	38.05	38.16	38.26	38.37	1	2	3	4	5	7	8	9	10
7.0	38.48	38.59	38.70	38.82	38.93	39.04	39.15	39.26	39.37	39.48	1	2	3	4	6	7	8	9	10
.1	39.59	39.70	39.82	39.93	40.04	40.15	40.26	40.38	40.49	40.60	1	2	3	4	6	7	8	9	10
.2	40.72	40.83	40.94	41.06	41.17	41.28	41.40	41.51	41.62	41.74	1	2	3	5	6	7	8	9	10
.3	41.85	41.97	42.08	42.20	42.31	42.43	42.54	42.66	42.78	42.89	1	2	3	5	6	7	8	9	10
.4	43.01	43.12	43.24	43.36	43.47	43.59	43.71	43.83	43.94	44.06	1	2	4	5	6	7	8	9	11
7.5	44.18	44.30	44.41	44.53	44.65	44.77	44.89	45.01	45.13	45.25	1	2	4	5	6	7	8	9	11
.6	45.36	45.48	45.60	45.72	45.84	45.96	46.08	46.20	46.32	46.45	1	2	4	5	6	7	8	10	11
.7	46.57	46.69	46.81	46.93	47.05	47.17	47.29	47.42	47.54	47.66	1	2	4	5	6	7	9	10	11
.8	47.78	47.91	48.03	48.15	48.27	48.40	48.52	48.65	48.77	48.89	1	2	4	5	6	7	9	10	11
.9	49.02	49.14	49.27	49.39	49.51	49.64	49.76	49.89	50.01	50.14	1	2	4	5	6	7	9	10	11
8.0	50.27	50.39	50.52	50.64	50.77	50.90	51.02	51.15	51.28	51.40	1	3	4	5	6	8	9	10	11
.1	51.53	51.66	51.78	51.91	52.04	52.17	52.30	52.42	52.55	52.68	1	3	4	5	6	8	9	10	12
.2	52.81	52.94	53.07	53.20	53.33	53.46	53.59	53.72	53.85	53.98	1	3	4	5	6	8	9	10	12
.3	54.11	54.24	54.37	54.50	54.63	54.76	54.89	55.02	55.15	55.29	1	3	4	5	7	8	9	10	12
.4	55.42	55.55	55.68	55.81	55.95	56.08	56.21	56.35	56.48	56.61	1	3	4	5	7	8	9	11	12
8.5	56.75	56.88	57.01	57.15	57.28	57.41	57.55	57.68	57.82	57.95	1	3	4	5	7	8	9	11	12
.6	58.09	58.22	58.36	58.49	58.63	58.77	58.90	59.04	59.17	59.31	1	3	4	5	7	8	10	11	12
.7	59.45	59.58	59.72	59.86	59.99	60.13	60.27	60.41	60.55	60.68	1	3	4	5	7	8	10	11	12
.8	60.82	60.96	61.10	61.24	61.38	61.51	61.65	61.79	61.93	62.07	1	3	4	6	7	8	10	11	13
.9	62.21	62.35	62.49	62.63	62.77	62.91	63.05	63.19	63.33	63.48	1	3	4	6	7	8	10	11	13
9.0	63.62	63.76	63.90	64.04	64.18	64.33	64.47	64.61	64.75	64.90	1	3	4	6	7	9	10	11	13
.1	65.04	65.18	65.33	65.47	65.61	65.76	65.90	66.04	66.19	66.33	1	3	4	6	7	9	10	11	13
.2	66.48	66.62	66.77	66.91	67.06	67.20	67.35	67.49	67.64	67.78	1	3	4	6	7	9	10	12	13
.3	67.93	68.08	68.22	68.37	68.51	68.66	68.81	68.96	69.10	69.25	1	3	4	6	7	9	10	12	13
.4	69.40	69.55	69.69	69.84	69.99	70.14	70.29	70.44	70.58	70.73	1	3	4	6	7	9	10	12	13
9.5	70.88	71.03	71.18	71.33	71.48	71.63	71.78	71.93	72.08	72.23	2	3	5	6	8	9	11	12	14
.6	72.38	72.53	72.68	72.84	72.99	73.14	73.29	73.44	73.59	73.75	2	3	5	6	8	9	11	12	14
.7	73.90	74.05	74.20	74.36	74.51	74.66	74.82	74.97	75.12	75.28	2	3	5	6	8	9	11	12	14
.8	75.43	75.58	75.74	75.89	76.05	76.20	76.36	76.51	76.67	76.82	2	3	5	6	8	9	11	12	14
.9	76.98	77.13	77.29	77.44	77.60	77.76	77.91	78.07	78.23	78.38	2	3	5	6	8	9	11	13	14

TABLE XXXV

These two pages give the **squares** of all numbers between 1 and 10, correct to four significant figures. For example: $(6.283)^2 = 39.48$. (Values obtained by interpolation may be in error by 1 in the last figure.)

To obtain the square of any number outside the range from 1 to 10, write the number as the product of: (a) some number between 1 and 10, and (b) some (positive or negative) power of 10; then proceed as in the following examples:

$$(62\,830)^2 = (6.283 \times 10^4)^2 = (6.283)^2 \times (10^4)^2 = 39.48 \times 10^8 = 3\,948\,000\,000.$$
$$(.06283)^2 = (6.283 \times 10^{-2})^2 = (6.283)^2 \times (10^{-2})^2 = 39.48 \times 10^{-4} = 0.003\,948.$$

In short, moving the decimal point ONE *place in column* n *is equivalent to moving it* TWO *places in the body of the table.*

The table used **inversely** gives the **square roots** of all numbers between 1 and 10^2. For example:

$$\sqrt{3.142} = 1.7725; \qquad \sqrt{31.42} = 5.606.$$

To obtain the square root of any number outside the range from 1 to 10^2, write the number as the product of: (a) some number between 1 and 10^2, and (b) some (positive or negative) power of 10^2; thus:

$$\sqrt{3\,142\,000} = \sqrt{3.142 \times 10^6} = \sqrt{3.142} \times \sqrt{10^6} = 1.7725 \times 10^3 = 1772.5$$
$$\sqrt{0.3142} = \sqrt{31.42 \times 10^{-2}} = \sqrt{31.42} \times \sqrt{10^{-2}} = 5.606 \times 10^{-1} = 0.5606$$

The bold-faced figures in the body of the table facilitate its use as an inverse table.

n^2 SQUARES

n	0	1	2	3	4	5	6	7	8	9	Tenths of the Tabular Difference 1 2 3	4 5 6	7 8 9
1.0	**1**.000	1.020	1.040	1.061	1.082	1.103	1.124	1.145 −	1.166	1.188			
.1	1.210	1.232	1.254	1.277	1.300	1.323	1.346	1.369	1.392	1.416			
.2	1.440	1.464	1.488	1.513	1.538	1.563	1.588	1.613	1.638	1.664			
.3	1.690	1.716	1.742	1.769	1.796	1.823	1.850 −	1.877	1.904	1.932			
.4	1.960	1.988	**2**.016	2.045 −	2.074	2.103	2.132	2.161	2.190	2.220			
1.5	2.250	2.280	2.310	2.341	2.372	2.403	2.434	2.465 −	2.496	2.528			
.6	2.560	2.592	2.624	2.657	2.690	2.723	2.756	2.789	2.822	2.856			
.7	2.890	2.924	2.958	2.993	**3**.028	3.063	3.098	3.133	3.168	3.204			
.8	3.240	3.276	3.312	3.349	3.386	3.423	3.460	3.497	3.534	3.572			
.9	3.610	3.648	3.686	3.725 −	3.764	3.803	3.842	3.881	3.920	3.960			
2.0	**4**.000	4.040	4.080	4.121	4.162	4.203	4.244	4.285 −	4.326	4.368			
.1	4.410	4.452	4.494	4.537	4.580	4.623	4.666	4.709	4.752	4.796			
.2	4.840	4.884	4.928	4.973	**5**.018	5.063	5.108	5.153	5.198	5.244			
.3	5.290	5.336	5.382	5.429	5.476	5.523	5.570	5.617	5.664	5.712			
.4	5.760	5.808	5.856	5.905 −	5.954	**6**.003	6.052	6.101	6.150	6.200			
2.5	6.250	6.300	6.350 +	6.401	6.452	6.503	6.554	6.605 −	6.656	6.708			
.6	6.760	6.812	6.864	6.917	6.970	**7**.023	7.076	7.129	7.182	7.236			
.7	7.290	7.344	7.398	7.453	7.508	7.563	7.618	7.673	7.728	7.784			
.8	7.840	7.896	7.952	**8**.009	8.066	8.123	8.180	8.237	8.294	8.352			
.9	8.410	8.468	8.526	8.585 −	8.644	8.703	8.762	8.821	8.880	8.940			
3.0	**9**.000	9.060	9.120	9.181	9.242	9.303	9.364	9.425 −	9.486	9.548			
.1	9.610	9.672	9.734	9.797	9 860	9.923	9.986	10.049					
.1								**10**.05 −	10.11	10.18	1 1 2	3 3 4	4 5 6
.2	10.24	10.30	10.37	10.43	10.50 −	10.56	10.63	10.69	10.76	10.82	1 1 2	3 3 4	5 5 6
.3	10.89	10.96	11.02	11.09	11.16	11.22	11.29	11.36	11.42	11.49	1 1 2	3 3 4	5 5 6
.4	11.56	11.63	11.70	11.76	11.83	11.90	11.97	12.04	12.11	12.18	1 1 2	3 3 4	5 6 6
3.5	12.25	12.32	12.39	12.46	12.53	12.60	12.67	12.74	12.82	12.89	1 1 2	3 4 4	5 6 6
.6	12.96	13.03	13.10	13.18	13.25 −	13.32	13.40	13.47	13.54	13.62	1 1 2	3 4 4	5 6 7
.7	13.69	13.76	13.84	13.91	13.99	14.06	14.14	14.21	14.29	14.36	1 1 2	3 4 4	5 6 7
.8	14.44	14.52	14.59	14.67	14.75 −	14.82	14.90	14.98	15.05 +	15.13	1 2 2	3 4 5	5 6 7
.9	15.21	15.29	15.37	15.44	15.52	15.60	15.68	15.76	15.84	15.92	1 2 2	3 4 5	6 6 7
4.0	16.00	16.08	16.16	16.24	16.32	16.40	16.48	16.56	16.65 −	16.73	1 2 2	3 4 5	6 6 7
.1	16.81	16.89	16.97	17.06	17.14	17.22	17.31	17.39	17.47	17.56	1 2 2	3 4 5	6 7 7
.2	17.64	17.72	17.81	17.89	17.98	18.06	18.15 −	18.23	18.32	18.40	1 2 3	3 4 5	6 7 8
.3	18.49	18.58	18.66	18.75 −	18.84	18.92	19.01	19.10	19.18	19.27	1 2 3	3 4 5	6 7 8
.4	19.36	19.45 −	19.54	19.62	19.71	19.80	19.89	19.98	**20**.07	20.16	1 2 3	4 4 5	6 7 8
4.5	20.25	20.34	20.43	20.52	20.61	20.70	20.79	20.88	20.98	21.07	1 2 3	4 5 5	6 7 8
.6	21.16	21.25 +	21.34	21.44	21.53	21.62	21.72	21.81	21.90	22.00	1 2 3	4 5 6	7 7 8
.7	22.09	22.18	22.28	22.37	22.47	22.56	22.66	22.75 +	22.85 −	22.94	1 2 3	4 5 6	7 8 9
.8	23.04	23.14	23.23	23.33	23.43	23.52	23.62	23.72	23.81	23.91	1 2 3	4 5 6	7 8 9
.9	24.01	24.11	24.21	24.30	24.40	24.50 +	24.60	24.70	24.80	24.90	1 2 3	4 5 6	7 8 9

To avoid interpolation in these twenty-two lines, see special table on pages 468, 469, and 470.

TABLE XXXV — *Continued*

n^2 **SQUARES**

TENTHS OF THE TABULAR DIFFERENCE

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5.0	25.00	25.10	25.20	25.30	25.40	25.50+	25.60	25.70	25.81	25.91	1	2	3	4	5	6	7	8	9
.1	26.01	26.11	26.21	26.32	26.42	26.52	26.63	26.73	26.83	26.94	1	2	3	4	5	6	7	8	9
.2	27.04	27.14	27.25−	27.35+	27.46	27.56	27.67	27.77	27.88	27.98	1	2	3	4	5	6	7	8	9
.3	28.09	28.20	28.30	28.41	28.52	28.62	28.73	28.84	28.94	29.05+	1	2	3	4	5	6	7	9	10
.4	29.16	29.27	29.38	29.48	29.59	29.70	29.81	29.92	**30**.03	30.14	1	2	3	4	5	7	8	9	10
5.5	30.25	30.36	30.47	30.58	30.69	30.80	30.91	31.02	31.14	31.25−	1	2	3	4	6	7	8	9	10
.6	31.36	31.47	31.58	32.70	31.81	31.92	32.04	32.15−	32.26	32.38	1	2	3	5	6	7	8	9	10
.7	32.49	32.60	32.72	32.83	32.95−	33.06	33.18	33.29	33.41	33.52	1	2	3	5	6	7	8	9	10
.8	33.64	33.76	33.87	33.99	34.11	34.22	34.34	34.46	34.57	34.69	1	2	4	5	6	7	8	9	11
.9	34.81	34.93	35.05−	35.16	35.28	35.40	35.52	35.64	35.76	35.88	1	2	4	5	6	7	8	10	11
6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09	1	2	4	5	6	7	8	10	11
.1	37.21	37.33	37.45+	37.58	37.70	37.82	37.95−	38.07	38.19	38.32	1	2	4	5	6	7	9	10	11
.2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56	1	2	4	5	6	7	9	10	11
.3	39.69	39.82	39.94	**40**.07	40.20	40.32	40.45−	40.58	40.70	40.83	1	3	4	5	6	8	9	10	11
.4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12	1	3	4	5	6	8	9	10	12
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43	1	3	4	5	7	8	9	10	12
.6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.76	1	3	4	5	7	8	9	11	12
.7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.83	45.97	46.10	1	3	4	5	7	8	9	11	12
.8	46.24	46.38	46.51	46.65−	46.79	46.92	47.06	47.20	47.33	47.47	1	3	4	5	7	8	10	11	12
.9	47.61	47.75−	47.89	48.02	48.16	48.30	48.44	48.58	48.72	48.86	1	3	4	6	7	8	10	11	13
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	**50**.13	50.27	1	3	4	6	7	8	10	11	13
.1	50.41	50.55+	50.69	50.84	50.98	51.12	51.27	51.41	51.55+	51.70	1	3	4	6	7	9	10	11	13
.2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85+	53.00+	53.14	1	3	4	6	7	9	10	12	13
.3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.46	54.61	1	3	4	6	7	9	10	12	13
.4	54.76	54.91	55.06	55.20	55.35+	55.50+	55.65+	55.80	55.95+	56.10	1	3	4	6	7	9	10	12	13
7.5	56.25	56.40	56.55+	56.70	56.85+	57.00	57.15+	57.30	57.46	57.61	2	3	5	6	8	9	11	12	14
.6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.98	59.14	2	3	5	6	8	9	11	12	14
.7	59.29	59.44	59.60	59.75+	59.91	**60**.06	60.22	60.37	60.53	60.68	2	3	5	6	8	9	11	12	14
.8	60.84	61.00	61.15+	61.31	61.47	61.62	61.78	61.94	62.09	62.25+	2	3	5	6	8	9	11	13	14
.9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.68	63.84	2	3	5	6	8	10	11	13	14
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45−	2	3	5	6	8	10	11	13	14
.1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75−	66.91	67.08	2	3	5	7	8	10	11	13	15
.2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72	2	3	5	7	8	10	12	13	15
.3	68.89	69.06	69.22	69.39	69.56	69.72	69.89	**70**.06	70.22	70.39	2	3	5	7	8	10	12	13	15
.4	70.56	70.73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.08	2	3	5	7	8	10	12	14	15
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79	2	3	5	7	9	10	12	14	15
.6	73.96	74.13	74.30	74.48	74.65−	74.82	75.00−	75.17	75.34	75.52	2	3	5	7	9	10	12	14	16
.7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	77.09	77.26	2	3	5	7	9	10	12	14	16
.8	77.44	77.62	77.79	77.97	78.15−	78.32	78.50−	78.68	78.85+	79.03	2	4	5	7	9	11	12	14	16
.9	79.21	79.39	79.57	79.74	79.92	**80**.10	80.28	80.46	80.64	80.82	2	4	5	7	9	11	13	14	16
9.0	81.00	81.18	81.36	81.54	81.72	81.90	82.08	82.26	82.45−	82.63	2	4	5	7	9	11	13	14	16
.1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	84.27	84.46	2	4	5	7	9	11	13	15	16
.2	84.64	84.82	85.01	85.19	85.38	85.56	85.75−	85.93	86.12	86.30	2	4	6	7	9	11	13	15	17
.3	86.49	86.68	86.86	87.05−	87.24	87.42	87.61	87.80	87.98	88.17	2	4	6	7	9	11	13	15	17
.4	88.36	88.55−	88.74	88.92	89.11	89.30	89.49	89.68	89.87	**90**.06	2	4	6	8	9	11	13	15	17
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97	2	4	6	8	10	11	13	15	17
.6	92.16	92.35+	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90	2	4	6	8	10	12	14	15	17
.7	94.09	94.28	94.48	94.67	94.87	95.06	95.26	95.45+	95.65−	95.84	2	4	6	8	10	12	14	16	18
.8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81	2	4	6	8	10	12	14	16	18
.9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80	2	4	6	8	10	12	14	16	18
10.0	**100**.0																		

TABLE XXXVI

These three pages give the **square roots** of all numbers between 1 and 10^2, correct to four significant figures. For example: $\sqrt{3.142} = 1.773$; $\sqrt{31.42} = 5.606$. (Values obtained by interpolation may be in error by 1 in the last figure.)

To obtain the square root of any number outside the range from 1 to 10^2, write the number as the product of: (a) some number between 1 and 10^2, and (b) some (positive or negative) power of 10^2; then proceed as in the following examples:

$$\sqrt{3\,14\,20\,00} = \sqrt{3.142 \times 10^6} = \sqrt{3.142} \times \sqrt{10^6} = 1.773 \times 10^3 = 1773.$$

$$\sqrt{0.31\,42} = \sqrt{31.42 \times 10^{-2}} = \sqrt{31.42} \times \sqrt{10^{-2}} = 5.606 \times 10^{-1} = 0.5606.$$

In short, moving the decimal point TWO *places in column* n *is equivalent to moving it* ONE *place in the body of the table.*

$\sqrt{n}$ SQUARE ROOTS

n	0	1	2	3	4	5	6	7	8	9	Tenths of the Tabular Difference 1 2 3	4 5 6	7 8 9
1.0	1.000	1.005−	1.010	1.015−	1.020	1.025−	1.030	1.034	1.039	1.044	0 1 1	2 2 3	3 4 4
.1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091	0 1 1	2 2 3	3 4 4
.2	1.095+	1.100	1.105−	1.109	1.114	1.118	1.122	1.127	1.131	1.136	0 1 1	2 2 3	3 4 4
.3	1.140	1.145−	1.149	1.153	1.158	1.162	1.166	1.170	1.175−	1.179	0 1 1	2 2 3	3 3 4
.4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221	0 1 1	2 2 2	3 3 4
1.5	1.225−	1.229	1.233	1.237	1.241	1.245−	1.249	1.253	1.257	1.261	0 1 1	2 2 2	3 3 4
.6	1.265−	1.269	1.273	1.277	1.281	1.285−	1.288	1.292	1.296	1.300	0 1 1	2 2 2	3 3 4
.7	1.304	1.308	1.311	1.315+	1.319	1.323	1.327	1.330	1.334	1.338	0 1 1	2 2 2	3 3 3
.8	1.342	1.345+	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.375−	0 1 1	1 2 2	3 3 3
.9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411	0 1 1	1 2 2	3 3 3
2.0	1.414	1.418	1.421	1.425−	1.428	1.432	1.435+	1.439	1.442	1.446	0 1 1	1 2 2	2 3 3
.1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480	0 1 1	1 2 2	2 3 3
.2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513	0 1 1	1 2 2	2 3 3
.3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546	0 1 1	1 2 2	2 3 3
.4	1.549	1.552	1.556	1.559	1.562	1.565+	1.568	1.572	1.575−	1.578	0 1 1	1 2 2	2 3 3
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609	0 1 1	1 2 2	2 3 3
.6	1.612	1.616	1.619	1.622	1.625−	1.628	1.631	1.634	1.637	1.640	0 1 1	1 2 2	2 2 3
.7	1.643	1.646	1.649	1.652	1.655+	1.658	1.661	1.664	1.667	1.670	0 1 1	1 2 2	2 2 3
.8	1.673	1.676	1.679	1.682	1.685+	1.688	1.691	1.694	1.697	1.700	0 1 1	1 1 2	2 2 3
.9	1.703	1.706	1.709	1.712	1.715−	1.718	1.720	1.723	1.726	1.729	0 1 1	1 1 2	2 2 3
3.0	1.732	1.735−	1.738	1.741	1.744	1.746	1.749	1.752	1.755−	1.758	0 1 1	1 1 2	2 2 3
.1	1.761	1.764	1.766	1.769	1.772	1.775−	1.778	1.780	1.783	1.786	0 1 1	1 1 2	2 2 3
.2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814	0 1 1	1 1 2	2 2 2
.3	1.817	1.819	1.822	1.825−	1.828	1.830	1.833	1.836	1.838	1.841	0 1 1	1 1 2	2 2 2
.4	1.844	1.847	1.849	1.852	1.855−	1.857	1.860	1.863	1.865+	1.868	0 1 1	1 1 2	2 2 2
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895−	0 1 1	1 1 2	2 2 2
.6	1.897	1.900	1.903	1.905+	1.908	1.910	1.913	1.916	1.918	1.921	0 1 1	1 1 2	2 2 2
.7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947	0 1 1	1 1 2	2 2 2
.8	1.949	1.952	1.954	1.957	1.960	1.962	1.965−	1.967	1.970	1.972	0 1 1	1 1 2	2 2 2
.9	1.975−	1.977	1.980	1.982	1.985−	1.987	1.990	1.992	1.995−	1.997	0 1 1	1 1 2	2 2 2
4.0	2.000	2.002	2.005−	2.007	2.010	2.012	2.015−	2.017	2.020	2.022	0 0 1	1 1 1	2 2 2
.1	2.025−	2.027	2.030	2.032	2.035−	2.037	2.040	2.042	2.045−	2.047	0 0 1	1 1 1	2 2 2
.2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071	0 0 1	1 1 1	2 2 2
.3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095+	0 0 1	1 1 1	2 2 2
.4	2.098	2.100	2.102	2.105−	2.107	2.110	2.112	2.114	2.117	2.119	0 0 1	1 1 1	2 2 2
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135+	2.138	2.140	2.142	0 0 1	1 1 1	2 2 2
.6	2.145−	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166	0 0 1	1 1 1	2 2 2
.7	2.168	2.170	2.173	2.175−	2.177	2.179	2.182	2.184	2.186	2.189	0 0 1	1 1 1	2 2 2
.8	2.191	2.193	2.195+	2.198	2.200	2.202	2.205−	2.207	2.209	2.211	0 0 1	1 1 1	2 2 2
.9	2.214	2.216	2.218	2.220	2.223	2.225−	2.227	2.229	2.232	2.234	0 0 1	1 1 1	2 2 2
5.0	2.236	2.238	2.241	2.243	2.245−	2.247	2.249	2.252	2.254	2.256	0 0 1	1 1 1	2 2 2
.1	2.258	2.261	2.263	2.265−	2.267	2.269	2.272	2.274	2.276	2.278	0 0 1	1 1 1	2 2 2
.2	2.280	2.283	2.285−	2.287	2.289	2.291	2.293	2.296	2.298	2.300	0 0 1	1 1 1	2 2 2
.3	2.302	2.304	2.307	2.309	2.311	2.313	2.315+	2.317	2.319	2.322	0 0 1	1 1 1	2 2 2
.4	2.324	2.326	2.328	2.330	2.332	2.335−	2.337	2.339	2.341	2.343	0 0 1	1 1 1	1 2 2
5.5	2.345+	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364	0 0 1	1 1 1	1 2 2
.6	2.366	2.369	2.371	2.373	2.375−	2.377	2.379	2.381	2.383	2.385+	0 0 1	1 1 1	1 2 2
.7	2.387	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406	0 0 1	1 1 1	1 2 2
.8	2.408	2.410	2.412	2.415−	2.417	2.419	2.421	2.423	2.425−	2.427	0 0 1	1 1 1	1 2 2
.9	2.429	2.431	2.433	2.435+	2.437	2.439	2.441	2.443	2.445+	2.447	0 0 1	1 1 1	1 2 2

$\sqrt{n}$ SQUARE ROOTS

n	0	1	2	3	4	5	6	7	8	9	Tenths of the Tabular Difference 1	2	3	4	5	6	7	8	9
6.0	2.449	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468	0	0	1	1	1	1	1	2	
.1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488	0	0	1	1	1	1	1	2	
.2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508	0	0	1	1	1	1	1	2	
.3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528	0	0	1	1	1	1	1	2	
.4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548	0	0	1	1	1	1	1	2	
6.5	2.550−	2.551	2.553	2.555+	2.557	2.559	2.561	2.563	2.565+	2.567	0	0	1	1	1	1	1	2	
.6	2.569	2.571	2.573	2.575−	2.577	2.579	2.581	2.583	2.585−	2.587	0	0	1	1	1	1	1	2	
.7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606	0	0	1	1	1	1	1	2	
.8	2.608	2.610	2.612	2.613	2.615+	2.617	2.619	2.621	2.623	2.625−	0	0	1	1	1	1	1	2	
.9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644	0	0	1	1	1	1	1	2	
7.0	2.646	2.648	2.650−	2.651	2.653	2.655+	2.657	2.659	2.661	2.663	0	0	1	1	1	1	1	2	2
.1	2.665−	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681	0	0	1	1	1	1	1	1	2
.2	2.683	2.685+	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700	0	0	1	1	1	1	1	1	2
.3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715−	2.717	2.718	0	0	1	1	1	1	1	1	2
.4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735−	2.737	0	0	1	1	1	1	1	1	2
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750−	2.751	2.753	2.755−	0	0	1	1	1	1	1	1	2
.6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773	0	0	1	1	1	1	1	1	2
.7	2.775−	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791	0	0	1	1	1	1	1	1	2
.8	2.793	2.795−	2.796	2.798	2.800	2.802	2.804	2.805+	2.807	2.809	0	0	1	1	1	1	1	1	2
.9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825−	2.827	0	0	1	1	1	1	1	1	2
8.0	2.828	2.830	2.832	2.834	2.835+	2.837	2.839	2.841	2.843	2.844	0	0	1	1	1	1	1	1	2
.1	2.846	2.848	2.850−	2.851	2.853	2.855−	2.857	2.858	2.860	2.862	0	0	1	1	1	1	1	1	2
.2	2.864	2.865+	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879	0	0	1	1	1	1	1	1	2
.3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895−	2.897	0	0	1	1	1	1	1	1	2
.4	2.898	2.900	2.902	2.903	2.905+	2.907	2.909	2.910	2.912	2.914	0	0	1	1	1	1	1	1	2
8.5	2.915+	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931	0	0	1	1	1	1	1	1	2
.6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948	0	0	1	1	1	1	1	1	2
.7	2.950−	2.951	2.953	2.955−	2.956	2.958	2.960	2.961	2.963	2.965−	0	0	1	1	1	1	1	1	2
.8	2.966	2.968	2.970	2.972	2.973	2.975−	2.977	2.978	2.980	2.982	0	0	1	1	1	1	1	1	2
.9	2.983	2.985−	2.987	2.988	2.990	2.992	2.993	2.995−	2.997	2.998	0	0	1	1	1	1	1	1	2
9.0	3.000	3.002	3.003	3.005−	3.007	3.008	3.010	3.012	3.013	3.015−	0	0	0	1	1	1	1	1	1
.1	3.017	3.018	3.020	3.022	3.023	3.025−	3.027	3.028	3.030	3.032	0	0	0	1	1	1	1	1	1
.2	3.033	3.035−	3.036	3.038	3.040	3.041	3.043	3.045−	3.046	3.048	0	0	0	1	1	1	1	1	1
.3	3.050−	3.051	3.053	3.055−	3.056	3.058	3.059	3.061	3.063	3.064	0	0	0	1	1	1	1	1	1
.4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081	0	0	0	1	1	1	1	1	1
9.5	3.082	3.084	3.085+	3.087	3.089	3.090	3.092	3.094	3.095+	3.097	0	0	0	1	1	1	1	1	1
.6	3.098	3.100	3.102	3.103	3.105−	3.106	3.108	3.110	3.111	3.113	0	0	0	1	1	1	1	1	1
.7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129	0	0	0	1	1	1	1	1	1
.8	3.130	3.132	3.134	3.135+	3.137	3.138	3.140	3.142	3.143	3.145−	0	0	0	1	1	1	1	1	1
.9	3.146	3.148	3.150−	3.151	3.153	3.154	3.156	3.158	3.159	3.161	0	0	0	1	1	1	1	1	1
10	3.162	3.178	3.194	3.209	3.225−	3.240	3.256	3.271	3.286	3.302	2	3	5	6	8	9	11	12	14
1	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435+	3.450−	1	3	4	6	7	9	10	12	13
2	3.464	3.479	3.493	3.507	3.521	3.536	3.550−	3.564	3.578	3.592	1	3	4	6	7	8	10	11	13
3	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715−	3.728	1	3	4	5	7	8	10	11	12
4	3.742	3.755−	3.768	3.782	3.795−	3.808	3.821	3.834	3.847	3.860	1	3	4	5	7	8	9	11	12
15	3.873	3.886	3.899	3.912	3.924	3.937	3.950−	3.962	3.975−	3.987	1	3	4	5	6	8	9	10	11
6	4.000	4.012	4.025−	4.037	4.050−	4.062	4.074	4.087	4.099	4.111	1	2	4	5	6	7	9	10	11
7	4.123	4.135+	4.147	4.159	4.171	4.183	4.195+	4.207	4.219	4.231	1	2	4	5	6	7	8	10	11
8	4.243	4.254	4.266	4.278	4.290	4.301	4.113	4.324	4.336	4.347	1	2	3	5	6	7	8	9	10
9	4.359	4.370	4.382	4.393	4.405−	4.416	4.427	4.438	4.450−	4.461	1	2	3	5	6	7	8	9	10
20	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550−	4.561	4.572	1	2	3	4	6	7	8	9	10
1	4.583	4.593	4.604	4.615+	4.626	4.637	4.648	4.658	4.669	4.680	1	2	3	4	5	6	8	9	10
2	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775−	4.785+	1	2	3	4	5	6	7	8	9
3	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889	1	2	3	4	5	6	7	8	9
4	4.899	4.909	4.919	4.930	4.940	4.950−	4.960	4.970	4.980	4.990	1	2	3	4	5	6	7	8	9
25	5.000	5.010	5.020	5.030	5.040	5.050−	5.060	5.070	5.079	5.089	1	2	3	4	5	6	7	8	9
6	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187	1	2	3	4	5	6	7	8	9
7	5.196	5.206	5.215+	5.225−	5.235−	5.244	5.254	5.263	5.273	5.282	1	2	3	4	5	6	7	8	9
8	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376	1	2	3	4	5	6	7	7	8
9	5.385+	5.394	5.404	5.413	5.422	5.431	5.441	5.450−	5.459	5.468	1	2	3	4	5	6	6	7	8
30	5.477	5.486	5.495+	5.505−	5.514	5.523	5.532	5.541	5.550−	5.559	1	2	3	4	5	5	6	7	8
1	5.568	5.577	5.586	5.595−	5.604	5.612	5.621	5.630	5.639	5.648	1	2	3	4	4	5	6	7	8
2	5.657	5.666	5.675−	5.683	5.692	5.701	5.710	5.718	5.727	5.736	1	2	3	4	4	5	6	7	8
3	5.745−	5.753	5.762	5.771	5.779	5.788	5.797	5.805+	5.814	5.822	1	2	3	3	4	5	6	7	8
4	5.831	5.840	5.848	5.857	5.865+	5.874	5.882	5.891	5.899	5.908	1	2	3	3	4	5	6	7	8
35	5.916	5.925−	5.933	5.941	5.950−	5.958	5.967	5.975−	5.983	5.992	1	2	3	3	4	5	6	7	8
6	6.000	6.008	6.017	6.025−	6.033	6.042	6.050−	6.058	6.066	6.075−	1	2	2	3	4	5	6	7	7
7	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156	1	2	2	3	4	5	6	7	7
8	6.164	6.173	6.181	6.189	6.197	6.205−	6.213	6.221	6.229	6.237	1	2	2	3	4	5	6	6	7
9	6.245−	6.253	6.261	6.269	6.277	6.285−	6.293	6.301	6.309	6.317	1	2	2	3	4	5	6	6	7

TABLE XXXVI — *Continued*

$\sqrt{n}$ SQUARE ROOTS

TENTHS OF THE TABULAR DIFFERENCE

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
40	6.325−	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395+	1	2	2	3	4	5	5	6	7
1	6.403	6.411	6.419	6.427	6.434	6.442	6.450−	6.458	6.465+	6.473	1	2	2	3	4	5	5	6	7
2	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535−	6.542	6.550−	1	2	2	3	4	5	5	6	7
3	6.557	6.565+	6.573	6.580	6.588	6.595+	6.603	6.611	6.618	6.626	1	2	2	3	4	5	5	6	7
4	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701	1	1	2	3	4	4	5	6	7
45	6.708	6.716	6.723	6.731	6.738	6.745+	6.753	6.760	6.768	6.775−	1	1	2	3	4	4	5	6	7
6	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848	1	1	2	3	4	4	5	6	7
7	6.856	6.863	6.870	6.877	6.885−	6.892	6.899	6.907	6.914	6.921	1	1	2	3	4	4	5	6	7
8	6.928	6.935+	6.943	6.950−	6.957	6.964	6.971	6.979	6.986	6.993	1	1	2	3	4	4	5	6	6
9	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050−	7.057	7.064	1	1	2	3	4	4	5	6	6
50	7.071	7.078	7.085+	7.092	7.099	7.106	7.113	7.120	7.127	7.134	1	1	2	3	4	4	5	6	6
1	7.141	7.148	7.155+	7.162	7.169	7.176	7.183	7.190	7.197	7.204	1	1	2	3	3	4	5	6	6
2	7.211	7.218	7.225−	7.232	7.239	7.246	7.253	7.259	7.266	7.273	1	1	2	3	3	4	5	6	6
3	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335−	7.342	1	1	2	3	3	4	5	5	6
4	7.348	7.355+	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409	1	1	2	3	3	4	5	5	6
55	7.416	7.423	7.430	7.436	7.443	7.450−	7.457	7.463	7.470	7.477	1	1	2	3	3	4	5	5	6
6	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543	1	1	2	3	3	4	5	5	6
7	7.550−	7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609	1	1	2	3	3	4	5	5	6
8	7.616	7.622	7.629	7.635+	7.642	7.649	7.655+	7.662	7.668	7.675−	1	1	2	3	3	4	5	5	6
9	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740	1	1	2	3	3	4	5	5	6
60	7.746	7.752	7.759	7.765+	7.772	7.778	7.785−	7.791	7.797	7.804	1	1	2	3	3	4	4	5	6
1	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855−	7.861	7.868	1	1	2	3	3	4	4	5	6
2	7.874	7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925−	7.931	1	1	2	3	3	4	4	5	6
3	7.937	7.944	7.950−	7.956	7.962	7.969	7.975−	7.981	7.987	7.994	1	1	2	3	3	4	4	5	6
4	8.000	8.006	8.012	8.019	8.025−	8.031	8.037	8.044	8.050−	8.056	1	1	2	2	3	4	4	5	6
65	8.062	8.068	8.075−	8.081	8.087	8.093	8.099	8.106	8.112	8.118	1	1	2	2	3	4	4	5	6
6	8.124	8.130	8.136	8.142	8.149	8.155−	8.161	8.167	8.173	8.179	1	1	2	2	3	4	4	5	6
7	8.185+	8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240	1	1	2	2	3	4	4	5	5
8	8.246	8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295−	8.301	1	1	2	2	3	4	4	5	5
9	8.307	8.313	8.319	8.325−	8.331	8.337	8.343	8.349	8.355−	8.361	1	1	2	2	3	4	4	5	5
70	8.367	8.373	8.379	8.385−	8.390	8.396	8.402	8.408	8.414	8.420	1	1	2	2	3	4	4	5	5
1	8.426	8.432	8.438	8.444	8.450−	8.456	8.462	8.468	8.473	8.479	1	1	2	2	3	4	4	5	5
2	8.485+	8.491	8.497	8.503	8.509	8.515−	8.521	8.526	8.532	8.538	1	1	2	2	3	4	4	5	5
3	8.544	8.550−	8.556	8.562	8.567	8.573	8.579	8.585−	8.591	8.597	1	1	2	2	3	3	4	5	5
4	8.602	8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654	1	1	2	2	3	3	4	5	5
75	8.660	8.666	8.672	8.678	8.683	8.689	8.695−	8.701	8.706	8.712	1	1	2	2	3	3	4	5	5
6	8.718	8.724	8.729	8.735−	8.741	8.746	8.752	8.758	8.764	8.769	1	1	2	2	3	3	4	5	5
7	8.775−	8.781	8.786	8.792	8.798	8.803	8.809	8.815−	8.820	8.826	1	1	2	2	3	3	4	5	5
8	8.832	8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883	1	1	2	2	3	3	4	5	5
9	8.888	8.894	8.899	8.905+	8.911	8.916	8.922	8.927	8.933	8.939	1	1	2	2	3	3	4	4	5
80	8.944	8.950−	8.955+	8.961	8.967	8.972	8.978	8.983	8.989	8.994	1	1	2	2	3	3	4	4	5
1	9.000	9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050−	1	1	2	2	3	3	4	4	5
2	9.055+	9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105−	1	1	2	2	3	3	4	4	5
3	9.110	9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160	1	1	2	2	3	3	4	4	5
4	9.165+	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214	1	1	2	2	3	3	4	4	5
85	9.220	9.225−	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268	1	1	2	2	3	3	4	4	5
6	9.274	9.279	9.284	9.290	9.295+	9.301	9.306	9.311	9.317	9.322	1	1	2	2	3	3	4	4	5
7	9.327	9.333	9.338	9.343	9.349	9.354	9.359	9.365−	9.370	9.375+	1	1	2	2	3	3	4	4	5
8	9.381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429	1	1	2	2	3	3	4	4	5
9	9.434	9.439	9.445−	9.450−	9.455+	9.460	9.466	9.471	9.476	9.482	1	1	2	2	3	3	4	4	5
90	9.487	9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534	1	1	2	2	3	3	4	4	5
1	9.539	9.545−	9.550−	9.555+	9.560	9.566	9.571	9.576	9.581	9.586	1	1	2	2	3	3	4	4	5
2	9.592	9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638	1	1	2	2	3	3	4	4	5
3	9.644	9.649	9.654	9.659	9.664	9.670	9.675−	9.680	9.685+	9.690	1	1	2	2	3	3	4	4	5
4	9.695+	9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742	1	1	2	2	3	3	4	4	5
95	9.747	9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793	1	1	2	2	3	3	4	4	5
6	9.798	9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844	1	1	2	2	3	3	4	4	5
7	9.849	9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894	1	1	2	2	3	3	4	4	5
8	9.899	9.905−	9.910	9.915−	9.920	9.925−	9.930	9.935−	9.940	9.945−	1	1	2	2	3	3	4	4	5
9	9.950−	9.955−	9.960	9.965−	9.970	9.975−	9.980	9.985−	9.990	9.995−	1	1	2	2	3	3	4	4	5

TABLE XXXVII

These two pages give the **three-halves powers** of all numbers between 1 and 10^2, correct to four significant figures. For example: $(2.718)^{\frac{3}{2}} = 4.482$; $(27.18)^{\frac{3}{2}} = 141.7$. (Values obtained by interpolation may be in error by 1 in the last figure.)

To obtain three-halves power of any number outside the range from 1 to 10^2, write the number as the product of: (a) some number between 1 and 10^2, and (b) some (positive or negative) power of 10^2; then proceed as in the following examples:

$$(27180.)^{\frac{3}{2}} = (2.718 \times 10^4)^{\frac{3}{2}} = (2.718)^{\frac{3}{2}} \times (10^4)^{\frac{3}{2}} = 4.482 \times 10^6 = 4\,482\,000$$

$$(0.2718)^{\frac{3}{2}} = (27.18 \times 10^{-2})^{\frac{3}{2}} = (27.18)^{\frac{3}{2}} \times (10^{-2})^{\frac{3}{2}} = 141.7 \times 10^{-3} = 0.1417$$

In short, moving the decimal point two *places in column* n *is equivalent to moving it* three *places in the body of the table.*

$n^{\frac{3}{2}}$ THREE-HALVES POWERS

n	0	1	2	3	4	5	6	7	8	9
1	1.000	1.154	1.315 −	1.482	1.657	1.837	2.024	2.217	2.415 −	2.619
2	2.828	3.043	3.263	3.488	3.718	3.953	4.192	4.437	4.685 +	4.939
3	5.196	5.458	5.724	5.995 −	6.269	6.548	6.831	7.117	7.408	7.702
4	8.000	8.302	8.607	8.917	9.230	9.546	9.866	10.190		
4								10.19	10.52	10.85 −
5	11.18	11.52	11.86	12.20	12.55 −	12.90	13.25 +	13.61	13.97	14.33
6	14.70	15.07	15.44	15.81	16.19	16.57	16.96	17.34	17.73	18.12
7	18.52	18.92	19.32	19.72	20.13	20.54	20.95 +	21.37	21.78	22.20
8	22.63	23.05 +	23.48	23.91	24.35 −	24.78	25.22	25.66	26.11	26.55 +
9	27.00	27.45 +	27.90	28.36	28.82	29.28	29.74	30.21	30.68	31.15 −
10	31.62	32.10	32.58	33.06	33.54	34.02	34.51	35.00 +	35.49	35.99
1	36.48	36.98	37.48	37.99	38.49	39.00	39.51	40.02	40.53	41.05 +
2	41.57	42.09	42.61	43.14	43.66	44.19	44.73	45.26	45.79	46.33
3	46.87	47.41	47.96	48.50 +	49.05 +	49.60	50.15 +	50.71	51.26	51.82
4	52.38	52.95 −	53.51	54.08	54.64	55.21	55.79	56.36	56.94	57.51
15	58.09	58.68	59.26	59.85 −	60.43	61.02	61.62	62.21	62.80	63.40
6	64.00	64.60	65.20	65.81	66.41	67.02	67.63	68.25 −	68.86	69.48
7	70.09	70.71	71.33	71.96	72.58	73.21	73.84	74.47	75.10	75.73
8	76.37	77.00	77.64	78.28	78.93	79.57	80.22	80.87	81.51	82.17
9	82.82	83.47	84.13	84.79	85.45 −	86.11	86.77	87.44	88.10	88.77
20	89.44	90.11	90.79	91.46	92.14	92.82	93.50 −	94.18	94.86	95.55 −
1	96.23	96.92	97.61	98.30	99.00	99.69	100.39			
1							100.4	101.1	101.8	102.5 −
2	103.2	103.9	104.6	105.3	106.0	106.7	107.4	108.2	108.9	109.6
3	110.3	111.0	111.7	112.5 −	113.2	113.9	114.6	115.4	116.1	116.8
4	117.6	118.3	119.0	119.8	120.5 +	121.3	122.0	122.8	123.5 +	124.3
25	125.0	125.8	126.5 +	127.3	128.0	128.8	129.5 +	130.3	131.0	131.8
6	132.6	133.3	134.1	134.9	135.6	136.4	137.2	138.0	138.7	139.5 +
7	140.3	141.1	141.9	142.6	143.4	144.2	145.0 −	145.8	146.6	147.4
8	148.2	149.0	149.8	150.5 +	151.3	152.1	152.9	153.8	154.6	155.4
9	156.2	157.0	157.8	158.6	159.4	160.2	161.0	161.9	162.7	163.5 −
30	164.3	165.1	166.0	166.8	167.6	168.4	169.3	170.1	170.9	171.8
1	172.6	173.4	174.3	175.1	176.0	176.8	177.6	178.5 −	179.3	180.2
2	181.0	181.9	182.7	183.6	184.4	185.3	186.1	187.0	187.8	188.7
3	189.6	190.4	191.3	192.2	193.0	193.9	194.8	195.6	196.5 +	197.4
4	198.3	199.1	200.0	200.9	201.8	202.6	203.5 +	204.4	205.3	206.2
35	207.1	208.0	208.8	209.7	210.6	211.5 +	212.4	213.3	214.2	215.1
6	216.0	216.9	217.8	218.7	219.6	220.5 +	221.4	222.3	223.2	224.2
7	225.1	226.0	226.9	227.8	228.7	229.6	230.6	231.5 −	232.4	233.3
8	234.2	235.2	236.1	237.0	238.0	238.9	239.8	240.8	241.7	242.6
9	243.6	244.5 −	245.4	246.4	247.3	248.3	249.2	250.1	251.1	252.0
40	253.0	253.9	254.9	255.8	256.8	257.7	258.7	259.7	260.6	261.6
1	262.5 +	263.5 −	264.5 −	265.4	266.4	267.3	268.3	269.3	270.2	271.2
2	272.2	273.2	274.1	275.1	276.1	277.1	278.0	279.0	280.0	281.0
3	282.0	283.0	283.9	284.9	285.9	286.9	287.9	288.9	289.9	290.9
4	291.9	292.9	293.9	294.9	295.9	296.9	297.9	298.9	299.9	300.9
45	301.9	302.9	303.9	304.9	305.9	306.9	307.9	308.9	310.0	311.0
6	312.0	313.0	314.0	315.0 +	316.1	317.1	318.1	319.1	320.2	321.2
7	322.2	323.2	324.3	325.3	326.3	327.4	328.4	329.4	330.5 −	331.5 +
8	332.6	333.6	334.6	335.7	336.7	337.8	338.8	339.9	340.9	342.0
9	343.0	344.1	345.1	346.2	347.2	348.3	349.3	350.4	351.4	352.5 −

Tenths of the Tabular Difference

1	2	3	4	5	6	7	8
To avoid interpolation in these four lines, see special table on page 356							
3	7	10	13	17	20	23	26
4	7	11	14	18	21	25	28
4	8	11	15	19	23	27	31
4	8	12	16	21	25	29	33
4	9	13	17	22	26	31	35
5	9	14	18	23	28	32	37
5	10	15	19	24	29	34	39
5	10	15	20	25	31	36	41
5	11	16	21	27	32	37	42
6	11	17	22	28	33	39	44
6	11	17	23	29	34	40	46
6	12	18	24	30	35	41	47
6	12	18	24	30	37	43	49
6	13	19	25	31	38	44	50
6	13	19	26	32	39	45	52
7	13	20	26	33	40	46	53
7	14	20	27	34	41	48	54
7	14	21	28	35	42	48	55
1	1	2	3	4	4	5	6
1	1	2	3	4	4	5	6
1	1	2	3	4	4	5	6
1	1	2	3	4	4	5	6
1	2	2	3	4	5	5	6
1	2	2	3	4	5	6	6
1	2	2	3	4	5	6	6
1	2	2	3	4	5	6	6
1	2	2	3	4	5	6	7
1	2	2	3	4	5	6	7
1	2	3	3	4	5	6	7
1	2	3	3	4	5	6	7
1	2	3	3	4	5	6	7
1	2	3	4	4	5	6	7
1	2	3	4	5	5	6	7
1	2	3	4	5	6	6	7
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8

Moving the decimal point TWO *places in column* n *is equivalent to moving it* THREE *places in the body of the table.*

$n^{\frac{3}{2}}$ THREE-HALVES POWERS

TENTHS OF THE TABULAR DIFFERENCE

n	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	353.6	354.6	355.7	356.7	357.8	358.9	359.9	361.0	362.1	363.1	1	2	3	4	5	6	7	9	10
1	364.2	365.3	366.4	367.4	368.5+	369.6	370.7	371.7	372.8	373.9	1	2	3	4	5	6	8	9	10
2	375.0−	376.1	377.1	378.2	379.3	380.4	381.5−	382.6	383.7	384.8	1	2	3	4	5	7	8	9	10
3	385.8	386.9	388.0	389.1	390.2	391.3	392.4	393.5+	394.6	395.7	1	2	3	4	5	7	8	9	10
4	396.8	397.9	399.0	400.1	401.2	402.3	403.4	404.6	405.7	406.8	1	2	3	4	6	7	8	9	10
55	407.9	409.0	410.1	411.2	412.3	413.5−	414.6	415.7	416.8	417.9	1	2	3	4	6	7	8	9	10
6	419.1	420.2	421.3	422.4	423.6	424.7	425.8	426.9	428.1	429.2	1	2	3	5	6	7	8	9	10
7	430.3	431.5−	432.6	433.7	434.9	436.0	437.2	438.3	439.4	440.6	1	2	3	5	6	7	8	9	10
8	441.7	442.9	444.0	445.1	446.3	447.4	448.6	449.7	450.9	452.0	1	2	3	5	6	7	8	9	10
9	453.2	454.3	455.5−	456.6	457.8	459.0	460.1	461.3	462.4	463.6	1	2	3	5	6	7	8	9	10
60	464.8	465.9	467.1	468.2	469.4	470.6	471.7	472.9	474.1	475.3	1	2	4	5	6	7	8	9	11
1	476.4	477.6	478.8	479.9	481.1	482.3	483.5−	484.6	485.8	487.0	1	2	4	5	6	7	8	9	11
2	488.2	489.4	490.6	491.7	492.9	494.1	495.3	496.5−	497.7	498.9	1	2	4	5	6	7	8	9	11
3	500.0+	501.2	502.4	503.6	504.8	506.0	507.2	508.4	509.6	510.8	1	2	4	5	6	7	8	10	11
4	512.0	513.2	514.4	515.6	516.8	518.0	519.2	520.4	521.6	522.8	1	2	4	5	6	7	8	10	11
65	524.0	525.3	526.5−	527.7	528.9	530.1	531.3	532.5+	533.8	535.0−	1	2	4	5	6	7	8	10	11
6	536.2	537.4	538.6	539.8	541.1	542.3	543.5+	544.7	546.0	547.2	1	2	4	5	6	7	9	10	11
7	548.4	549.6	550.9	552.1	553.3	554.6	555.8	557.0	558.3	559.5+	1	2	4	5	6	7	9	10	11
8	560.7	562.0	563.2	564.5−	565.7	566.9	568.2	569.4	570.7	571.9	1	2	4	5	6	7	9	10	11
9	573.2	574.4	575.7	576.9	578.1	579.4	580.6	581.9	583.2	584.4	1	3	4	5	6	8	9	10	11
70	585.7	586.9	588.2	589.4	590.7	591.9	593.2	594.5−	595.7	597.0	1	3	4	5	6	8	9	10	11
1	598.3	599.5+	600.8	602.1	603.3	604.6	605.9	607.1	608.4	609.7	1	3	4	5	6	8	9	10	11
2	610.9	612.2	613.5−	614.8	616.0	617.3	618.6	619.9	621.2	622.4	1	3	4	5	6	8	9	10	11
3	623.7	625.0−	626.3	627.6	628.8	630.1	631.4	632.7	634.0	635.3	1	3	4	5	6	8	9	10	12
4	636.6	637.9	639.2	640.4	641.7	643.0	644.3	645.6	646.9	648.2	1	3	4	5	6	8	9	10	12
75	649.5+	650.8	652.1	653.4	654.7	656.0	657.3	658.6	659.9	661.2	1	3	4	5	7	8	9	10	12
6	662.6	663.9	665.2	666.5−	667.8	669.1	670.4	671.7	673.0	674.4	1	3	4	5	7	8	9	10	12
7	675.7	677.0	678.3	679.6	680.9	682.3	683.6	684.9	686.2	687.6	1	3	4	5	7	8	9	11	12
8	688.9	690.2	691.5+	692.9	694.2	695.5+	696.8	698.2	699.5+	700.8	1	3	4	5	7	8	9	11	12
9	702.2	703.5+	704.8	706.2	707.5+	708.8	710.2	711.5+	712.9	714.2	1	3	4	5	7	8	9	11	12
80	715.5+	716.9	718.2	719.6	720.9	722.3	723.6	725.0−	726.3	727.7	1	3	4	5	7	8	9	11	12
1	729.0	730.4	731.7	733.1	734.4	735.8	737.1	738.5−	739.8	741.2	1	3	4	5	7	8	9	11	12
2	742.5+	743.9	745.3	746.6	748.0	749.3	750.7	752.1	753.4	754.8	1	3	4	5	7	8	10	11	12
3	756.2	757.5+	758.9	760.3	761.6	763.0	764.4	765.8	767.1	768.5−	1	3	4	5	7	8	10	11	12
4	769.9	771.2	772.6	774.0	775.4	776.8	778.1	779.5+	780.9	782.3	1	3	4	6	7	8	10	11	12
85	783.7	785.0+	786.4	787.8	789.2	790.6	792.0	793.4	794.8	796.1	1	3	4	6	7	8	10	11	12
6	797.5+	798.9	800.3	801.7	803.1	804.5−	805.9	807.3	808.7	810.1	1	3	4	6	7	8	10	11	13
7	811.5−	812.9	814.3	815.7	817.1	818.5−	819.9	821.3	822.7	824.1	1	3	4	6	7	8	10	11	13
8	825.5+	826.9	828.3	829.7	831.1	832.6	834.0	835.4	836.8	838.2	1	3	4	6	7	8	10	11	13
9	839.6	841.0	842.5−	843.9	845.3	846.7	848.1	849.5+	851.0	852.4	1	3	4	6	7	9	10	11	13
90	853.8	855.2	856.7	858.1	859.5+	860.9	862.4	863.8	865.2	866.7	1	3	4	6	7	9	10	11	13
1	868.1	869.5+	870.9	872.4	873.8	875.2	876.7	878.1	879.6	881.0	1	3	4	6	7	9	10	11	13
2	882.4	883.9	885.3	886.8	888.2	889.6	891.1	892.5+	894.0	895.4	1	3	4	6	7	9	10	12	13
3	896.9	898.3	899.8	901.2	902.7	904.1	905.6	907.0	908.5−	909.9	1	3	4	6	7	9	10	12	13
4	911.4	912.8	914.3	915.7	917.2	918.6	920.1	921.6	923.0	924.5−	1	3	4	6	7	9	10	12	13
95	925.9	927.4	928.9	930.3	931.8	933.3	934.7	936.2	937.7	939.1	1	3	4	6	7	9	10	12	13
6	940.6	942.1	943.5+	945.0+	946.5−	948.0	949.4	950.9	952.4	953.9	1	3	4	6	7	9	10	12	13
7	955.3	956.8	958.3	959.8	961.3	962.7	964.2	965.7	967.2	968.7	1	3	4	6	7	9	10	12	13
8	970.2	971.6	973.1	974.6	976.1	977.6	979.1	980.6	982.1	983.5+	1	3	4	6	7	9	10	12	13
9	985.0+	986.5+	988.0	989.5+	991.0	992.5+	994.0	995.5+	997.0	998.5+	1	3	4	6	7	9	10	12	13
100	1000.0																		

SPECIAL TABLE

Moving the decimal point TWO *places in column* n *is equivalent to moving it* THREE *places in the body of the table.*

$n^{\frac{3}{2}}$ THREE–HALVES POWERS

n	0	1	2	3	4	5	6	7	8	9	Tenths of the Tabular Difference 1	2	3	4	5	6	7	8	9
1.0	1.000	1.015+	1.030	1.045+	1.061	1.076	1.091	1.107	1.122	1.138	2	3	5	6	8	9	11	12	14
.1	1.154	1.169	1.185+	1.201	1.217	1.233	1.249	1.266	1.282	1.298	2	3	5	6	8	10	11	13	14
.2	1.315−	1.331	1.348	1.364	1.381	1.398	1.414	1.431	1.448	1.465+	2	3	5	7	8	10	12	13	15
.3	1.482	1.499	1.517	1.534	1.551	1.569	1.586	1.604	1.621	1.639	2	3	5	7	9	10	12	14	15
.4	1.657	1.674	1.692	1.710	1.728	1.746	1.764	1.782	1.800	1.819	2	4	5	7	9	11	13	14	16
1.5	1.837	1.856	1.874	1.893	1.911	1.930	1.948	1.967	1.986	2.005−	2	4	6	7	9	11	13	15	17
.6	2.024	2.043	2.062	2.081	2.100	2.119	2.139	2.158	2.178	2.197	2	4	6	8	10	12	13	15	17
.7	2.217	2.236	2.256	2.275+	2.295+	2.315+	2.335−	2.355−	2.375−	2.395−	2	4	6	8	10	12	14	16	18
.8	2.415−	2.435+	2.455+	2.476	2.496	2.516	2.537	2.557	2.578	2.598	2	4	6	8	10	12	14	16	18
.9	2.619	2.640	2.660	2.681	2.702	2.723	2.744	2.765+	2.786	2.807	2	4	6	8	10	13	15	17	19
2.0	2.828	2.850−	2.871	2.892	2.914	2.935+	2.957	2.978	3.000	3.021	2	4	6	9	11	13	15	17	19
.1	3.043	3.065−	3.087	3.109	3.131	3.153	3.175−	3.197	3.219	3.241	2	4	7	9	11	13	15	18	20
.2	3.263	3.285+	3.308	3.330	3.353	3.375	3.398	3.420	3.443	3.465+	2	4	7	9	11	13	16	18	20
.3	3.488	3.511	3.534	3.557	3.580	3.602	3.626	3.649	3.672	3.695−	2	5	7	9	11	14	16	18	21
.4	3.718	3.741	3.765−	3.788	3.811	3.835−	3.858	3.882	3.906	3.929	2	5	7	9	12	14	16	19	21
2.5	3.953	3.977	4.000	4.024	4.048	4.072	4.096	4.120	4.144	4.168	2	5	7	10	12	14	17	19	22
.6	4.192	4.217	4.241	4.265+	4.289	4.314	4.338	4.363	4.387	4.412	2	5	7	10	12	15	17	20	22
.7	4.437	4.461	4.486	4.511	4.536	4.560	4.585+	4.610	4.635+	4.660	2	5	7	10	12	15	17	20	22
.8	4.685+	4.710	4.736	4.761	4.786	4.811	4.837	4.862	4.888	4.913	3	5	8	10	13	15	18	20	23
.9	4.939	4.964	4.990	5.015+	5.041	5.067	5.093	5.118	5.144	5.170	3	5	8	10	13	15	18	21	23
3.0	5.196	5.222	5.248	5.274	5.300	5.327	5.353	5.379	5.405+	5.432	3	5	8	10	13	16	18	21	24
.1	5.458	5.485−	5.511	5.538	5.564	5.591	5.617	5.644	5.671	5.698	3	5	8	11	13	16	19	21	24
.2	5.724	5.751	5.778	5.805+	5.832	5.859	5.886	5.913	5.940	6.968	3	5	8	11	14	16	19	22	24
.3	5.995−	6.022	6.049	6.077	6.104	6.132	6.159	6.186	6.214	6.242	3	5	8	11	14	16	19	22	25
.4	6.269	6.297	6.325−	6.352	6.380	6.408	6.436	6.464	6.492	6.520	3	6	8	11	14	17	20	22	25
3.5	6.548	6.576	6.604	6.632	6.660	6.689	6.717	6.745+	6.774	6.802	3	6	8	11	14	17	20	23	25
.6	6.831	6.859	6.888	6.916	6.945−	6.973	7.002	7.031	7.059	7.088	3	6	9	11	14	17	20	23	26
.7	7.117	7.146	7.175−	7.204	7.233	7.262	7.291	7.320	7.349	7.378	3	6	9	12	15	17	20	23	26
.8	7.408	7.437	7.466	7.495+	7.525−	7.554	7.584	7.613	7.643	7.672	3	6	9	12	15	18	21	24	27
.9	7.702	7.732	7.761	7.791	7.821	7.850+	7.880	7.910	7.940	7.970	3	6	9	12	15	18	21	24	27
4.0	8.000	8.030	8.060	8.090	8.120	8.150+	8.181	8.211	8.241	8.272	3	6	9	12	15	18	21	24	27
.1	8.302	8.332	8.363	8.393	8.424	8.454	8.485−	8.515+	8.546	8.577	3	6	9	12	15	18	21	24	27
.2	8.607	8.638	8.669	8.700	8.731	8.762	8.793	8.824	8.855−	8.886	3	6	9	12	15	19	22	24	28
.3	8.917	8.948	8.979	9.010	9.041	9.073	9.104	9.135+	9.167	9.198	3	6	9	13	16	19	22	25	28
.4	9.230	9.261	9.293	9.324	9.356	9.387	9.419	9.451	9.482	9.514	3	6	9	13	16	19	22	25	28
4.5	9.546	9.578	9.610	9.642	9.674	9.705+	9.737	9.770	9.802	9.834	3	6	10	13	16	19	22	26	29
.6	9.866	9.898	9.930	9.963	9.995−	10.027					3	6	10	13	16	19	23	26	29

TABLE XXXVIII

Condensed Table of Trigonometric Functions

°	Radian	sin	tan	sec	vers	
0°	0.0000	0.0000	0.0000	1.000	0.0000	**90°**
1	.0175	.0175	.0175	1.000	.0002	89
2	.0349	.0349	.0349	1.001	.0006	88
3	.0524	.0523	.0524	1.001	.0014	87
4	.0698	.0698	.0699	1.002	.0024	86
5°	0.0873	0.0872	0.0875	1.004	0.0038	85°
6	.1047	.1045	.1051	1.006	.0055	84
7	.1222	.1219	.1228	1.008	.0075	83
8	.1396	.1392	.1405	1.010	.0097	82
9	.1571	.1564	.1584	1.012	.0123	81
10°	0.1745	0.1736	0.1763	1.015	0.0152	**80°**
11	.1920	.1908	.1944	1.019	.0184	79
12	.2094	.2079	.2126	1.022	.0219	78
13	.2269	.2250	.2309	1.026	.0256	77
14	.2443	.2419	.2493	1.031	.0297	76
15°	0.262	0.259	0.268	1.035	0.0341	75°
16	.279	.276	.287	1.040	.0387	74
17	.297	.292	.306	1.046	.0437	73
18°	.314	.309	.325	1.051	.0489	72
19	.332	.326	.344	1.058	.0545	71
20°	0.349	0.342	0.364	1.064	0.0603	**70°**
21	.367	.358	.384	1.071	.0664	69
22	.384	.375	.404	1.079	.0728	68
23	.401	.391	.424	1.086	.0795	67
24	.419	.407	.445	1.095	.0856	66
25°	0.436	0.423	0.466	1.103	0.0937	65°
26	.454	.438	.488	1.113	.1012	64
27	.471	.454	.510	1.122	.1090	63
28	.489	.469	.532	1.133	.1171	62
29	.506	.485	.554	1.143	.1254	61
30°	0.524	0.500	0.577	1.155	0.1340	**60°**
31	.541	.515	.601	1.167	.1428	59
32	.559	.530	.625	1.179	.1520	58
33	.576	.545	.649	1.192	.1613	57
34	.593	.559	.675	1.206	.1710	56
35°	0.611	0.574	0.700	1.221	0.1808	55°
36	.628	.588	.727	1.236	.1910	54
37	.646	.602	.754	1.252	.2014	53
38	.663	.616	.781	1.269	.2120	52
39	.681	.629	.810	1.287	.2229	51
40°	0.698	0.643	0.839	1.305	0.234	**50°**
41	.716	.656	.869	1.325	.245	49
42	.733	.669	.900	1.346	.257	48
43	.750	.682	.933	1.367	.269	47
44	.768	.695	0.966	1.390	.281	46
45°	0.785	0.707	1.000	1.414	0.293	**45°**
		cos	ctn	csc	cvrs	°

°	Radian	sin	tan	sec	vers	
45°	0.785	0.707	1.000	1.414	0.293	**45°**
46	.803	.719	1.036	1.440	.305	44
47°	.820	.731	1.072	1.466	.318	43
48	.838	.743	1.111	1.494	.331	42
49	.855	.755	1.150	1.524	.344	41
50°	0.873	0.766	1.192	1.556	0.357	**40°**
51	.890	.777	1.235	1.589	.371	39
52	.908	.788	1.280	1.624	.384	38
53	.925	.799	1.327	1.662	.398	37
54	.942	.809	1.376	1.701	.412	36
55°	0.960	0.819	1.428	1.743	0.426	35°
56	.977	.829	1.483	1.788	.441	34
57	0.995	.839	1.540	1.836	.455	33
58	1.012	.848	1.600	1.887	.470	32
59	1.030	.857	1.664	1.942	.485	31
60°	1.047	0.866	1.732	2.00	0.500	**30°**
61	1.065	.875	1.804	2.06	.515	29
62	1.082	.883	1.881	2.13	.531	28
63	1.100	.891	1.963	2.20	.546	27
64	1.117	.899	2.050	2.28	.562	26
65°	1.134	0.906	2.14	2.37	0.577	25°
66	1.152	.914	2.25	2.46	.593	24
67	1.169	.921	2.36	2.56	.609	23
68	1.187	.927	2.48	2.67	.625	22
69	1.204	.934	2.61	2.79	.642	21
70°	1.222	0.940	2.75	2.92	0.658	**20°**
71	1.239	.946	2.90	3.07	.674	19
72	1.257	.951	3.08	3.24	.691	18
73	1.274	.956	3.27	3.42	.708	17
74	1.292	.961	3.49	3.63	.724	16
75°	1.309	0.966	3.73	3.86	0.741	15°
76	1.326	.970	4.01	4.13	.758	14
77	1.344	.974	4.33	4.45	.775	13
78	1.361	.978	4.70	4.81	.792	12
79	1.379	.982	5.14	5.24	.809	11
80°	1.396	0.985	5.67*	5.76*	0.826	**10°**
81	1.414	.988	6.31*	6.39*	.844	9
82	1.431	.990	7.12*	7.19*	.861	8
83	1.449	.993	8.14*	8.21*	.878	7
84	1.466	.995	9.51*	9.57*	.895	6
85°	1.484	0.996	11.43*	11.47*	0.913	5°
86	1.501	.998	14.30*	14.34*	.930	4
87	1.518	.999	19.08*	19.11*	.948	3
88	1.536	0.999	28.64*	28.65*	.965	2
89	1.553	1.000	57.29*	57.30*	0.983	1
90°	1.571	1.000	∞	∞	1.000	**0°**
		cos	ctn	csc	cvrs	°

* Interpolation at points marked * may be inaccurate.

TABLE XXXIX

These two pages give the common logarithms of numbers between 1 and 10, correct to four places. Moving the decimal point n places to the right (or left) in the number is equivalent to adding n (or $-n$) to the logarithm. Thus, $\log 0.017453 = 0.2419 - 2$ $[= \bar{2}.2419]$.

For any Angle x,	When x is between 0° and 1°.1	When x is between 88°.9 and 90°
$\log (x \text{ in radians} =$	$\log \sin x = \bar{2}.2419 + \log (x°)$	$\log \cos x = \bar{2}.2419 + \log (90° - x°)$
$\bar{2}.2419 + \log (x°)$	$\log \tan x = \bar{2}.2419 + \log (x°)$	$\log \cot x = \bar{2}.2419 + \log (90° - x°)$
	$\log \cot x = 1.7581 - \log (x°)$	$\log \tan x = 1.7581 - \log (90° - x°)$

Log

	0	1	2	3	4	5	6	7	8	9	10	Tenths of the Tabular Difference 1	2	3	4	5
1.0	0.0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	0414					
1.1	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	0792					
1.2	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	1139					
1.3	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	1461					
1.4	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	1761					
1.5	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	2041					
1.6	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	2304					
1.7	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2553					
1.8	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2788					
1.9	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	3010					
2.0	0.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	3222	2	4	6	8	11
2.1	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	3424	2	4	6	8	10
2.2	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	3617	2	4	6	8	10
2.3	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	3802	2	4	5	7	9
2.4	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	3979	2	4	5	7	9
2.5	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	4150	2	3	5	7	9
2.6	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	4314	2	3	5	7	8
2.7	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	4472	2	3	5	6	8
2.8	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	4624	2	3	5	6	8
2.9	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	4771	1	3	4	6	7
3.0	0.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	4914	1	3	4	6	7
3.1	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	5051	1	3	4	6	7
3.2	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	5185	1	3	4	5	7
3.3	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	5315	1	3	4	5	6
3.4	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	5441	1	3	4	5	6
3.5	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	5563	1	2	4	5	6
3.6	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	5682	1	2	4	5	6
3.7	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	5798	1	2	3	5	6
3.8	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	5911	1	2	3	5	6
3.9	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	6021	1	2	3	4	6
4.0	0.6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	6128	1	2	3	4	5
4.1	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	6232	1	2	3	4	5
4.2	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	6335	1	2	3	4	5
4.3	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	6435	1	2	3	4	5
4.4	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	6532	1	2	3	4	5
4.5	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	6628	1	2	3	4	5
4.6	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	6721	1	2	3	4	5
4.7	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	6812	1	2	3	4	5
4.8	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	6902	1	2	3	4	4
4.9	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	6990	1	2	3	4	4

To avoid interpolation in the first ten lines, use the special table on pages 496 and 497.

$\log \pi = 0.4971$ $\quad \log \pi/2 = 0.1961$ $\quad \log \pi^2 = 0.9943$ $\quad \log \sqrt{\pi} = 0.2486$

$\log e = 0.4343$ $\quad \log (0.4343) = \bar{1}.6378$

TABLE XXXIX — *Continued*

	0	1	2	3	4	5	6	7	8	9	10	Tenths of the Tabular Difference 1	2	3	4	5
5.0	0.6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	7076	1	2	3	3	4
5.1	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	7160	1	2	3	3	4
5.2	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	7243	1	2	2	3	4
5.3	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	7324	1	2	2	3	4
5.4	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	7404	1	2	2	3	4
5.5	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	7482	1	2	2	3	4
5.6	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	7559	1	2	2	3	4
5.7	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	7634	1	2	2	3	4
5.8	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	7709	1	1	2	3	4
5.9	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	7782	1	1	2	3	4
6.0	0.7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7853	1	1	2	3	4
6.1	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	7924	1	1	2	3	4
6.2	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7993	1	1	2	3	3
6.3	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	8062	1	1	2	3	3
6.4	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	8129	1	1	2	3	3
6.5	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	8195	1	1	2	3	3
6.6	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	8261	1	1	2	3	3
6.7	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	8325	1	1	2	3	3
6.8	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	8388	1	1	2	3	3
6.9	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	8451	1	1	2	3	3
7.0	0.8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	8513	1	1	2	2	3
7.1	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	8573	1	1	2	2	3
7.2	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	8633	1	1	2	2	3
7.3	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	8692	1	1	2	2	3
7.4	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	8751	1	1	2	2	3
7.5	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	8808	1	1	2	2	3
7.6	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	8865	1	1	2	2	3
7.7	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	8921	1	1	2	2	3
7.8	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	8976	1	1	2	2	3
7.9	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	9031	1	1	2	2	3
8.0	0.9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	9085	1	1	2	2	3
8.1	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	9138	1	1	2	2	3
8.2	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	9191	1	1	2	2	3
8.3	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	9243	1	1	2	2	3
8.4	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	9294	1	1	2	2	3
8.5	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	9345	1	1	2	2	3
8.6	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	9395	1	1	2	2	3
8.7	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	9445	0	1	1	2	2
8.8	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	9494	0	1	1	2	2
8.9	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	9542	0	1	1	2	2
9.0	0.9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	9590	0	1	1	2	2
9.1	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	9638	0	1	1	2	2
9.2	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	9685	0	1	1	2	2
9.3	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	9731	0	1	1	2	2
9.4	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	9777	0	1	1	2	2
9.5	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	9823	0	1	1	2	2
9.6	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	9868	0	1	1	2	2
9.7	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	9912	0	1	1	2	2
9.8	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	9956	0	1	1	2	2
9.9	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996		0	1	1	2	2

Log

	0	1	2	3	4	5	6	7	8	9	10
1.00	0.0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	0043
1.01	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082	0086
1.02	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	0128
1.03	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	0170
1.04	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208	0212
1.05	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249	0253
1.06	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290	0294
1.07	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330	0334
1.08	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370	0374
1.09	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410	0414
1.10	0.0414	0418	0422	0426	0430	0434	0438	0441	0445	0449	0453
1.11	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488	0492
1.12	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527	0531
1.13	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565	0569
1.14	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603	0607
1.15	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641	0645
1.16	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678	0682
1.17	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715	0719
1.18	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752	0755
1.19	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788	0792
1.20	0.0792	0795	0799	0803	0806	0810	0813	0817	0821	0824	0828
1.21	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860	0864
1.22	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896	0899
1.23	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931	0934
1.24	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966	0969
1.25	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000	1004
1.26	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035	1038
1.27	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069	1072
1.28	1072	1075	1079	1082	1086	1089	1092	1096	1099	1103	1106
1.29	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136	1139
1.30	0.1139	1143	1146	1149	1153	1156	1159	1163	1166	1169	1173
1.31	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202	1206
1.32	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235	1239
1.33	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268	1271
1.34	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300	1303
1.35	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332	1335
1.36	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364	1367
1.37	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396	1399
1.38	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427	1430
1.39	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458	1461
1.40	0.1461	1464	1467	1471	1474	1477	1480	1483	1486	1489	1492
1.41	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520	1523
1.42	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550	1553
1.43	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581	1584
1.44	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611	1614
1.45	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641	1644
1.46	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670	1673
1.47	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700	1703
1.48	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729	1732
1.49	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758	1761

Log

	0	1	2	3	4	5	6	7	8	9	10
1.50	0.1761	1764	1767	1770	1772	1775	1778	1781	1784	1787	1790
1.51	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816	1818
1.52	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844	1847
1.53	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872	1875
1.54	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	1903
1.55	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	1931
1.56	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	1959
1.57	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984	1987
1.58	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011	2014
1.59	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	2041
1.60	0.2041	2044	2047	2049	2052	2055	2057	2060	2063	2066	2068
1.61	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092	2095
1.62	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	2122
1.63	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	2148
1.64	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	2175
1.65	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	2201
1.66	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	2227
1.67	2227	2230	2232	2235	2238	2240	2243	2245	2248	2251	2253
1.68	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276	2279
1.69	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	2304
1.70	0.2304	2307	2310	2312	2315	2317	2320	2322	2325	2327	2330
1.71	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353	2355
1.72	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378	2380
1.73	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403	2405
1.74	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428	2430
1.75	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	2455
1.76	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	2480
1.77	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	2504
1.78	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	2529
1.79	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	2553
1.80	0.2553	2555	2558	2560	2562	2565	2567	2570	2572	2574	2577
1.81	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598	2601
1.82	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622	2625
1.83	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646	2648
1.84	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	2672
1.85	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	2695
1.86	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	2718
1.87	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	2742
1.88	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	2765
1.89	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	2788
1.90	0.2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	2810
1.91	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	2833
1.92	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	2856
1.93	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	2878
1.94	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898	2900
1.95	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	2923
1.96	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	2945
1.97	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	2967
1.98	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	2989
1.99	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	3010

TABLE XL *

VALUES OF C AND f CORRESPONDING TO FANNING'S COEFFICIENT $\left(\text{WHICH WAS EQUAL TO } \frac{f}{4}\right)$, APPLICABLE ONLY TO NEW SMOOTH STRAIGHT PIPES OR CHANNELS

DIAMETERS			MEAN VELOCITY (V) IN FEET PER SECOND								
Inches	Feet		0.5	1	2	3	4	5	10	15	20
$\frac{1}{2}$	.0417	C	78	82	87	90	93	95	101	104	106
		f	.0418	.0381	.0340	.0317	.0300	.0287	.0250	.0237	.0231
$\frac{3}{4}$	.0625	C	80	84	88	91	94	96	102	105	106
		f	.0405	.0366	.0329	.0308	.0292	.0280	.0247	.0235	.0229
1	.0833	C	80	85	90	93	95	97	103	105	106
		f	.0398	.0353	.0317	.0300	.0285	.0274	.0245	.0234	.0228
$1\frac{1}{2}$	.125	C	82	87	91	94	96	98	103	106	107
		f	.0384	.0343	.0310	.0292	.0278	.0268	.0241	.0231	.0226
2	.167	C	84	88	93	95	97	99	104	106	107
		f	.0364	.0330	.0301	.0284	.0272	.0263	.0237	.0228	.0223
3	.250	C	85	90	95	97	99	101	105	107	108
		f	.0354	.0317	.0288	.0273	.0263	.0254	.0232	.0224	.0219
4	.333	C	87	92	96	99	100	102	107	108	110
		f	.0340	.0306	.0279	.0265	.0255	.0247	.0226	.0219	.0214
6	.500	C	90	94	99	101	103	104	108	110	111
		f	.0317	.0289	.0264	.0252	.0243	.0236	.0219	.0212	.0208
8	.667	C	93	97	101	103	105	106	110	111	113
		f	.0296	.0275	.0253	.0242	.0234	.0227	.0212	.0207	.0202
10	.833	C	95	99	103	105	107	108	112	113	114
		f	.0283	.0262	.0242	.0232	.0225	.0220	.0206	.0201	.0197
12	1.00	C	98	101	105	107	109	110	113	115	116
		f	.0268	.0250	.0233	.0224	.0218	.0213	.0201	.0196	.0192

14	1.17	C	100	103	107	109	110	111	115	116	117
		f	.0256	.0241	.0225	.0217	.0211	.0207	.0196	.0192	.0188
16	1.33	C	103	105	109	111	112	113	116	117	118
		f	.0244	.0232	.0218	.0210	.0205	.0201	.0192	.0188	.0184
18	1.50	C	104	107	110	112	114	115	117	119	119
		f	.0236	.0224	.0211	.0204	.0199	.0196	.0188	.0183	.0181
20	1.67	C	106	109	112	114	115	116	118	119	120
		f	.0229	.0216	.0204	.0198	.0194	.0191	.0184	.0180	.0177
24	2.00	C	110	113	116	117	118	119	121	122	123
		f	.0212	.0202	.0193	.0187	.0184	.0182	.0176	.0173	.0170
30	2.50	C	115	118	120	121	122	123	125	126	126
		f	.0194	.0186	.0179	.0175	.0173	.0171	.0166	.0163	.0161
36	3.00	C	120	123	125	125	126	126	128	129	130
		f	.0177	.0171	.0166	.0164	.0162	.0161	.0156	.0154	.0152
42	3.50	C	125	127	128	129	130	130	132	133	133
		f	.0164	.0160	.0156	.0154	.0153	.0152	.0148	.0146	.0145
48	4.00	C	130	131	132	133	133	134	135	136	137
		f	.0153	.0150	.0147	.0146	.0145	.0144	.0141	.0139	.0138
54	4 50	C	134	135	136	136	137	137	139	139	140
		f	.0144	.0142	.0140	.0138	.0137	.0137	.0134	.0133	.0132
60	5.00	C	137	138	139	140	140	140	142	143	143
		f	.0137	.0135	.0133	.0132	.0131	.0131	.0128	.0127	.0125
72	6.00	C	143	144	145	146	147	147	148	148	148
		f	.0126	.0124	.0122	.0120	.0120	.0119	.0117	.0117	.0117
84	7.00	C	148	150	151	151	152	152	153	154	154
		f	.0117	.0115	.0113	.0112	.0112	.0111	.0109	.0109	.0108
96	8.00	C	154	155	156	156	156	157	157	157	158
		f	.0109	.0107	.0106	.0106	.0105	.0105	.0104	.0104	.0103

* Tables XL and XLI are from the original edition of Hughes & Safford, *Hydraulics*, but were not included in the Revised and Abridged Edition of 1926. It has proved desirable to add them and they will now appear in the Revised and Abridged Edition following page 361.

TABLE XLI *

Values of C by Kutter's Formula

$s = .0010$

Mean Hydraulic Radius R	Coefficient of Rugosity, n													
	.009	.01	.011	.012	.013	.015	.017	.02	.025	.03	.035	.04	.045	.05
Feet														
.1	108	94	82	73	65	53	45	35	26	20	16	14	12	10
.2	130	113	100	89	80	66	56	45	34	26	21	18	16	14
.3	142	124	111	99	89	75	64	52	39	31	25	21	18	16
.4	151	132	118	106	96	80	69	56	42	34	28	24	20	18
.6	162	143	128	115	105	89	77	63	48	39	32	27	24	21
.8	170	150	135	122	112	95	82	68	52	42	35	30	26	23
1	175	156	140	127	117	99	86	71	55	45	38	33	28	25
1.5	185	166	150	136	125	108	94	78	61	50	43	37	33	29
2	191	171	155	141	130	112	98	83	65	54	46	40	35	31
3	199	180	163	149	138	119	105	89	71	59	51	44	39	35
3.28	201	181	165	151	139	121	107	91	72	60	52	45	40	36
4	205	184	168	154	143	124	110	93	75	63	54	48	42	38
6	211	190	174	160	149	130	115	99	80	68	59	52	46	42
8	215	195	178	165	153	134	119	103	84	71	62	55	50	45
10	218	198	181	167	155	137	122	105	87	74	64	58	52	47
15	222	202	186	171	160	141	127	110	91	78	68	62	56	51
20	226	205	188	174	163	144	129	113	94	80	72	64	59	54
30	229	209	192	178	167	147	133	116	97	84	75	67	62	58
40	231	211	194	180	169	150	135	119	99	87	77	70	65	60
60	233	213	197	183	171	153	138	121	102	89	81	73	67	63
80	235	215	198	184	173	154	139	122	104	91	82	75	70	65

* Computed; but checked from Trautwine's *Pocket Book*, and *The Flow of Water*, etc.

TABLE XLI — *Continued*

Values of C by Kutter's Formula

$s = .0002$

Mean Hydraulic Radius R	Coefficient of Rugosity, n													
	.009	.01	.011	.012	.013	.015	.017	.02	.025	.03	.035	.04	.045	.05
Feet														
.1	99	86	75	66	59	48	41	32	24	18	15	13	11	9
.15	112	97	85	76	68	56	47	38	28	22	18	15	13	11
.2	121	105	92	83	74	61	52	42	31	24	20	17	14	13
.3	134	117	104	93	84	70	59	48	36	29	24	20	17	15
.4	143	126	112	101	91	76	65	53	40	32	26	22	19	17
.6	156	138	123	111	101	85	73	60	46	37	31	26	23	20
.8	164	146	131	118	108	91	79	65	50	41	34	29	25	22
1	171	152	137	124	113	96	83	69	54	44	37	31	27	24
1.5	182	163	147	134	123	105	92	77	60	49	41	36	32	28
2	189	170	154	140	129	111	97	82	64	54	45	39	35	31
3	200	179	163	149	137	119	105	89	72	59	51	45	40	36
3.28	201	183	165	151	139	121	107	91	72	60	52	45	40	36
4	205	185	168	155	143	125	111	94	76	63	55	48	43	38
6	213	193	176	163	151	132	117	100	82	69	60	53	47	43
8	218	198	181	168	155	136	122	105	86	73	64	57	51	46
10	222	202	185	171	159	140	125	108	89	76	66	59	53	49
15	228	207	190	176	164	145	130	114	94	81	72	64	58	53
20	231	209	194	181	168	149	134	117	98	85	75	67	62	57
30	236	215	198	184	172	153	139	122	103	89	79	72	66	61
40	237	218	201	187	175	156	142	125	105	92	82	75	68	64
60	242	221	204	190	179	160	145	128	109	95	86	78	73	68
80	243	222	206	193	180	162	147	130	111	98	88	81	75	70

TABLE XLI — *Continued*

Values of C by Kutter's Formula

$s = .0001$

Mean Hydraulic Radius R	Coefficient of Rugosity, n													
	.009	.01	.011	.012	.013	.015	.017	.02	.025	.03	.035	.04	.045	.05
Feet														
.1	91	78	69	61	54	44	37	30	22	17	14	12	10	9
.15	103	90	79	70	63	51	43	35	26	20	17	14	12	11
.2	112	98	87	77	69	57	48	39	29	23	19	16	14	12
.3	125	110	98	88	79	65	56	45	34	27	22	19	16	14
.4	136	119	106	96	86	71	61	50	38	30	25	21	18	16
.6	150	132	118	106	96	81	70	57	44	35	29	25	22	19
.8	159	141	126	114	104	88	76	63	48	39	32	28	24	21
1	166	147	132	120	109	93	81	67	52	42	35	31	27	24
1.5	180	160	144	131	120	103	90	75	59	48	41	35	31	28
2	188	168	151	138	127	109	96	81	64	52	45	39	34	30
3	198	178	162	149	137	119	104	89	71	59	51	45	39	35
3.28	201	181	165	151	139	121	107	90	72	60	52	45	40	36
4	206	186	169	155	143	125	111	94	76	64	55	49	43	39
6	215	195	178	164	152	133	118	102	83	70	61	54	48	44
8	220	201	185	170	158	139	124	107	88	75	65	58	52	47
10	226	205	188	174	162	143	128	111	92	78	69	61	55	50
15	233	212	195	182	169	150	135	118	98	84	74	67	61	56
20	237	217	200	186	174	154	139	122	102	89	79	71	64	59
30	243	222	206	191	179	160	145	128	107	94	84	76	70	65
40	248	226	210	195	183	163	149	132	111	98	88	80	73	68
60	251	230	213	199	188	168	153	136	115	103	92	85	78	74
80	254	232	215	202	190	171	156	139	119	106	96	88	81	76

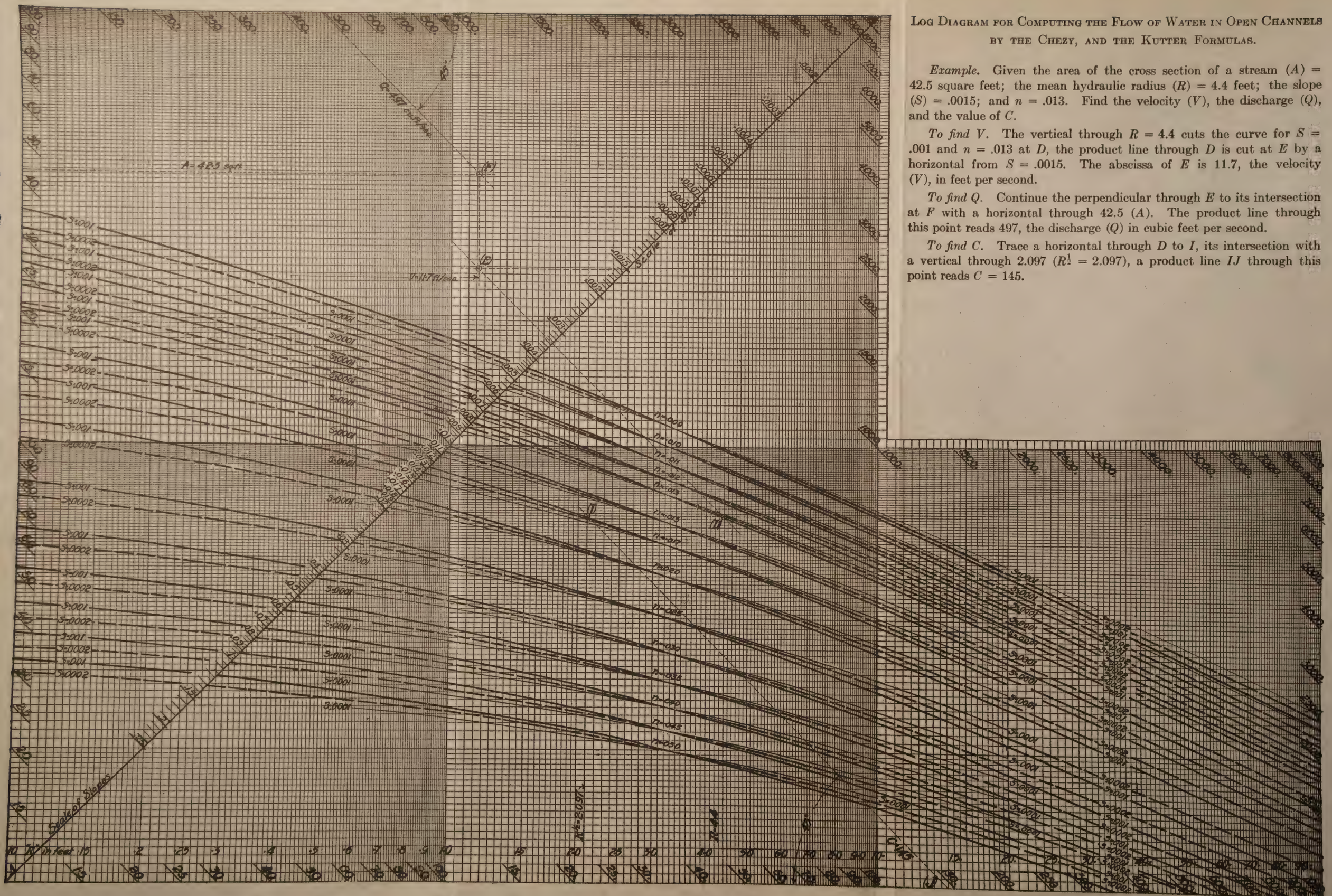
Log Diagram for Computing the Flow of Water in Open Channels by the Chezy, and the Kutter Formulas.

Example. Given the area of the cross section of a stream (A) = 42.5 square feet; the mean hydraulic radius (R) = 4.4 feet; the slope (S) = .0015; and n = .013. Find the velocity (V), the discharge (Q), and the value of C.

To find V. The vertical through $R = 4.4$ cuts the curve for S = .001 and n = .013 at D, the product line through D is cut at E by a horizontal from S = .0015. The abscissa of E is 11.7, the velocity (V), in feet per second.

To find Q. Continue the perpendicular through E to its intersection at F with a horizontal through 42.5 (A). The product line through this point reads 497, the discharge (Q) in cubic feet per second.

To find C. Trace a horizontal through D to I, its intersection with a vertical through 2.097 ($R^{\frac{1}{2}} = 2.097$), a product line IJ through this point reads C = 145.


A=425 sqft
Q=497 cu ft per sec
V=11.7 ft/sec
Slopes of Stream
Hydraulic Radius in feet
Velocity in feet per second
Discharge in cubic feet per second
Area of Cross Section in square feet
Coefficient C

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ANSWERS TO PROBLEMS IN HYDRAULICS

Solutions (except as noted) have been made by four-place logarithms. Answers (unless otherwise defined) are given in the following units: Heads, dimensions, and distances, in feet; Areas, in square feet; Volumes, in cubic feet; Discharges (volumes of flow), in cubic feet per second; Velocities, in feet per second; Intensities of pressure, in pounds per square inch; Forces, pressures, and weights, in pounds.

Coefficients used in solutions are stated, if latitude of choice exists, to indicate what seems reasonable; the selection of a coefficient is often the most difficult part of a hydraulic problem.

Chap. II. Page 12

1. 529.7 cu. ft./sec. **2.** (*a*) 6283 gal./min. (*b*) 9,049,000 gal./day. **3.** (*a*) 80.85 lbs./sq. in. (*b*) 11,640 lbs./sq. ft. (*c*) 186.6 ft. (*d*) 182.0 ft. (*e*) 164.7 in. (*f*) 56.87 meters. **4.** (*a*) 60,170 gal. (*b*) 502,100 lbs. (*c*) 227,800 liters. (*d*) 485,500 lbs. (Taking $W = 60.35$ lbs. at 190° F.) **5.** 4,141,000 lbs.

Chap. III. Page 30

1. (*a*) 191.5 lbs./sq. in. (*b*) 1,267,000. **2.** (*a*) 187,200 lbs. (*b*) 249,600 lbs. **3.** (*a*) 114.7 lbs./sq. in. at top; 119.9 at bottom. (*b*) 1,131,000 lbs. at top; 1,190,000 at bottom. **4.** 0.97 in. **5.** 0.23 in. **6.** 256,000 lbs. **7.** (*a*) 1.83 in. (*b*) 57.7 in. (*c*) 24.3 in. **8.** 0.00953 lbs./sq. in. **9.** 35.2 lbs. **10.** 26,460 lbs.

Chap. IV. Page 46

1. (*a*) 8985 lbs. (*b*) 3.11 ft. above bottom. **2.** Higher than 49.15 ft. **3.** 7323 lbs. **4.** $H = 6839$ lbs.; $V = 3891$ lbs. **5.** If vertical, $P = 4493$ lbs.; $y_p = 4$. **6.** 3.28 ft. **7.** $R = 24{,}830$ lbs.; $V = 21{,}472$ lbs.; R cuts base 3.53 ft. downstream from vertical through center of gravity; not stable. **8.** $H = 25{,}700$ lbs.; $V = 18{,}000$ lbs. **9.** 35,270 lbs.; $y_p = 20.15$ ft. **10.** (*a*) 1.35 (*b*) 20.0 ft. (*c*) same at both toes. 3647 lbs./sq. in. Upward pressure neglected. **11.** Beginning at bottom 0.14 in., 0.10 in., 0.07 in., 0.035 in.; too thin, use about 0.25 in. throughout. **12.** Spacing at bottom 1.9 in.; at top, 26 in., required 96 hoops.

Chap. V. Page 57

1. (*a*) 1123 lbs. (*b*) $4 \times 3 \times 1.5 \times 62.4 = 1123$ lbs. (*c*) 2194 lbs. **2.** .164 ft. **3.** Weight of steel 490 lbs./cu. ft. 2.13 ft.; 6.16 half full; 3.90; 16.2 half full. **4.** 3.33 ft. **5.** 4.5 ft. **6.** (*a*) $x_1 = 4.0$; $y_1 = 0.67$. (*b*) $M_1 = 9.67$ ft. (*c*) 3,709,000 ft. lbs.

Chap. VI. Page 65

1. (*a*) 16.4 M.P.H. (*b*) 21.9 M.P.H. **2.** —— (left for pupil). **3.** Range 27.56 ft., greatest height 8.21 ft. **4.** 4.52 H.P. **5.** 230,800 ft. lbs./sec.; 419.6 H.P. **6.** 466.4 lbs. **7.** 13,610 H.P.

Chap. VIII. Page 87

1. 1.25 spec. gr. — 0.818, 1.16, 2.31, 3.38; 1.50 spec. gr. — 1.16, 1.64, 3.27, 4.77 **2.** 4.96 ft./sec. **3.** (*a*) 0.908 cu. ft./sec., 1.67 ft./sec., pipe coef. 0.85 (*b*) 1.63 cu. ft./sec., 2.98 ft./sec., pipe coef. 0.86

Chap. IX. Page 94

1. (*a*) $C = 0.98$ (*b*) $C = 0.965$ **2.** 30,250 gal./min. **3.** Dia. = 6″, $Q = 14$ cu. ft./sec. **4.** $V_a = 2.30$ ft./sec., $V_b = 9.20$ ft./sec., $h_b = 20.14$ ft.

Chap. X. Page 111

1. (*a*) 0.41 cu. ft./sec. (*b*) 0.41 cu. ft./sec. **2.** (*a*) 0.41 cu. ft./sec. (*b*) 0.414 cu. ft./sec. **3.** 0.37 cu. ft./sec. **4.** 2.4 cu. ft./sec. **5.** $15\frac{3}{4}$ sec.

Chap. XI. Page 121

1. $Q = 0.262$ cu. ft./sec. **2.** $Q = 0.274$ **3.** $Q = 0.203$, pressure = 14.1 lbs./sq. in. **4.** $P_0/w = 33.9 - 0.514\, h_c$. **5.** Bell mouthed: $Q = 23.9$ cu. ft./sec. $C = 0.94$. Sharp edged: $Q = 15.2$ cu. ft./sec. $C = 0.60$ **6.** Vel. = 53.3 ft./sec.; $P_0/w = 0$; $Q = 1.67$ cu. ft./sec. $C = 0.94$

Chap. XII. Page 140

1. $C_v = C = 0.962$; $V = 67.1$ ft./sec. **2.** 438 gal./min. ($C = 0.959$ from table VII.) **3.** 417 gal./min. ($C = 0.971$). **4.** 380 gal./min. ($C = 0.971$). **5.** $p = 45$ lbs./sq. in., $Q = 247$ gal./min. ($C = 0.976$). **6.** 295 gal./min., height = 88 ft. **7.** 99.4 lbs./sq. in. **8.** 181.2 lbs./sq. in. **9.** 199 gal./min. ($C = 0.974$). **10.** 248 gal./min. ($C = 0.971$). **11.** 123.4 lbs./sq. in.; 288 gal./min.

Chap. XIII. Page 187

1. 53.73 cu. ft./sec. **2.** 49.68 cu. ft./sec. **3.** (*a*) 2.60 cu. ft./sec. (*b*) 2.59 cu. ft./sec. **4.** 45 ft. long. **5.** (*a*) 47.62 cu. ft./sec. (*b*) 0.255 ft./sec. (*c*) $h_v = 0.001$ ft. $Q = 47.57$ cu. ft./sec. by Fteley-Stearns formula.

Chap. XIII. Page 188

6. (*a*) 5.77 cu. ft./sec. (*b*) 5.86 cu. ft./sec. **7.** (*a*) 66.71 cu. ft./sec. (*b*) 66.27 cu. ft./sec. (*c*) 67.89 cu. ft./sec. (*d*) 66.47 cu. ft./sec.

8. (*a*) 36.96 cu. ft./sec. Francis. (*b*) 37.48 Francis
36.80 F. & S. 37.57 F. & S.
36.87 Bazin. 37.99 Bazin.

9. 17.25 cu. ft./sec. **10.** 0.865 cu. ft./sec. **11.** (*a*) 106.5 cu. ft./sec. (*b*) 107.8 (*c*) 109.4 ($P = 7.2$ ft.) **12.** 2.08 ft. (F. &. S, $h = 2.92$, $h^1 = 1.92$)

Chap. XIV. Page 206

1. 639.3 cu. ft./sec. **2.** (*a*) 2.52 ft./sec. by vertical velocity curve. (*b*) 2.54 ft./sec. by 0.2 and 0.8 method.

Chap. XV. Page 251

1. 0.00635 ($C = 123$). **2.** 10.4 ft. ($C = 118$). **3.** (*a*) 37.8 ft. ($C = 113$); (*b*) 75.5 ft. ($C = 80$). **4.** 103.46 ft. ($C = 122$). **5.** $Q = 16.22$ cu. ft./sec. Elev. 66.64 ($C = 124$). **6.** 1121 gal./min. at 1 mi. water stands; 22 ft. above pipe, at 2 mi., 32 ($C = 108$); 22 ft. = 9.53 lbs./sq. in., 32 ft. = 13.87 lbs./sq. in. **7.** 4 inch wrought iron pipe; about 36 times as much as is required for 60 min. **8.** Loss of head for successive 5000 ft. sections, beginning at inlet: (*a*) 0.545; 0.479; 0.360; 0.173; 0.0767; ($C = 108$). (*b*) 0.442; 0.388; 0.292; 0.140; 0.0621; ($C = 120$). **9.** (*a*) 3.03 + 100 ft. ($C = 132$). (*b*) 109.9 + 100 ft. ($C = 124$). **10.** 26 inch dia. ($C = 90$). Actually use 24 in. or 30 in. depending upon conditions. **11.** (*a*) 5.50 ft./sec. ($C = 110$). (*b*) 4.50 ft./sec. ($C = 90$).

Chap. XV. Page 252

12. 1.6 lbs./sq. in. ($C = 88$). **13.** 0.31 ft. **14.** $h_f + h_b = 0.17$ ft. for 80 diameters long. (280 ft.) $C = 130$, Table XXII. **15.** 24 inch dia. ($C = 110$). **16.** 1.42; 1.16; 6.87 ft. loss. **17.** Q into upper reservoir 12.4 cu. ft./sec., into lower 89.3 cu. ft./sec. At point of divergence water in piezometer stands 21.9 ft. above water surface in lower reservoir. $C = 145$ for 48 in. and 124 for 24 in. **18.** Water may be assumed to be supplied by seven routes each of which consists of half of 8 in. and half of 6 in. Pressure at $A = 74.1$ lbs./sq. in.

Chap. XVI. Page 276

1. (*a*) 0.293, 0.5, 0.603, 0.5 (*b*) 0.333, 0.5, 0.6, 0.5 **2.** By Bazin, $V = 2.12$, $Q = 33.8$ By Kutter, $V = 2.22$, $Q = 35.4$ **3.** By Kutter, $n = 0.13$, $Q = 2.49$, when $d = 0.95\ D$, $Q = 2.31$, when $d = 0.81\ D$ **4.** By Kutter, $n = 0.013$ $Q = 7.51$ **5.** $D = 8$ ft. **6.** (*a*) 0.00033 (*b*) 2.41 cu. ft./sec. **7.** By Kutter, $n = 0.02$, $Q = 825$ cu. ft./sec. **8.** By Kutter, $n = 0.02$, $Q = 3051$ cu. ft./sec.

Chap. XVI. Page 276

9. The best section is that in which R is a maximum; this may be found by making

$$\frac{\delta A}{\delta(\text{w.p.})} = 0.$$

	Slope of sides	*Depth*	*Bottom width*	*Value of C*
(*a*)	$1\frac{1}{2} : 1$	11.8 ft.	7.14 ft.	100
(*b*)	2 : 1	12.1	5.7	80
(*c*)	Vertical	11.25	22.5	118
(*d*)	Vertical	10.25	21.0	140

Section *b* would probably scour. This would depend upon exactly what sort of sand it was. A 2 : 1 slope is too steep for most sands, moreover.

Chap. XVII. Page 309

1. Tension in each string 242.6 lbs. **2.** 914.3 lbs.; 527.8 lbs. **3.** (*a*) 186.3 lbs. (*b*) 82.8 lbs. (*c*) 1324 ft. lbs./sec. **4.** (*a*) 51.1 ft./sec. (*b*) 48.8 r.p.m. **5.** (*a*) 36.4 lbs. (*b*) 23.2 lbs. **6.** (*a*) 1230 lbs. (*b*) 192.3 H.P. **7.** 703.9 H.P. **8.** (*a*) This is necessarily solved by trial and error. E for steel = 28,000,000; joint efficiency 80%. Note errata for correction in formula. h = 789 ft. = 341 lbs./sq. in. $t = 1\frac{3}{8}$ in. (*b*) Same as *a*. (*c*) 145.1 ft. for 1000 ft. long. 1451.0 ft. for 10,000 ft. long.

Chap. XVIII. Page 329

48″ diameter (running at 150 R.P.M. or for relative velocity = 0.783). 1426 H.P. maximum if run at 171 R.P.M. or for relative velocity = 0.894 At this speed discharge = 608 cu. ft./sec. Or 1404 H.P. maximum if run at 150 R.P.M. for relative velocity = 0.783 150 R.P.M. 964 KW 77%. 538.8 KW 69.9% 1.88¢ per KWH.